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OPTIMIZATION OF PREMATURED TREASURY BONDS REDEMPTION ASSOCIATED WITH SELLING OF NEW BONDS

Abstract

The paper presents an approach to optimization of prematured bond redemption connected with an issue of new bonds. Both transactions are assumed in the auction form. The decision variables are: redemption level and new issue level, the criterion function expresses total servicing cost of the debt. The constraints of the problem comprise budgetary requirements and other features of the debt issued. The approach has been applied with the use of actual data. The paper is based on the research made in Ministry of Finance (Poland).

Keywords

Optimization of debt management, multiple criteria: bond redemption and bond issue, minimization of debt servicing costs

Introduction

Redemption of prematured bonds is a tool for debt management, which allows replacement of old bonds (issued in the past) having some non-desirable features by new ones with appropriate properties. Such replacement can be done with the use of two simultaneous auctions; the first auction determines the value and structure of redeemed bonds, while the second – the value and structure of bonds providing financial means for redemption purpose. The result of both auctions can be optimized; the criterion function comprises two factors, so the problem has multiple criteria form. The purpose of the paper is presentation of the problem and an example of its optimal solution. The optimization approach for debt management problems in Poland is presented in Klukowski 2003, 2010 (see also Cleassens et al 1995).

The components of optimization task under consideration are: • decision variables expressing sale of new bonds and redemption of prematured bonds, • criterion function expressing combined servicing costs of both bonds and • constraints expressing budgetary requirements and other features of debt issued, e.g. maturity period. The criterion function has typically non-linear form; the optimal solution cannot be determined without optimization methods.

In Poland auctions aimed at replacement of treasury securities are performed in simpler form, i.e. redemption - with constant (arbitrary) price, while the sale is usual multi-price auction. It is suggested to replace the constant (redemption) price by multi-price auction. The system of two simultaneous multi-price auctions allows obtaining the optimal solution (portfolio), which minimizes the total servicing costs of redeemed and sold bonds. It is so, because the feasible set of combined auctions is broadened, in relation to current auction. Moreover, an average price of redeemed bonds can be lower, than those determined in arbitrary way. Optimal solution can also assume a zero-form, i.e. nothing to redeem, nothing to sell.

The example of the approach proposed is based on data from two actual auctions: sell and redeem, performed in two consecutive days (not simultaneously). The financial market was "stable" in these days;

therefore the optimal solution has features of actual solution of the problem.

The paper consists of four sections. The main results – formulation of the problem and empirical results are presented in Sections 1 – 2. Last Section summarizes the results.

1. Problem formulation

The optimization problem under consideration can be formulated as follows. To optimize the result of two simultaneous multi-price auctions for treasury bonds: prematured redemption and issuance of new bonds under the following assumptions:

- criterion function expresses servicing costs of redeemed and new bonds,
- the new bonds have to provide means for redemption purpose.

The constraints of the problem comprise some features of the debt structure after the auctions. The optimization problem, i.e. criterion function (minimized) and constraints (inequalities) can be formulated in the following form.

The criterion function - minimized:

$$\min_{x,w}[\sum_{i\in I_t} x_i(M-d^{(t)}(x_i))\varphi_i(x_i)+$$

$$+\sum_{j\in I_{w,t+t}}\sum_{\tau\in\Lambda_t}(W_{j,t+t+\tau}-w_{j,t+t+\tau})P_{j,t+t+\tau}(W_{j,t+t+\tau}-w_{j,t+t+\tau})\times \tag{1}$$

$$\times\phi_{j,t+t+\tau}(W_{j,t+t+\tau}-w_{j,t+t+\tau})],$$

where:

$$\phi_{j,t+t+\tau}(W_{j,t+t+\tau}-w_{j,t+t+\tau})=$$

$$=\left(\sum_{\omega=0}^{cm(t+\tau)}R_{i,j+\omega}^{(w)}\prod_{k=\omega+1}^{H}\left(1+r_{j,t+k}^{(w)}\right)+M\prod_{k=cm(t+\tau+1)}^{H}\left(1+r_{j,t+k}^{(w)}\right)\right)^{1/H}\times$$

$$\times(1/P_{j,t+\iota+\tau}(W_{j,t+\iota+\tau}-w_{j,t+\iota+\tau}))^{1/H}-1$$

and:

ent(μ) – integer part of number μ;

t – current year;

ι - time from the start of a (current) year t to a current moment (ι<1); $t+\iota+\tau$ $(\tau>0)$ a time moment in future;

x_i ($i\in I_x$) – a decision variable - number of nominals of i-th bond (sale auction); I_x - the set of bonds issued;

x – vector of decision variables x_i, $i\in I_x$;

$d^{(i)}(x_i)$ - average discount of one nominal of i-th bond, corresponding to x_i nominals;

$\varphi_i(x_i)$ - componed rate of return (profitability) of i-th bond, corresponding to x_i nominals, for optimization horizon H;

$I_{w,t+\iota}$ - the set of bonds redempted;

$w_{j,t+\iota+\tau}$ $(j\in I_{w,t+\iota+\tau})$ – decision variable - number of nominals of j-th bond, with (original) redemption time $t+\iota+\tau$; $\tau\in\Lambda_j$, Λ_j - a set of redemption times of j-th bond;

w – vector of decision variables $(w_{j,t+\iota+\tau};\ j\in I_{w,t+\iota+\tau})$;

$P_{j,t+\iota+\tau}(W_{j,t+\iota+\tau}-w_{j,t+\iota+\tau})$ – average price of non-redeemed bonds ($W_{j,t+\iota+\tau}-w_{j,t+\iota+\tau}$ nominals) of j-th type ($\tau\in\Lambda_j$), under assumption of increasing bids on auction;

$P^{(w)}_{j,t+\iota+\tau}(w_{j+\iota+\tau})$ - average price of redeemed bonds ($w_{j+\iota+\tau}$ nominals) of j-th type ($\tau\in\Lambda_j$), under assumption of increasing bids on auction;

$\Phi_{j,t+\iota+\tau}(W_{j,t+\iota+\tau}-w_{j,t+\iota+\tau})$ – compound rate of return (average) of non-redeemed bonds of j-th type refinanced in optimization horizon ($\tau\in\Lambda_j$);

$R^{(w)}_{j,t+\iota+\tau}$ - amount of interests of j-th redeemed bond with maturity time $t+\iota+\tau$ ($\tau\in\Lambda_j$) paid in a year $t+\omega$ ($\omega\geq 0$), for actual redemption time;

H – optimization horizon (in years);

$r^{(w)}_{j,t+\iota+\tau}$ - (market) interest rates of j-th bond ($\tau\in\Lambda_j$), with maturity time $t+\iota+\tau$, in the year $t+\omega$ ($\omega\geq 0$) in the case of actual redemption time;

$\Delta_{j,t+\iota+\tau}$ - discount of j-th redeemed bond with maturity time $t+\iota+\tau$ ($\tau\in\Lambda_j$), in the case of actual redemption time;

$R_{i,t+\omega}$ ($\omega\geq 1$) - amount of interests of i-th obond ($i\in I_x$), sold for redemption purpose, paid in the year $t+\omega$;

M – nominal value of the bond.

The constraints - specific for the problem under consideration - can be formulated as follows:

a) the constraint for the capital value necessary for redemption purpose:

$$\sum_{i\in I_x} x_i(M-d^{(i)}(x_i)) \geq \sum_{j\in I_{w,t+\iota}} \sum_{\tau\in\Lambda_j} w_{j,t+\iota+\tau} P^{(w)}_{j,t+\iota+\tau}(w_{j,t+\iota+\tau}); \tag{2}$$

b) the constraint for change of nominal value of debt – as a result of the auctions:

$$\vartheta_{\min} \leq M(\sum_{i\in I_x} x_i - \sum_{j\in I_{w,t+\iota}} \sum_{\tau\in\Lambda_j} w_{j,t+\iota+\tau}) \leq \vartheta_{\max}; \tag{3}$$

c) the constraints for changes of servicing costs of the debt in future years – as a result of the auctions:

$\omega=0$ and $\iota+\tau<1$:

$$\begin{aligned}&\sum_{j\in I_{w,t+\iota}} \sum_{\iota+\tau<1} w_{j,t+\iota+\tau}(M - P^{(w)}_{j,t+\iota+\tau}(w_{j,t+\iota+\tau})) -\\ &- \sum_{j\in I_{w,t+\iota}} \sum_{\iota+\tau<1} w_{j,t+\iota+\tau} R^{(w)}_{j,t+\iota+\tau} \leq b_t;\end{aligned} \tag{4}$$

$\omega=1$ oraz $1<\iota+\tau\leq 2$:

$$\begin{aligned}&\sum_{i\in I_x} x_i R_{i,t+1} - \sum_{j\in I_{w,t+\iota+\tau}} \sum_{1\leq\iota+\tau<2} w_{j,t+\iota+\tau} R^{(w)}_{j,t+\iota+\tau} -\\ &- \sum_{j\in I_{w,t+\iota}} \sum_{1<\iota+\tau\leq 2} w_{j,t+\iota+\tau} \Delta_{j,t+\iota+\tau} \leq b_{t+1};\end{aligned} \tag{5}$$

(the sum $\sum_{j \in I_{w,t+t}} \sum_{1 \le t+\tau < 2} w_{j,t+t+\tau} R^{(w)}_{j,t+t+\tau}$ comprises components satisfying conjunction of conditions, i.e. index $j \in I_{w,t+t}$ and time moments $t+\tau \in [1, 2)$; constraints of costs for the next years are similar).

Other constraints can be formulated similarly as optimization tasks for servicing costs, e.g.:

d) the constrain for maturity period of new debt:

$$\mu_{\min} \le \sum_{i \in I_x} \nu_i x_i / \sum_{i \in I_x} x_i \le \mu_{\max}, \tag{6}$$

where:

ν_i - maturity of i–th bond,

$\mu_{\min}, \mu_{\max}$ - minimal and maximal maturity period of issued bonds.

The criterion function comprises two types of variables: • redeemed bonds $w_{j,t+t+\tau}$ and • bonds x_i providing capital for redemption. The optimal solution of the task is the mixture of old and new bonds; of course, some of the decision variables can be equal zero. The solution determines the debt structure with minimal servicing costs and other - necessary features of the debt.

The solution of the problem under consideration cannot be determined without optimization methods. Thus, the approach proposed in the paper allows achievement of actual basis for accurate decisions.

2. Example of application

An example of application of the task (1) – (6) is based on two actual auctions: sale and prematured redemption. Two bonds were sold: two-years x_1 and five-years x_2 and two redeemed: two-years w_1 and three-years w_2 (assimilative to five years). The auctions were performed in two consecutive days – in stable situation on financial market. The components of criterion function (1) corresponding to the variables: x_1 and x_2, are not simple and has been approximated with polynomials (see Klukowski 2010). The bids corresponding to sale auction (variables x_i $(i = 1, 2)$) are ordered in accordance with increasing profitability, the bids corresponding to redemption auction (variables $W_i - w_i$ $(i = 1, ..., 2)$) – in accordance with increasing price. The approximations, corresponding to the variables x_i $(i = 1, 2)$ are convex increasing functions, the approximations corresponding to the variables $W_i - w_i$ $(i = 1, 2)$ are convex decreasing functions.

Numerical form of the task has been obtained on the basis actual data – forecasts of interest rates and auctions results. Two optimal solutions are presented below. The first one has been obtained for the following constraints:

$0 \le x_1 \le 1500000$,

$0 \le x_2 \le 3000000$,

$0 \le W_1 - w_1 \le 342197$,

$0 \le W_2 - w_2 \le 134734$.

The optimal solution of the task assumes the form $x_1^* = 0$, $x_2^* = 230\,918$, $w_1^* = 212\,945$, $w_2^* = 34\,568$; the value of criterion function equals 34 637 898 zł, the value of capital constraint (left-hand side of the inequality (2)) equals: 232 687 111 zł.

The second solution has been obtained for the constraints:

$0 \le W_1 - w_1 \le 130000$,

$0 \le W_2 - w_2 \le 110000$.

The solution assumes the form: $x_1^* = 115\ 501$, $x_2^* = 125\ 999$, $w_1^* = 212\ 197$, $w_2^* = 24\ 734$; the value of the criterion function - equal 34 871 551 zł.

Both optimal solutions have decision values w_i^* $(i = 1, 2)$ inside the constraints; they are non-trivial. The optimal solutions are sensitive on values of constraints, especially the constraints of the second task provide non-zero value of the variable x_1^*. Such feature cannot be detected without optimization methods.

3. Summary

The paper presents nonlinear, multiple criteria approach to optimization of prematured treasury bonds redemption associated with selling of new bonds. It is an element of optimization methodology of debt management developed by the author (Klukowski 2003). The approach exceeds current practice – it provides budgetary savings, decreases management costs, generates debt structure with optimal properties. Moreover, its complexity and computation cost is entirely acceptable in practice.

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