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Abstract

The paper deals with the problem of fuzzy optimisation in presence of flexible constraints that describe user preferences. Fuzzy optimisation procedure is used to find a certain reference value. Then, the set of admissible solutions that are in a certain sense indistinguishable from the best attainable reference value is calculated using the known from the theory of possibility concept of the Necessity of Strict Dominance index. Admissible solutions are evaluated and ranked using methods developed for solving the flexible constraint satisfaction problems. The proposed algorithm is applied for solving a practical problem of choosing a user-preferred inspection interval. This problem is often encountered in statistical quality control and reliability testing.

Keywords: Optimal inspection, Fuzzy optimality, Flexible constraint satisfaction problem, User preferences.

1 Introduction

Optimisation problems are frequently encountered in many practical situations. Specialists in optimisation problems have developed powerfull tools that are able to solve large and complicated problems. However, all these tools are surpisingly seldom used in a real practice. One of the main reasons of this situation is an inadequate description of real problems. Suppose, for example, that the problem under cosideration is stated as follows

$$\max_{x} f(x; \mathbf{w}) \tag{1}$$

where $x \in X$, $f : R \longrightarrow R$ is the optimised, with respect to some x, objective function, and $\mathbf{w} = (w_1, w_2, ..., w_m)$ is a vector of parameters. The solution of this unconstrained optimisation problem is not difficult, even in the case of non-smooth objective functions. When an additional set of constraints $C : g_i(x; \mathbf{w}_c) \geq 0,\ i = 1, ..., m_c$, where $\mathbf{w}_c$ is a set of parameters, is given, the optimisation problem may be more complicated, but its solution is still not very difficult. However, even in the case of well defined objective and constraints functions, the real practical problem consists very often in finding appropriate values of model's parameters, both for the objective function and the constraints. In addition, very often the user is not able to describe precisely the constraints for his/hers optimisation problem. It happens, when the constraints are formulated rather as vaguely defined user's preferences than rigorously described mathematical relations. All these problems result with difficulties in applying the results of classical optimisation procedures in a real practice.

In the next section we present a typical example of the problem described above - the optimal choice of an inspection interval. The choice of the inspection interval seems to be one of the most important problems in quality control and reliability testing. In the third section we describe critically some methods that might be used for finding the optimal inspection interval. We assume, that the objective function becomes a fuzzy objective function, and the satisfaction of user's preferences can be described as a simple flexible constraint satisfaction problem. The fourth section of the paper is devoted to a newly proposed method that might be used for finding a user-preferred solution. We propose to use the possibility theory in order to find a set of satisfactory solutions, and then we use the flexible constraint satisfaction methodology to find the user-preferred solution. A numerical example that illustrates the usage of the described optimisation methods is given in the fifth section of the paper. Finally, we discuss the results and propose further extensions and generalisations.

2 Optimal choice of inspection intervals

Finding of an appropriate inspection interval is one of the main problems of contemporary quality control and reliability testing. When the current state of a system is not continuously monitored it can be revealed only after performing some test procedures. Usually these test procedures are repeated cyclically, and the time between two consecutive tests is called an inspection interval. When failures of the monitored system appear randomly, and the monitoring procedure is not perfect (there is a positive probability that a failure of the system may not be detected during the inspection) there is a need to perform tests rather frequently in order to reveal the failure as quickly as possible. On the other hand, however, frequent tests are costly. Moreover, when the test are not perfect there exists also a possibility of many false alarms (when the inspection fasely reveals the failure of the system). Therefore, it is possible to formulate an optimisation problem with the aim to find a cost optimal inspection interval.

There are many papers devoted to the problem of the optimal choice of the inspection interval. In general, mathematical models that are used for the optimisation of the inspection interval may be very complicated, and may involve many input parameters of a different character (probabilities, costs, parameters of probabilities distributions, etc.). Thus, in a general case the optimisation procedure is not simple. However, in order to simplify it we may use the asymptotic result of Hryniewicz [10] who showed that the original optimisation procedure is equivalent to the minimisation of a simple objective function

$$G(h) = \frac{b \cdot h \cdot (A - 0.5)}{\tau} + (\delta + S)\frac{\tau}{h} \qquad (2)$$

where

h - optimised inspection interval,

b - standardised (related to a cost of a false alarm) average profit from one renewal of the system,

A - expected number of inspections while the system is failed,

τ - expected time to the system failure,

δ - probability of a false alarm,

S - standardised (related to a cost of a false alarm) expected cost of one inspection.

Parameters of the objective function (2) may be evaluated directly or calculated as functions of other parameters. More detailed description of the problem when applied in the statistical quality control may be found in Hryniewicz[9].

When the input parameters of $G(h)$ are known it is possible to calculate the optimal value of the inspection interval h. Simple calcutations show that it is equal to

$$h^{*} = \tau\sqrt{\frac{\delta + S}{b\,(A - 0.5)}} \qquad (3)$$

It is easy to show that if we insert (3) into (2) the minimal value of the objective function for any set of input parameters is equal to

$$G_{\min} = 2\sqrt{b\,(A - 0,5)\,(\delta + S)} \qquad (4)$$

Thus, the optimal inspection interval is very easy to calculate. Moreover, the expresion (4) shows how the values of input parameters influence the optimal cost. However, this apparently very simple result is hardly used in practice. One of the reasons for that is the lack of reliable input information. In practically all cases the input parameters (especially costs) of all optimisation models are very difficult to evaluate. This is also a case in the problem of the optimal choice of the inspection interval. Therefore, even in such a simple case, the optimal solutions are very seldom used in practice.

The objective function in the considered optimisation problem (2) has clear economic interpretation. However, there exist also other requirements which interpretation in terms of costs is difficult or even impossible. First of all, some values of the inspection interval are more preferred from an organisational point of view. For example, the inspection interval equal to one hour is much more preferred than the inspection interval equal to, say, 47 minutes. In certain circumstances, the inspection procedure is possible only during production breaks (in the night, for example), etc. In some cases the requirements of such a type can be formulated as classical constraints. However, in many cases they are presented in terms of preferreces, where some values are more preferred than some other values.

The preferences described above are formulated with respect to the optimised variable - the inspection interval. There exist, however, other preferences that are related to some functions of the optimised variable. In the cosidered case there exists also at least one such requirement. This requirement is related to the expected number of false alarms during a given time period. It has been observed in practice, that false alarms have a negative impact on managers. When they are too frequent, the managers begin to doubt in the efficiency of the inspection procedures. It is easy to show that the expected number of false alarms during a certain time period T is given by

$$A_f = \delta \frac{T}{h} \tag{5}$$

As the impact of false alarms on managers is rather a vague concept it is not clear how to describe it in terms of a classical constrained optimisation problem.

In general, in practical opitimisation problems sometimes is difficult to define one objective for the optimisation procedure. When it is possible to define other objectives the multi-objective programming may be applied. In other approaches we try to represent the values of different objective functions on the same scale in order to build one goal function which comprises different objectives. In such a case for each considered objective we need to construct a utility function that assigns utility values from a given interval to all possible values of the objective function. In the considered case of choosing an optimal value for the inspection interval such approaches are not natural, as the values of the objective function, and the preferences related to the values of the optimised variable cannot be measured on the same scale.

3 Optimal choice of the inspection interval - a fuzzy optimisation problem

In the previous section we noticed that the main reason of apparent difficulties with the practical implementation of the optimal inspection intervals is a lack of knowledge about the values of the optimisation model parameters. These values are seldom known to users, and there exist, at least, two reasons for such a situation. First, they may randomly vary in time. In such a case, in the optimisation procedure we may use their average values, as it is frequently used in practice. However, there exists also a second reason that seems to be much more important. The input parameters (or their average values) are very often imprecisely defined. Such lack of information has not, in our opinion, a stochastic nature, and has to be described in another way.

To describe imprecise information about the model parameters in (2) we propose to use the notion of fuzzy numbers. The fuzzy equivalents of the input parameters b, A, S, and τ we describe using the fuzzy sets $\widetilde{b}$, $\widetilde{A}$, $\widetilde{S}$, and $\widetilde{\tau}$ with the following α-cuts:

$$\left(b_{\min}^{\alpha}, b_{\max}^{\alpha}\right) = \left\{b \in R^{+}, \mu_b\left(b\right) \geq \alpha\right\} \tag{6}$$

$$\left(A_{\min}^{\alpha}, A_{\max}^{\alpha}\right) = \left\{A \in R^{+}, \mu_A\left(A\right) \geq \alpha\right\} \tag{7}$$

$$\left(S_{\min}^{\alpha}, S_{\max}^{\alpha}\right) = \left\{S \in R^{+}, \mu_S\left(S\right) \geq \alpha\right\} \tag{8}$$

$$\left(\tau_{\min}^{\alpha}, \tau_{\max}^{\alpha}\right) = \left\{\tau \in R^{+}, \mu_\tau\left(\tau\right) \geq \alpha\right\} \tag{9}$$

where $\mu_b\left(b\right), \mu_A\left(A\right), \mu_S\left(S\right), \mu_\tau\left(\tau\right)$ are the membership functions of $\widetilde{b}$, $\widetilde{A}$, $\widetilde{S}$, and $\widetilde{\tau}$, respectively. The probability of a false alarm δ is usually much easier to evaluate, so we will assume that its crisp value is known. However, this parameter may be also fuzzified in case of a lack of reliable information about its value.

The initial optimisation problem (2) becomes now a fuzzy optimisation problem with the fuzzy objective function

$$\widetilde{G}(h) = \frac{\widetilde{b} \cdot h \cdot (\widetilde{A} - 0.5)}{\widetilde{\tau}} + (\delta + \widetilde{S})\frac{\widetilde{\tau}}{h} \qquad (10)$$

In the simplest, and naive, optimisation procedure we can use the extension principle for the representation of the objective function (10) in a fuzzy form, and then to defuzzify it using, for example, the method proposed by Fortemps and Roubens [8] or any other defuzzification method. This procedure, however, is not well justified, and usually does not provide us with the optimal inspection interval which is preferred by a user. Instead, we propose to apply the result described in the paper of Canestrelli and Giove [2] that may be directly applied in the considered case. According to this result, the fuzzy-optimal solution to the unconstrained optimisation problem may be found by the application of Zadeh's extension principle to the non-fuzzy solution of the equivalent crisp optimisation problem. In the considered case the fuzzy-optimal solution can be obtained by applying the extension principle to (3). In our case we assume that fuzzy parameters $\widetilde{b}$, $\widetilde{A}$, $\widetilde{S}$, and $\widetilde{\tau}$ are not interactive. This assumption is not exactly true for $\widetilde{b}$ and $\widetilde{\tau}$. There exist mathematical models that describe the link between these parameters in a crisp case. Using these models we may evaluate the connection between $\widetilde{b}$ and $\widetilde{\tau}$. We decided, however, to assume that they are not interactive in order to keep the model as simple as possible. Some numerical experiments have shown, that in many cases this approximation is acceptable in practice. The fuzzy-optimal solution to our optimisation problem is given as a fuzzy number $\widetilde{h}^*$ defined by the set of α-cuts $({h^*}^{\alpha}_{min}, {h^*}^{\alpha}_{max})$ with

$${h^*}^{\alpha}_{min} = \tau^{\alpha}_{min}\sqrt{\frac{\delta + S^{\alpha}_{min}}{b^{\alpha}_{max}\,(A^{\alpha}_{max} - 0.5)}} \qquad (11)$$

and

$${h^*}^{\alpha}_{max} = \tau^{\alpha}_{max}\sqrt{\frac{\delta + S^{\alpha}_{max}}{b^{\alpha}_{min}\,(A^{\alpha}_{min} - 0.5)}} \qquad (12)$$

A crisp optimal inspection interval may be found by applying one of many defuzzication methods to $\widetilde{h}^*$. However, this value will probably be hardly preferred by the user.

If we want to take into account user's preferences we must introduce some constraints. As we have already noticed, usually such constraints cannot be precisely defined. To cope with this situation we may use the concept of flexible constraints. The idea of flexible constraints has its origins in the paper of Bellman and Zadeh [1] on decision making in a fuzzy environment. Since this seminal paper many results on the flexible constraints satisfaction problem have been published. For recent references the reader is advised to consult, for example, the paper of Dubois and Fortemps [6].

In the problem of the optimal choice of the inspection interval the fuzzy constraints represent user's preferences. They may be also interpreted as fuzzy restrictions in a sense of Zadeh (see [7] for discussion). Thus, a fuzzy constraint may be viewed as the association of a classical constraint (a set of admissible values of the decision variable) and a preference-type criterion which rank-orders solutions to the original optimisation problem. To describe user preference requirements for the inspection interval we may use any function $\nu : R^+ \rightarrow [0, 1]$. As we have noticed in the previous section, preferences may be expressed in two different ways. First, a user may directly evaluate his/hers preference for specific values (set of values) of the inspection interval. Second, a user may evaluate his/hers preferences for the values of quantities that are funtions of the decision variable. In this second case, the preferences for different values of the decision variable are given indirectly. In both cases, however, the final result is similar, the user defines one or more preference functions $\nu(h)$ that can be interpreted as the membership functions of fuzzy restrictions representing flexible fuzzy constraints. The most preferred solution is equivalent now to the solution of the flexibe constraint satisfaction problem (FCSP).

Several approaches to solve the constraint satisfaction problem have been proposed. The most effective are described, for example, in the paper by Dubois, Fargier, Fortemps, and Prade [3]. According to the simplest and most popular maxmin approach to combine different preference functions $\nu(h)$ $i = 1, ..., k$ we use a fuzzy decision set D defined by the following membership function

$$\mu_D(h) = \min_{i=1,\ldots,k} \nu(h) \tag{13}$$

It is worthy to note that this combination rule may be applied when all considered preferences $v(h)$ are measured on the same scale. Moreover, by assuming this type of aggregation we assume that the global level of satisfaction of a set of fuzzy constraints is the level of satisfaction of the least satisfied one(s). The most preferred solution is now defined as that which maximises $\mu_D(h)$.

As for now, we have shown that there exist sound mathematical methods to cope with the problem of finding the fuzzy optimal solution to the original optimisation problem and with the problem of finding the solution which is preferred by the user, separately. Unfortunately, we cannot aggregate fuzzy decision set D represented by (13) and the fuzzy-optimal solution $\widetilde{h^*}$ defined by α-cuts given by (11) and (12) as these values are not commensurable. A possible solution may be the following. We choose a certain α-cut of $\widetilde{h^*}$ as a set of admissible values of h. Then, we choose such h that maximises $\mu_D(h)$ over the set of such defined admissible values of h. This approach has, however, some disadvantages. First, the choice of α is arbitrary, and cannot be easily related to user's preferences. Second, we may unnecessarily restrict ourselves only to a relatively short interval of admissible values of h. To overcome these problems we propose in the next section another method for finding user-preferred optimal inspection intervals.

4 User-preferred optimal inspection intervals

In the second section of this paper we introduced the minimal value of the objective function (4). When the input parameters are fuzzy the objective function is also fuzzy, and its fuzzy minimal value can be obtained by fuzzification of (4). For further analysis we will define the following fuzzy *reference value* $\widetilde{G}_{ref}$ using the following α-cuts:

$$\Big(-2\sqrt{b^{\alpha}_{\max}\left(A^{\alpha}_{\max} - 0,5\right)\left(\delta + S^{\alpha}_{\max}\right)},\\ -2\sqrt{b^{\alpha}_{\min}\left(A^{\alpha}_{\min} - 0,5\right)\left(\delta + S^{\alpha}_{\min}\right)}\Big)$$

Now, denote by $\widetilde{G}_x$ the fuzzy equivalent of the function $-G(x)$, where $G(x)$ is given by (2) with $x = h/\tau$. We will look now for such *admissible* values of x, for which the value of $\widetilde{G}_x$ is not significantly worse than the reference value $\widetilde{G}_{ref}$. To quantify the difference between these two fuzzy sets we propose to use the Necessity of Strict Dominance index (NSD) defined by Dubois and Prade [7] as follows

$$\begin{aligned} NSD &= Ness\left(\widetilde{A} > \widetilde{B}\right) \\ &= 1 - \sup_{x,y;x\le y} \min\left\{\mu_A(x), \mu_B(y)\right\} \end{aligned} \tag{14}$$

where $\mu_A(x)$, and $\mu_B(y)$ are the membership functions of the fuzzy sets $\widetilde{A}$ and $\widetilde{B}$, respectively. In the considered case, we define a γ-*NSD-admissible* set of values of x, as the set of all values of x for which the following inequality holds:

$$NSD\left(\widetilde{G}_{ref} > \widetilde{G}_x\right) \le \gamma \tag{15}$$

For a fixed value of x the requirement (15) is equivalent to the following inequality

$$\begin{aligned} &-\left[b^{1-\gamma}_{\min}\left(A^{1-\gamma}_{\min} - 0,5\right)x + \left(\delta + S^{1-\gamma}_{\min}\right)\frac{1}{x}\right] \ge \\ &\ge -2\sqrt{b^{1-\gamma}_{\max}\left(A^{1-\gamma}_{\max} - 0,5\right)\left(\delta + S^{1-\gamma}_{\max}\right)} \end{aligned} \tag{16}$$

Solving this inequality with respect to x we obtain the γ-*NSD-admissible* set $(x_{\min}, x_{\max})$, where

$$x_{\min} = \frac{2\sqrt{b^{1-\gamma}_{\max}\left(A^{1-\gamma}_{\max} - 0,5\right)\left(\delta + S^{1-\gamma}_{\max}\right)} - \sqrt{\Delta}}{2b^{1-\gamma}_{\min}\left(A^{1-\gamma}_{\min} - 0,5\right)} \tag{17}$$

$$x_{\max} = \frac{2\sqrt{b^{1-\gamma}_{\max}\left(A^{1-\gamma}_{\max} - 0,5\right)\left(\delta + S^{1-\gamma}_{\max}\right)} + \sqrt{\Delta}}{2b^{1-\gamma}_{\min}\left(A^{1-\gamma}_{\min} - 0,5\right)} \tag{18}$$

and

$$\begin{aligned} \Delta = &\ 4b^{1-\gamma}_{\max}\left(A^{1-\gamma}_{\max} - 0,5\right)\left(\delta + S^{1-\gamma}_{\max}\right) \\ &-4b^{1-\gamma}_{\min}\left(A^{1-\gamma}_{\min} - 0,5\right)\left(\delta + S^{1-\gamma}_{\min}\right) \end{aligned} \tag{19}$$

Having the γ-***NSD-admissible*** set of values of x we may define the γ-***NSD-admissible*** set of values of h as the interval $\left(x_{\min}\tau_{\min}^{1-\gamma}, x_{\max}\tau_{\max}^{1-\gamma}\right)$. The values of the inspection interval h that belong to the set of γ-***NSD-admissible*** values will be further analysed with respect to other user preference measures. It has to be strongly stressed, however, that the set of γ-***NSD-admissible*** values of h is not the same as the $(1-\gamma)$-cut set of the fuzzy set obtained by the fuzzification (by the application of the extension principle) of h^* given by (3). The set of γ-***NSD-admissible*** values of h is definitely larger as it contains not only optimal values of h, but also those values of the inspection interval for which the values of the objective function cannot be distinguished (in the sense of the NSD concept) from the optimal ones.

The inspection interval h which may be acceptable for a user has to fulfil his/hers preference requirements. We will further assume that the costs connected with the application of the user-preferred inspection interval should be sufficiently close to the costs connected with the application of the optimal inspection interval. The notion of the γ-***NSD-admissible*** set of values of h introduced in this section let us indicate all these values of h for which the economic consequences, due to the imprecise values of input parameters, are difficult to be distinguished, and are - more or less - equivalent. Thus, we propose to choose the user-preferred inspection interval from this specific set of values of h. In such a case, for choosing an appropriate inspection interval we may take into account all other preference requirements having in mind that for all γ-***NSD-admissible*** values of h economic consequences are indistinguishable.

Let's denote by H^γ the γ-***NSD-admissible*** set of values of h. If we assume, as previously, that the user describes his/hers preferences by defining preference functions $\nu_i^\gamma : H^\gamma \to [0,1]$, $i = 1,...,m$, and if we apply the FCSP technique described in the previous section, then the user-preferred optimal inspection interval may be calculated as

$$h_\gamma^* = \arg\max_{h\in H^\gamma}\min_{i=1,\dots,m} \nu_i^\gamma(h) \qquad (20)$$

Thus, the user-preferred optimal inspection interval maximises the minimal preference for the set of the considered criteria.

5 Numerical example

Let us find the inspection interval for a production machine which works 16 hours per day during two eight-hours working shifts. To find the user-preferred inspection interval we need the input information described in the second section of this paper. Suppose now that the input information is imprecise, and is described by fuzzy numbers of a triangular form with the membership function $\mu(z)$ such that $\mu(z^*) = 1$, $\mu(\omega_L z^*) = 0$, and $\mu(\omega_R z^*) = 0$. The left arm of such a triangle is, thus, described by the function

$$\mu_L(z) = \left(\frac{1}{(1-\omega_L)\,z^*}\right) z - \frac{\omega_L}{1-\omega_L}\ ,\ z \in [\omega_L z^*, z^*] \qquad (21)$$

and the right arm is described by the function

$$\mu_R(z) = \left(\frac{1}{(1-\omega_R)\,z^*}\right) z - \frac{\omega_R}{1-\omega_R}\ ,\ z \in [z^*, \omega_R z^*] \qquad (22)$$

We can use this representation when the input information is given in a form of imprecise statements like 'the value of z is about z^*'.

Suppose now, that for the considered machine the expected time to failure τ is described by the statement "τ equals about 2000 hours". To model this fuzzy number we use (21) and (22) taking $z^* = \tau^* = 2000$, $\omega_L = 0,8$, and $\omega_R = 1,2$. The inspection cost S (with the cost unit equal to the cost of a false alarm) is expressed as "S equals about $0,02$ cost units". Also in this case to model this imprecise number we use (21) and (22) with $z^* = S^* = 0,02$, $\omega_L = 0,8$, and $\omega_R = 1,2$. The standardised average profit from one renewal of the system b is evaluated as "b equals about 20 cost units". The membership function in this case may be described by (21) and (22) with $z^* = b^* = 20$, $\omega_L = 0,8$, and $\omega_R = 1,2$. To find the expected number of inspections while the system is failed A we need to analyse the probability that the failure is revealed during the inspection which can be evaluated only imprecisely. Suppose, that after such an analysis we arrive at the fuzzy version of A described by (21) and (22) with $z^* = A^* = 1,1$, $\omega_L = 0,95$, and $\omega_R = 1,07$. Note, that in this case the membership function is not symmetric. It stems from a fact that A must not be smaller than 1. The value of the remaining parameter namely, the probability of a false alarm δ, usually can be estimated rather precisely, and in our case is set to $0,01$.

Having the input information in the form described above let us find the γ-*NSD-admissible* set of values of h. Assume that γ is equal to $0,5$. Hence, we can find the following input values for (17) and (18): $b_{\min}^{0,5} = 18$, $b_{\max}^{0,5} = 22$, $S_{\min}^{0,5} = 0,018$, $S_{\max}^{0,5} = 0,022$, $A_{\min}^{0,5} = 1,0725$, $A_{\max}^{0,5} = 1,1385$, and $\delta = 0,01$. When we insert these values into (17) , (18) and (19) we find that

$$x_{\min} = 0,026, \quad x_{\max} = 0,104$$

Taking in mind that $\tau_{\min}^{0,5} = 1800$, and $\tau_{\max}^{0,5} = 2200$ we arrive at the following $0,5$-*NSD-admissible* set of values of h: $H^{0,5} = (46,8hrs.,228,8hrs.,)$. It is worth noting that the simple fuzzification of (3) gives the following α-cut set for the fuzzy-optimal inspection interval

$$\left(\tau_{\min}^{\alpha}\sqrt{\frac{\delta+S_{\min}^{\alpha}}{b_{\max}^{\alpha}(A_{\max}^{\alpha}-0.5)}}, \tau_{\max}^{\alpha}\sqrt{\frac{\delta+S_{\max}^{\alpha}}{b_{\min}^{\alpha}\left(A_{\min}^{\alpha}-0.5\right)}}\right) \tag{23}$$

When we take $\alpha = 1 - \gamma = 0,5$ and insert the other input data into (23) we obtain the interval $(81hrs.,123,2hrs.)$. We can see that this interval is much narrower than the $0,5$-*NSD-admissible* set of values of h. It means th aking the values of h from a wider $0,5$-*NSD-admissible* set we admit such values of h that may not be optimal. However, the values of the objective function for those non-optimal values of h cannot be distingushed (in the sense of the NSD index) from the values of this function for the fuzzy-optimal values of h.

Let us consider now other preference requirements that may be expressed for the inspection intervals. First, let us notice that in the considered case some values of h are definitely more preferred than the others. It seems rather obvious that inspections performed after 16 hours of work, i.e. during the night break, are much more preferable than the inspections performed every, say, 1 hour. To describe formally the preferences of this type denote by $\nu^j \in [0,1]$, $j = 1, ..., k$ the preference assigned to the values of h that fulfil certain requirements. Denote by $I(*)$ the set-indicator function that indicates those values of h that fulfil a certain requirement. The overall preference function may be now expressed as follows

$$\nu_1(h) = \sup\left(\nu^1 I(h \bmod h^{(1)} = 0)\right), \nu^2 I(h \bmod h^{(2)} = 0), .)$$

Assume that in our case we have $\left(\nu^1 = 1, h^{(1)} = 16\right)$, $\left(\nu^2 = 0,8, h^{(2)} = 8\right)$, and $\left(\nu^3 = 0,2, h^{(3)} = 4\right)$. Thus, the sampling intervals equal to $(48, 64, 80, ..., 224)$ hours are equally preferred with $\nu_1(h) = 1$.

Consider second requirement for the inspection interval that was mentioned in the second section of the paper. This requirement is related to the expected number of false alarms during a given time period. As any false alarms are undesirable we may assume that the preference function $\nu(A_f)$ is a non-increasing function of A_f. Thus, the respective preference function $\nu_2(h)$ is a non-decreasing function of h. In the considered case this information, together with the knowledge of $\nu_1(h)$, is sufficient to choose the user-preferred inspection interval which is equal to 224 hours. Thus, we should inspect the machine every 14 working days.

6 Conclusions and open problems

In the paper we have found a relatively simple solution to an important practical problem. This was the goal of the research. Despite the apparent simplicity of this problem the solution appeared to be not obvious. Both optimisation problems considered, i.e. the optimisation of a fuzzy objective function and the solution of the flexible constraint satisfaction problem treated separately are well described in literature. However, when these two approaches have to be dealt with together there are still problems to be solved.

The methodology for finding the user-preferred optimal inspection interval can be viewed as an alternative method for treating two aforementioned optimisation problems together. The proposed method for the evaluation of the possible solutions by comparing them to the best attainable has its origins in the approach proposed by Bellman and Zadeh [1]. Therefore, it seems to be consistent with the flexible constraint satisfaction problem that was used for choosing the most preferred solution.

Numerical example considered in this paper explains - in some way - the problems with the application of "optimal" solutions in practice. It is clearly seen that even in a case of relatively small imprecision in setting the values of model's parameters the interval which contains admissible values is - from a practical point of view - so wide that it is sufficient to look only for the most preferred solution. In practice it means that experienced users choose the most preferred solutions ignoring possible optimisation problems.

The proposed method can be easily generalised. For example, in the majority of optimisation problems the optimal solution is not explicitly given as in the case cosidered in this paper. In a forthcoming paper of Hryniewicz [11]a simple algorithm is proposed to find the γ-*NSD-admissible* set of values of the optimised variable in such a case. A further generalisation to a multidimesional case is possible, but numerically not efficient.

In the considered case the preference structure is very simple, and the maxmin approach to solve the flexible constraint satisfaction problem is quite natural. In more complicated cases with many preference relations it may be useful to consider some other approaches. For example a *discrimin* approach, described in the paper of Dubois, Fargier and Prade [5] may be used to choose a better solution. When the preference constraints have different priorities we may use the concept of *prioritized constraints* proposed by by Dubois, Fargier and Prade [4]. Finally, the concept of γ-*NSD-admissibility* that was introduced in this paper may be used to rank different solutions. In suchγmin t may be viewed upon as an additional flexible constraint. This approach, however, may be too complicated numerically to be effectively used in practice.

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