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Uncertainty and Dynamics under Kyoto Obligations

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Abstract

Implementation of the Kyoto protocol raises the question of verification of the Kyoto obligations. Estimates show that uncertainty of the emissions reported by countries is high, mainly due to the methodology of the report preparation.

This indicates that the final verification of the obligations should take into account uncertainty of the reported emissions. This problem is considered in the paper.

Our proposition for verification is based on setting a (small enough) risk of not satisfying obligations. It leads to the easy to check conditions, presented in the paper. This can be done either within deterministic or stochastic setup.

Acceptation of this idea will influence rules of emission trading. This problem is also addressed in the paper.

We propose a method for empirical estimation of the standard deviation of the reported errors, necessary for verification procedure. It uses a smoothing procedure based on the spline functions.

Then we introduce a model of the emission process and show its good performance on historical data. This model can be used to assess roughly possibility of satisfying the obligations, under assumption of constant emission conditions in the future.

Contents

1. Introduction

Global change calls for the development of prevention, mitigation, and adaptation strategies as envisaged by relevant international conventions including the Kyoto Protocol.

The Protocol contains the first legally binding commitments to limit or reduce the emissions of six greenhouse gases or groups of gases (CO_2, CH_4, N_2O, HFCs, PFCs, and SF_6). For Annex I Parties, the targets agreed upon under the Protocol by the first commitment period (2008 to 2012) add up to a decrease in greenhouse gas emissions of 5.2% below 1990 levels[1] in terms of CO_2 equivalents. Non-Annex I Parties are not required to take on specific commitments for emission reductions. Articles 3.3 and 3.4 of the Protocol stipulate that human activities related to land use, land-use change and forestry (afforestation, reforestation, deforestation, forest management and agricultural activities) since 1990 can also be used to meet 2008–2012 commitments. In addition, the Protocol endorses emissions trading (Article 17), joint fulfilment and implementation between Annex I Parties (Articles 4 and 6), and a clean development mechanism (Article 12) that allows Annex I and non-Annex I Parties to act together to reduce emissions (FCCC, 1998; see also Jonas *et al.*, 1999; Jonas and Nilsson, 2001).

The Kyoto Protocol also mentions uncertainty. However, it does not put uncertainty (and, thus, verification) at the centerpiece of its efforts to slow global warming (Nilsson *et al.*, 2001, 2002). So far, the number of countries that have made, or will soon make, their uncertainty assessments available, is limited to Austria, Great Britain, Netherlands, Norway, Poland, and Russia (Jonas and Nilsson, 2001; Charles *et al.*, 1998; van Amstel *et al.*, 2000; Rypdal and Zhang, 2000; Gawin, 2002; Nilsson *et al.*, 2000). It can be easily noticed that the uncertainty exceed decidedly the greenhouse gas reduction obligations agreed upon in the Kyoto protocol.

These findings signal difficulties in verification of the Kyoto obligations, connected with credibility of the reported emissions. Under big uncertainties, hitting the Kyoto targets gives actually little information, as it is almost equally probable that the real emission lies not close to the reported value but somewhere far above or below of the target value. The situation is even more difficult because also the target value is not known exactly due to the uncertainty of the emission in the base year, see Fig. 1.1.

The concept of uncertainty put forward by the IPCC (2000a, b) and envisaged for use under the Kyoto Protocol (FCCC, 2001) is defined with regard to two predefined points in time but disregards how the signal evolves dynamically in time.

In this paper we discuss the problem by considering all data reported yearly in the Kyoto period and by introducing dynamics of the emission system. We propose solutions to the problem of verification of the emission data in the context of the Kyoto Protocol based on statistical empirical distribution of the emission data and risk analysis. We propose also some rules for trading consistent with the verification suggested.

Continuing our considerations we propose a model for describing evolution of the emission data in time. It is in the form of the exponential growth function with the variable growth coefficient. It is observed that the model with constant but different growth coefficients in different time periods can well explain the data. This kind of a model can be easily used to assess the possibility of fulfilling the obligations by a country.

[1] For some countries the base year was agreed to be different from 1990.

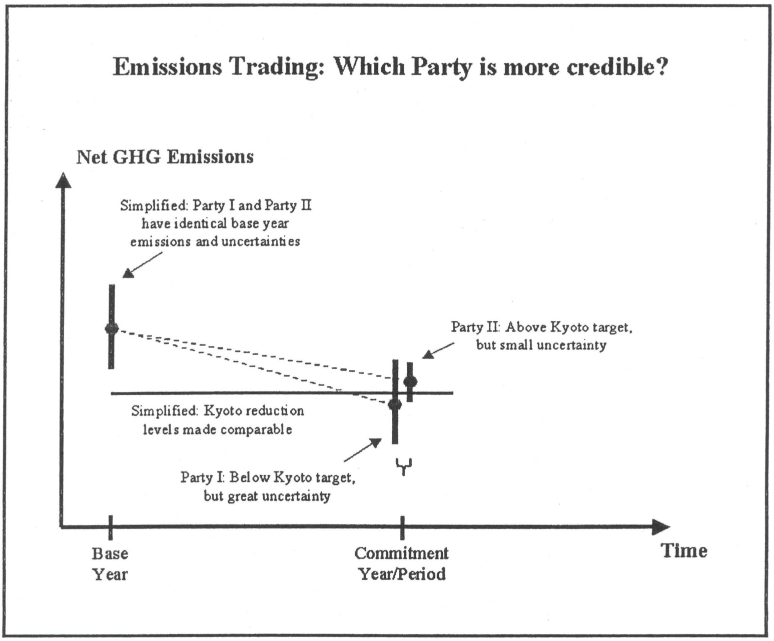


Figure 1.1: Simplified graphical representation to illustrate the importance of uncertainty and verification in the context of the Kyoto Protocol, here addressing the crucial question of credibility. The uncertainty intervals of both Party I and Party II encompass the same Kyoto target, but which Party is more credible for ET? Party I reveals a greater uncertainty interval, the mean of which undershoots the Kyoto target, while Party II reveals a smaller uncertainty interval, the mean of which, however, does not comply with the Kyoto target.

2. Verification of obligations

Treatment of the uncertainty problem depends on admitted description of the uncertain variables. Using some kind of stochastic distribution, like for example the normal distribution, we can only talk on a probability of fulfilling of the obligations. In such case, the condition for the fulfilment can be only expressed in terms of probabilities and of the related risks of not fulfilling the obligations, see Fig. 2.1. This idea will be explored in this section.

Let us introduce some basic notations. By $x(t)$ we denote the real emission, as a function of time. The emission in the basic year t_0 will be denoted $x(t_0) = x_0$. The important years, like 2005, 2008–2010, will be denoted T_i, where i = 5, 8–10. The real emission is unknown and can be only estimated. Hats will mark the estimated values. Thus $\hat{x}(t)$ is the estimated emission. The simple estimation can be made by calculation of the total emission according to methodology used in preparing the reports. These reported (observed) values are contaminated with the observation errors. In the sequel we propose more advanced estimation of the emission.

By δ we denote the fraction of the emission to be reduced until the commitment period. Thus at the commitment period the emission should be not greater than $(1-\delta)x_0$. Obviously, 100δ is the percentage reduction required by the Kyoto protocol.

Finally, we want to take risk not greater than α that the reduction in the year T_i is not fulfilled. We discuss three approaches.

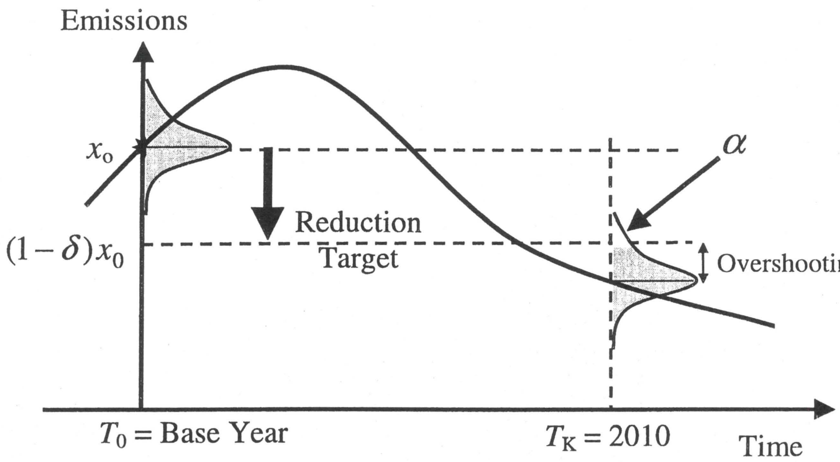


Figure 2.1: Graphical presentation of checking fulfilment of the reduction obligations. The value of α represents the risk that the actual emission does not fulfil the obligations

2.1. Interval uncertainty of both x_0 and $x(T_i)$

Let us assume the independent interval uncertainty of both x_0 and $x(T_i)$

$$|x_0 - \hat{x}_0| \le \Delta_0, \quad |x(T_i) - \hat{x}(T_i)| \le \Delta_i \tag{2.1}$$

Then the uncertainty of the estimate of the fulfillment of the obligations is

$$\left|\hat{x}_0(1-\delta) - \hat{x}(T_i) - \left(x_0(1-\delta) - x(T_i)\right)\right| \le \Delta_i + (1-\delta)\Delta_0 = \Delta_{0i} \tag{2.2}$$

As we agree to take the risk α that the value $x(T_i)$ may be actually greater then $x_0(1-\delta)$ then it must hold

$$\hat{x}_0(1-\delta) - \hat{x}(T_i) \ge (1-\alpha)\Delta_{0i}$$

from which we get the condition

$$\hat{x}(T_i) \le \hat{x}_0(1-\delta) - (1-\alpha)\Delta_{0i} \tag{2.3}$$

It requires that the value $\hat{x}_0(1-\delta)$ is overshot with almost two uncertainty intervals.

Dividing by $\hat{x}_0$ and taking logarithms of both sides yields

$$\hat{X}(T_i) \le \ln\left(1-\delta-(1-\alpha)\frac{\Delta_{0i}}{\hat{x}_0}\right) \cong -\delta-(1-\alpha)\frac{\Delta_{0i}}{\hat{x}_0} \tag{2.4}$$

where $\hat{X}(T_i) = \ln\frac{\hat{x}(T_i)}{\hat{x}_0}$ is the logarithm of the normalised emission. For $\frac{\hat{x}(T_i)}{\hat{x}_0}$ close to 1 it approximately holds

$$\hat{X}(T_i) = \ln\frac{\hat{x}(T_i)}{\hat{x}_0} \cong \frac{\hat{x}(T_i)}{\hat{x}_0} - 1 = \frac{\hat{x}(T_i)-\hat{x}_0}{\hat{x}_0} \tag{2.5}$$

Thus, $\hat{X}(T_i)$ may be interpreted as the relative change of $\hat{x}(T_i)$ with respect to $\hat{x}_0$.

2.2. Interval uncertainty of x_0 and stochastic uncertainty of $x(T_i)$

We assume now that our knowledge of uncertainty of x_0 is in the form of the uncertainty interval $|x_0 - \hat{x}_0| \le \Delta_0$ while $x(T_i)$ is stochastic, described by a given distribution with unknown parameters. We require now that the probability of not satisfying the obligations be not bigger than some small value α. This can be written as, see Fig. 2.1

$$P\left(\frac{x_0(1-\delta)-\hat{x}(T_i)}{\sigma_{\hat{x}}(T_i)} \ge q_{1-\alpha}\right) \le \alpha$$

where $q_{1-\alpha}$ is the 1-α quantile of the distribution of the variable $\hat{x}(T_i)$ and $\sigma_{\hat{x}}(T_i)$ is its standard deviation. This gives the following condition for the estimate $\hat{x}(T_i)$

$$\hat{x}(T_i) \le x_0(1-\delta) - q_{1-\alpha}\sigma_{\hat{x}}(T_i)$$

Taking into account that the frequently used quantile for the normal distribution is close to 2, this condition requires overshooting with approximately two standard deviations. Thus this condition and that of (2.3) are approximately equivalent.

Dividing now both sides by $\hat{x}_0$ we have

$$\frac{\hat{x}(T_i)}{\hat{x}_0} \le \frac{x_0}{\hat{x}_0}\left[1-\delta-q_{1-\alpha}\frac{\sigma_{\hat{x}}(T_i)}{x_0}\right] \tag{2.6}$$

Now, taking logarithms of both sides yields

$$\hat{X}(T_i) \le \ln\frac{x_0}{\hat{x}_0} + \ln\left[1-\delta-q_{1-\alpha}\frac{\sigma_{\hat{x}}(T_i)}{x_0}\right] \cong \frac{x_0-\hat{x}_0}{\hat{x}_0} - \delta - q_{1-\alpha}\frac{\sigma_{\hat{x}}(T_i)}{x_0}$$

To obtain the risk α related solely to the first term on the right hand side, the variable $\hat{x}(T_i)$ has to be not greater than $-(1-\alpha)\frac{\Delta_0}{\hat{x}_0}$. Then finally we get the condition

$$\hat{X}(T_i) \le -(1-\alpha)\frac{\Delta_0}{\hat{x}_0} - \delta - q_{1-\alpha}\frac{\sigma_{\hat{x}}(T_i)}{x_0} \tag{2.7}$$

2.3. Stochastic uncertainty of both x_0 and $x(T_i)$

This case is simpler than the previous one. Now the condition is

$$P\left(\frac{\hat{x}_0(1-\delta)-\hat{x}(T_i)}{\sigma_{\hat{x}}} \geq q_{1-\alpha}\right) \leq \alpha$$

where it holds

$$\sigma_{\hat{x}}^2 = \sigma_{\hat{x}}^2(0) + \sigma_{\hat{x}}^2(T_i) \tag{2.8}$$

and $\sigma_{\hat{x}}^2(0)$ is the variance of the $\hat{x}_0$ variable. This provides the condition

$$\hat{x}(T_i) \leq \hat{x}_0(1-\delta) - q_{1-\alpha}\sigma_{\hat{x}}$$

and in the consequence

$$\frac{\hat{x}(T_i)}{\hat{x}_0} \leq 1-\delta - q_{1-\alpha}\frac{\sigma_{\hat{x}}}{\hat{x}_0}$$

which yields

$$\hat{X}(T_i) \leq \ln\left(1-\delta - q_{1-\alpha}\frac{\sigma_{\hat{x}}}{\hat{x}_0}\right) \cong -\delta - q_{1-\alpha}\frac{\sigma_{\hat{x}}}{\hat{x}_0} \tag{2.9}$$

These are the basic dependencies that will be used in the sequel.

2.4. Choice of δ

The value 100δ may be conceived to be the percentage reduction fixed in the Kyoto protocol. Within the methodology proposed in the paper, it is required from the countries to overshoot their reduction target. Such additional requirement is costly and may cause objections.

Thus, our proposition consists in using a smaller δ than fixed in the Kyoto protocol. The proposed value of δ should fulfil the condition that a country with the emission value $\hat{x}(T_i)$ equal to the Kyoto target and with an "average" standard deviation should be considered as fulfilling the obligations, see Fig. 2.2. This way even a country which has not achieved the target value may be considered as fulfilling the obligations, if only standard deviation of its estimated emission $\hat{X}$ or $\hat{x}$ is small enough. On the contrary, a country with big standard deviation may be considered as not fulfilling the obligations although it actually reports even small overshooting of the Kyoto reduction, see Fig. 2.2.

Let δ_0 be the required Kyoto reduction, σ_s the reference standard deviation of the normal distribution (i.e. the reference distribution is $N_S\left(\hat{x}_0(1-\delta_0), \sigma_s^2\right)$). Then the proposed redefined value δ_D will satisfy

$$\frac{\hat{x}_0(1-\delta_D) - \hat{x}_0(1-\delta_0)}{\sigma_s} = q_{1-\alpha}^s$$

where $q_{1-\alpha}^0$ is the $(1-\alpha)$th quantile of the standard normal distribution N_S. At this point, it is convenient to use the "normalised standard deviation"

$$u_{ss} = \frac{\sigma_s}{\hat{x}_0}$$

Then, rearranging we get

$$\delta_D = \delta_0 - \frac{q_{1-\alpha}^s \sigma_s}{\hat{x}_0} = \delta_0 - q_{1-\alpha}^s u_{ss} \tag{2.10}$$

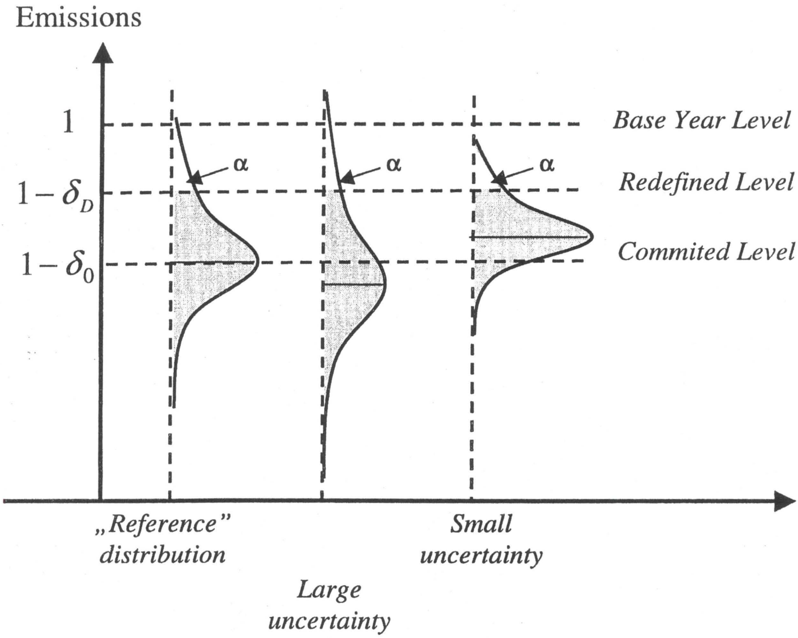


Figure 2.2: Graphical presentation of the redefined level concept.

2.5. Verification in 2005

The Kyoto protocol requires that in 2005 the emissions by the Annex I Parties should not be greater than the emission in the base year t_0. We consider now the three approaches discussed earlier. Let us define the year 2005 as T_5.

A. Interval uncertainties of both x_0 and $x(T_i)$

From (2.4), in the year 2005 the following condition has to be satisfied

$$\hat{X}(T_5) \le -(1-\alpha)\frac{\Delta_{0i}}{x_0} \tag{2.11}$$

B. Interval uncertainty of x_0 and stochastic uncertainty of $x(T_i)$

In the year 2005 the following condition has to be satisfied

$$\hat{X}(T_5) \le -(1-\alpha)\frac{\Delta_0}{\hat{x}_0} - q_{1-\alpha}\frac{\sigma_{\hat{X}}(T_5)}{x_0} \tag{2.12}$$

C. Stochastic uncertainty of both x_0 and $x(T_i)$

In analogy to two earlier cases we have

$$\hat{X}(T_5) \leq -q_{1-\alpha} \frac{\sigma_{\hat{x}}}{\hat{x}_0} \tag{2.13}$$

2.6. Verification in 2008 - 2012

The Kyoto protocol does not specify explicitly how to understand fulfilment of the emission reduction in the years 2008 – 2012. Taking it literally, the countries should fulfil the obligations in every year in this period. Thus, defining $T_i = 2000 + i$, $i = 8, 9, 10, 11, 12$, the following equations can be used at each year T_i. Here we use generally δ for the fractional reduction although according to the earlier proposition it should be understood that $\delta = \delta_D$.

A. Interval uncertainties of both x_0 and $x(T_i)$

$$\hat{X}(T_i) \leq -\delta - (1-\alpha)\frac{\Delta_{0i}}{\hat{x}_0} \tag{2.14}$$

B. Interval uncertainty of x_0 and stochastic uncertainty of $x(T_i)$

$$\hat{X}(T_i) \leq -(1-\alpha)\frac{\Delta_0}{\hat{x}_0} - \delta - q_{1-\alpha}\frac{\sigma_{\hat{x}}(T_i)}{x_0} \tag{2.15}$$

C. Stochastic uncertainty of both x_0 and $x(T_i)$

$$\hat{X}(T_i) \leq -\delta - q_{1-\alpha}\frac{\sigma_{\hat{x}}}{\hat{x}_0} \tag{2.16}$$

3. Effective emission in trading

3.1. Interval uncertainty

The proposed methodology of verification of the Kyoto obligations will influence the conditions of emission trading. Let us consider two countries: a country emitting $x_1(T_i)$ units of the carbon dioxide with the interval uncertainty Δ_i^1 wishing to buy E units from another country which is emitting $x_2(T_i)$ units of the carbon dioxide with the interval uncertainty Δ_i^2. That is, it holds

$$|x_1(T_i) - \hat{x}_1(T_i)| \leq \Delta_i^1, \qquad |E - \hat{E}| \leq r\Delta_i^2, \qquad r = \frac{\hat{E}}{\hat{x}_2(T_i)}$$

The dependence of the uncertainty interval on the emission level arises from the fact that the uncertainty is generally given as a constant fraction of the emission, in percents, i.e. we know rather r than Δ_i^2.

The corrected emission of the country 1 after purchasing will be $\hat{x}_1(T_i) - \hat{E}$. With respect to the original emission the following inequality should be satisfied

$$\hat{x}_1(T_i)+(1-\alpha)[\Delta_i^1+(1-\delta)\Delta_0^1]\le \hat{x}_0(1-\delta)$$

which is proposed to be changed after purchasing to

$$\hat{x}_1(T_i)-\hat{E}+(1-\alpha)[\Delta_i^1+r\Delta_i^2-\hat{E}v_s+(1-\delta)\Delta_0^1]\le \hat{x}_0(1-\delta)$$

where v_s is a "reference" uncertainty ratio (i.e. for the emission $\hat{x}(T_i)$ with the "reference" uncertainty ratio v_s it holds $v_s=\dfrac{\Delta_s}{\hat{x}(T_i)}$ where Δ_s is the "reference" interval uncertainty). Our intention for introduction of the "reference" ratio is to make the effective emission "symmetric", that is to make it bigger or smaller than $\hat{E}$, depending on whether $v_2 > v_s$ or $v_2 < v_s$, where v_2 is the uncertainty ratio of the country 2 defined below in (3.1).

Thus, purchasing caused subtraction of the following component from the left side

$$E_{eff}=\hat{E}-(1-\alpha)(r\Delta_i^2-\hat{E}v_s)=\hat{E}[1-(1-\alpha)(v_2-v_s)],\qquad v_2=\frac{\Delta_i^2}{\hat{x}_2(T_i)} \tag{3.1}$$

which will be called *the effective emission in trading*. This reduced value may form a basis for financial liabilities among countries.

3.2. Stochastic uncertainty

Likewise, using the formula for the sum of two normal distributions we have for the stochastic approach

$$\sigma^2_{\hat{x}_1-\hat{E}}(T_i)=\sigma^2_{\hat{x}_1}(T_i)+r^2\sigma^2_{\hat{x}_2}(T_i)$$

where, as before, $r=\dfrac{\hat{E}}{\hat{x}_2(T_i)}$. To fulfil the obligations, the original emission of the country 1 should satisfy the following condition

$$\hat{x}_1(T_i)-q_{1-\alpha}\sigma_{\hat{x}_1}\le \hat{x}_0(1-\delta)$$

where $\sigma_{\hat{x}_1}$ is given by (2.8). After purchasing $\hat{E}$ units from the country 2 this condition will be changed to

$$\hat{x}_1(T_i)-\hat{E}-q_{1-\alpha}\sqrt{\sigma^2_{\hat{x}_1}(0)+\sigma^2_{\hat{x}_1}(T_i)+r^2\sigma^2_{\hat{x}_2}(T_i)}\le \hat{x}_0(1-\delta)$$

This can be written in the form

$$\hat{x}_1(T_i)-q_{1-\alpha}\sigma_{\hat{x}_1}-\hat{E}-q_{1-\alpha}\left(\sqrt{\sigma^2_{\hat{x}_1}(0)+\sigma^2_{\hat{x}_1}(T_i)+r^2\sigma^2_{\hat{x}_2}(T_i)}-\sigma_{\hat{x}_1}\right)\le \hat{x}_0(1-\delta)$$

The component in the parenthesis, denoted as P, is transformed as follows

$$P=\sqrt{\sigma^2_{\hat{x}_1}(0)+\sigma^2_{\hat{x}_1}(T_i)+r^2\sigma^2_{\hat{x}_2}(T_i)}-\sigma_{\hat{x}_1}=$$

$$=r\sigma_{\hat{x}_2}(T_i)\left(\sqrt{\frac{\sigma^2_{\hat{x}_1}(0)}{r^2\sigma^2_{\hat{x}_2}(T_i)}+\frac{\sigma^2_{\hat{x}_1}(T_i)}{r^2\sigma^2_{\hat{x}_2}(T_i)}+1}-\frac{\sigma_{\hat{x}_1}}{r\sigma_{\hat{x}_2}(T_i)}\right)$$

We have

$$r\sigma_{\hat{x}_2}(T_i)=\hat{E}\frac{\sigma_{\hat{x}_2}(T_i)}{\hat{x}_2(T_i)}=\hat{E}v_2$$

where now

$$v_2 = \frac{\sigma_{\hat{x}_2}(T_i)}{\hat{x}_2(T_i)}$$

is the uncertainty ratio of the country 2 at time T_i. Although formally v_2 here is different from the uncertainty ratio used in the interval uncertainty case, it will be more convenient to keep the same notation for both cases. This should not cause any confusion. Moreover, it holds

$$\frac{\sigma^2_{\hat{x}_1}}{r^2\sigma^2_{\hat{x}_2}(T_i)} = \frac{\hat{x}_1^2(0)}{\hat{x}_1^2(T_i)} \frac{\dfrac{\sigma^2_{\hat{x}_1}(0)}{\hat{x}_1^2(0)}}{\dfrac{\sigma^2_{\hat{x}_2}(T_i)}{\hat{x}_2^2(T_i)}} \frac{\hat{x}_1^2(T_i)}{\hat{E}^2} + \frac{\dfrac{\sigma^2_{\hat{x}_1}(T_i)}{\hat{x}_1^2(T_i)}}{\dfrac{\sigma^2_{\hat{x}_2}(T_i)}{\hat{x}_2^2(T_i)}} \frac{\hat{x}_1^2(T_i)}{\hat{E}^2} =$$

$$= \frac{v_{10}^2}{v_2^2} \frac{1}{\eta^2 R^2} + \frac{v_1^2}{v_2^2} \frac{1}{R^2} = \frac{1}{R^2 v_2^2}\left(\frac{v_{10}^2}{\eta^2} + v_1^2\right) = S^2$$

where v_1 is the uncertainty ratio of the country 2 at time T_i, v_{10} is the uncertainty ratio of the country 1 at time 0. The reduction factor $\eta = \dfrac{\hat{x}_1(T_i)}{\hat{x}_1(0)}$ will be $(1-\delta)$, if the country 1 fulfils the obligations. As we know that it has not, we can assume that $\eta \approx 1$. Moreover

$$R = \frac{\hat{E}}{\hat{x}_1(T_i)}$$

is the purchased fraction of emission of the country 1. It will be close, possibly smaller than δ, and therefore of the order of few percent.

Thus

$$P = \frac{\hat{E} v_2}{\sqrt{\dfrac{1}{R^2 v_2^2}\left(\dfrac{v_{10}^2}{\eta^2} + v_1^2\right) + 1} + \sqrt{\dfrac{1}{R^2 v_2^2}\left(\dfrac{v_{10}^2}{\eta^2} + v_1^2\right)}}$$

As before, we want to refer to the reference uncertainty ratio v_s, so we insert $v_2 - v_s$ instead of v_2 in the numerator above. We can assume that $\dfrac{v_{10}}{\eta}$ does not differ significantly from v_1. Thus, the expression for P can be simplified to

$$P = \hat{E}\frac{\sqrt{2}}{4} R \frac{v_2}{v_1}(v_2 - v_s)$$

and therefore *the effective emission in trading between two countries* 1 and 2 is

$$E_{eff} = \hat{E}\left(1 - q_{1-\alpha}\frac{\sqrt{2}}{4} R \frac{v_2}{v_1}(v_2 - v_s)\right) \tag{3.2}$$

This expression resembles that of (3.1). However, now the effective emission depends on the uncertainty ratios of both countries and, moreover, on the purchased fraction. This is why we call it the effective emission in trading between two countries.

Let is notice that if we assume that $\hat{x}_1(0)$ is known exactly, then the coefficient $\dfrac{\sqrt{2}}{4}$ will be replaced by $\dfrac{1}{2}$.

4. Estimating the uncertainty parameters

An important parameter of the verification schemes in the previous section is the estimate of the uncertainty model parameter. At present, the estimates are calculated by aggregation of the uncertainties of the partial emissions. This method gives rather big estimates that may be never or only seldom attained. Here we propose another approach, obtained under assumption of stochastic errors. A method of estimation of the standard deviation of the normal distribution of the error from the reported emission data is proposed.

4.1 Basic assumptions and simplifications

4.1.1 Data treatment

We assume that the emission process can be described by a continuous and differentiable function $x(t)$ which represents the real emissions. The process can be then observed with errors in equally spaced time intervals $\Delta t = 1$ year. We introduce a simplified notation $x(t_i = i\Delta t) \equiv x_i$ and assume that $x_i > 0$.

4.1.2 Uncertainty treatment

We assume that the real process x_i is observed with errors $\varepsilon_i = u_i x_i$, where u_i, $i = 0,1,\ldots$, form a zero-mean stochastic process with independent variables, i.e. it holds

$$E(u_i) = 0\ ,$$
$$E(u_i^2) = \sigma_i^2\ ,$$
$$E(u_i u_j) = 0 \quad \text{for} \quad i \neq j$$

Thus, the observations can be presented in the following way

$$y_i = x_i + u_i x_i = (1 + u_i) x_i\ ,\ i = 0,1,\ldots,N\ .$$

where y_i are the observed emissions, x_i the (unknown) real emissions, and u_i their relative uncertainties.

The function $x(t)$ is generally not known. Therefore, we can also introduce the errors proportional to the observed value $\varepsilon_i = u_i y_i$. This yields

$$y_i = x_i + u_i y_i$$
$$y_i = \frac{1}{1 - u_i} x_i$$

For small u_i both models are approximately the same. However, for higher u_i it is not so.

There is an important difference between two above models. In the former y_i is linear in u_i, which is the only stochastic variable at the right hand side. This is not the case in the latter.

In both cases we obtained models with the multiplicative errors. The models can be turned into the additive form by logarithmic transformation of the both sides

$$\ln y_i = \ln x_i + \ln(1+u_i)$$

and

$$\ln y_i = \ln x_i - \ln(1-u_i)$$

For small u_i

$$\ln(1 \pm u_i) \cong \pm u_i$$

resulting in the identical equation

$$\ln y_i = \ln x_i + u_i \ .$$

in which $\ln y_i$ linearly depends on u_i. Moreover, the exact dependence of $\ln y_i$ on $\ln(1+u_i)$ or $\ln(1-u_i)$ is linear in both cases.

4.2. Smoothing and uncertainty analysis

4.2.1 Smoothing splines

Let us consider some abstract data z_i generated by the following system

$$z_i = f(t_i) + \varepsilon_i, \qquad i = 0,1,2,\ldots,N$$

The vector

$$\boldsymbol{\varepsilon} = (\varepsilon_0,\ldots,\varepsilon_N) \propto \mathrm{N}(0,\sigma^2\mathbf{I})$$

contains the set of observation errors. We want to recover the function $f(t)$, assumed to be smooth enough, knowing only the erroneous observations z_i, i = 0, 1, ..., N. For this we use splines.

In the interpolating splines an approximation $\hat{z}(t)$ to $f(t)$ is obtained assuming that $\hat{z}(t)$ is a polynomial of an order m (we use m = 3) on each segment $[t_i, t_{i+1})$, $i = 0,1,2,\ldots,N-1$, satisfying $\hat{z}(t) = z_i$ and having the continuous derivatives up to the order $m-1$ on the whole interval (t_1, t_N).

In the presence of noise the interpolating spline generally quickly varies in time, overshooting and undershooting very much the function $f(t)$.

Much better approximation can be then achieved using the smoothing splines. Their idea is to find the function $\hat{z}(t)$ that does not need to go directly through the observed points z_i, in order to get a function with smaller $(m-1)$th derivative.

If we restrict our attention to the third order polynomials, then the task is to find a smooth function $\hat{z}(t)$ which minimises the sum

$$\frac{1}{N+1}\sum_{i=0}^{N}(z_i - \hat{z}(t_i))^2 + \lambda \int_{t_0}^{t_N}\left(\hat{z}^{(2)}(t)\right)^2 dt \qquad (4.1)$$

where

$$\hat{z}(t) = a_i + b_i(t-t_i) + c_i(t-t_i)^2 + d_i(t-t_i)^3$$
$$t \in [t_i, t_{i+1}), \qquad i = 0,1,\ldots,N-1$$

The solution of the problem was delivered by Wahba (1990) and can be written in a general form

$$\hat{z}(t_i) = a_i = \sum_j A_{ij}(N,\lambda) z_j \qquad \frac{d\hat{z}(t_i)}{dt} = b_i = \sum_k B_{ik}(N,\lambda) z_k \tag{4.2}$$

see also Gu (2002), where A_{ij} and B_{ik} are coefficients which do not depend on data z_i and can be precomputed.

4.2.2 Uncertainty analysis

The solution depends on the value of λ. This value is estimated by the generalized cross validation method (Wahba, 1990) by minimising the criterion

$$V(N,\lambda) = \frac{\sum_{i=0}^{N} [z_i - \hat{z}_i(N,\lambda)]^2}{N+1-\sum_{i=0}^{N} A_{ii}(N,\lambda)} \tag{4.3}$$

The optimal value will be denoted $\hat{\lambda}$. The optimal value of the criterion can be used as an estimate of σ^2, i.e.

$$\hat{\sigma}^2(N) = V(N,\hat{\lambda}) \tag{4.4}$$

The expression in the denominator of (4.3) can be interpreted as the degree of freedom for the noise, in analogy to the degrees of freedom in the regression analysis. However, in contrast to the regression analysis, the good statistical properties of the estimate $\hat{\sigma}^2(N)$ for the smoothing splines have not been proved theoretically but only checked on numerical simulations.

The estimated variance of $\hat{z}(t_i)$ is now

$$\hat{\sigma}_{\hat{z}}^2(N,t_i) = \hat{\sigma}^2(N) A_{ii}(N,\hat{\lambda}) \tag{4.5}$$

The above analysis can be applied for smoothing the data $Y_i = \ln y_i$ to obtain the smoothed values $\hat{z}_i = \hat{X}_i$. Then, we assume that the following holds

$$\frac{\hat{x}_i}{\hat{x}_0} = e^{\hat{X}_i}$$

With the assumption of the additive errors in the observations y_i, when it holds $y_i = x_i + u_i$, it would be appropriate to smooth directly the series $\{x_i\}$ and calculate $\hat{X}_i = \ln \frac{\hat{x}_i}{\hat{x}_0}$. We use the former possibility, as it seems to us more relevant. Although the choice is important from the theoretical point of view, it seems to not provide much difference in the real calculations.

Equation (4.5) has been used to calculate the estimates of the standard deviations $\hat{\sigma}_{\hat{z}}(N,t_i)$ for the emission from the fossil fuels provided by Marland et al. (1999). As the values $A_{ii}(N,\hat{\lambda})$ do not change significantly in i except for few time points at the beginning and the end of the data, only one estimate of the standard deviation, called $\hat{\sigma}_{\hat{z}}(N)$, is presented for each country. As indicated in its denomination, the value $\hat{\sigma}_{\hat{z}}(N)$ depends on the number of data used. This dependence is visible, although mostly not crucial, in the results presented in Table 4.1 for different time periods. However, for few cases reduction of the number of data caused big drop of the standard deviation value.

Table 4.1. Optimal values of λ and estimated standard deviations of observation errors for different countries and two time periods

Years 1950-1998			Years 1970-1998		
Country	$\hat{\lambda}$	Std. Dev.	Country	$\hat{\lambda}$	Std. Dev.
ARGENTINA	0.06	2.4	ARGENTINA	0.00	0.5
AUSTRALIA	0.06	1.8	AUSTRALIA	7.44	1.1
AUSTRIA	0.15	2.7	AUSTRIA	232.4	1.7
BELGIUM	0.07	2.4	BELGIUM	0.12	2.3
BRAZIL	0.31	2.0	BRAZIL	0.19	1.3
CANADA	0.03	2.0	CANADA	0.00	0.4
CHINA	0.03	4.8	CHINA	0.13	1.4
CUBA	0.16	6.7	CUBA	43.08	2.6
EGYPT	1.16	3.5	EGYPT	0.32	2.6
FINLAND	0.03	4.9	FINLAND	0.04	3.9
FRANCE	0.14	2.4	FRANCE	0.55	2.3
GREECE	0.14	2.9	GREECE	0.16	2.3
ICELAND	1.64	3.7	ICELAND	1.68	2.9
IRELAND	0.11	4.4	IRELAND	0.30	2.3
ISRAEL	0.03	3.5	ISRAEL	0.22	2.0
ITALY	0.10	1.7	ITALY	0.86	1.3
JAPAN	0.01	2.8	JAPAN	0.07	1.8
LUXEMBOURG	0.05	3.0	LUXEMBOURG	0.13	2.8
MEXICO	0.77	1.8	MEXICO	2.22	1.8
NETHERLANDS	0.08	2.9	NETHERLANDS	0.04	3.8
NEW ZEALAND	5.11	2.0	NEW ZEALAND	0.05	3.0
NORWAY	3.44	4.6	NORWAY	3.49	5.8
POLAND	0.71	1.5	POLAND	0.67	1.8
PORTUGAL	3.35	2.1	PORTUGAL	3.96	2.1
ROMANIA	0.20	1.9	ROMANIA	0.32	2.2
SPAIN	0.03	3.1	SPAIN	0.82	1.8
SWEDEN	3.69	2.8	SWEDEN	3.60	2.6
SWITZERLAND	0.11	3.4	SWITZERLAND	1.05	2.0
TURKEY	0.11	3.2	TURKEY	0.02	3.5
UNITED KINGDOM	0.15	1.6	UNITED KINGDOM	1.09	1.4
USA	0.02	1.7	USA	0.00	0.4

The estimated values agree quite well in magnitudes with the common idea of the errors made in calculation of the fossil fuel emission, believed to be around 1-2%. A little bigger figures obtained in some of our calculations may be connected with some additional factors that might have influenced the calculated estimates, as year-to-year variations in the weather conditions or in the economic situation of the countries.

5. Empirical models for the net emission data

In this section we consider a set of values x_i which can be considered as a time series consisting of N elements. We introduce a difference model and a differential model to describe the time evolution of the data. Then we motivate the choice of the model and finally present some results for fitting the model to the emission data for some countries.

5.1. Difference model

As we assumed that x_i are non-negative we can define a new time series

$$g_i := \frac{x_{i+1}}{x_i} - 1 = \frac{x_{i+1} - x_i}{x_i} \quad , \quad i = 0, \ldots, \; N-1$$

Each element g_i of a new time series can be interpreted as a relative difference of the two consecutive elements x_{i+1} and x_i.

From the latter relation we can now formulate the following difference equation

$$x_{i+1} - x_i = g_i \, x_i, \qquad x_0 = x(t_0) \tag{5.1}$$

which can be then easily solved giving

$$x_N = x_0 \prod_{i=0}^{N-1} (1 + g_i)$$

As all x_i are positive we can convert this solution to an additive form

$$X_N = \sum_{i=0}^{N-1} \ln(1 + g_i) \cong \sum_{i=0}^{N-1} g_i \tag{5.2}$$

where we introduced the new variable $X_i = \ln \frac{x_i}{x_0}$.

5.2. Differential model

A similar way of reasoning can be provided directly for the function $x(t)$. Starting with the equation

$$\frac{dx(t)}{dt} = g(t) x(t), \qquad x(t_0) = x_0 \tag{5.3}$$

we obtain the solution for $x(t_i)$ which depends on the function $g(t)$

$$x(t_i) = x_0 \exp\left(\int_{t_0}^{t_i} g(\tau)d\tau\right)$$

This provides us with the formula

$$\ln\frac{x(t_i)}{x_0} = \int_{t_0}^{t_i} g(\tau)d\tau \cong \sum_{j=0}^{i-1} g(t_j)$$

Denoting $X(t_i) = \ln\frac{x(t_i)}{x_0}$ and taking $i = N$ we get

$$X(t_N) = \int_{t_0}^{t_N} g(\tau)d\tau \cong \sum_{i=0}^{N-1} g(t_i) \tag{5.4}$$

which is equivalent to (5.2).

The actual solution of the problem relies, as it was already mentioned, on the function g. From (5.3) this function can be written in the form

$$g(t) = \frac{1}{\frac{x(t)}{x_0}} \frac{d\frac{x(t)}{x_0}}{dt} = \frac{d}{dt}\ln\frac{x(t)}{x_0} = \frac{dX(t)}{dt} \tag{5.5}$$

From this expression we find that the function g can be conceived as the rate of changes of the variable $X(t)$.

Estimated value of the signal in the year t_N, after modelling by the smoothing splines, is given by

$$\frac{\hat{x}_N}{\hat{x}_0} = \exp(\hat{X}_N) = \exp a_N$$

or

$$\frac{\hat{x}_N}{\hat{x}_0} = \exp\left(\int_{t_0}^{t_N} \frac{d\hat{X}(\tau)}{d\tau}d\tau\right) \cong \exp\sum_{i=0}^{N-1} b_i = \exp\left(\int_{t_0}^{t_N} \hat{g}(\tau)d\tau\right) \tag{5.6}$$

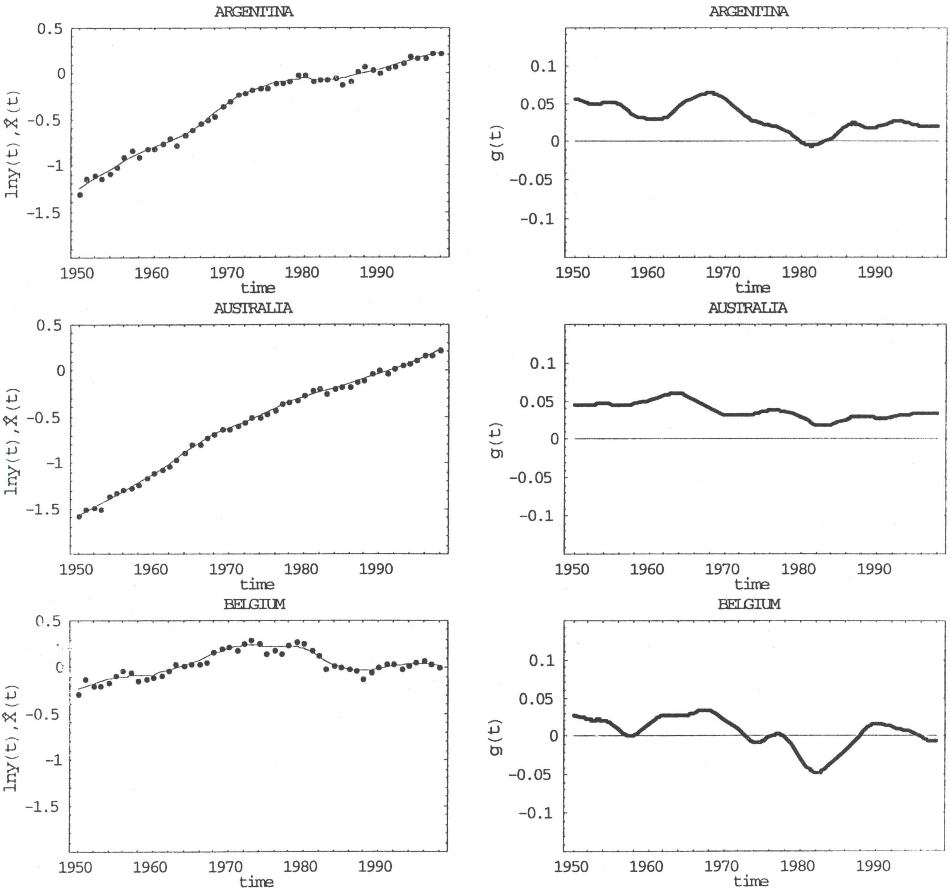
This can be also expressed in the form

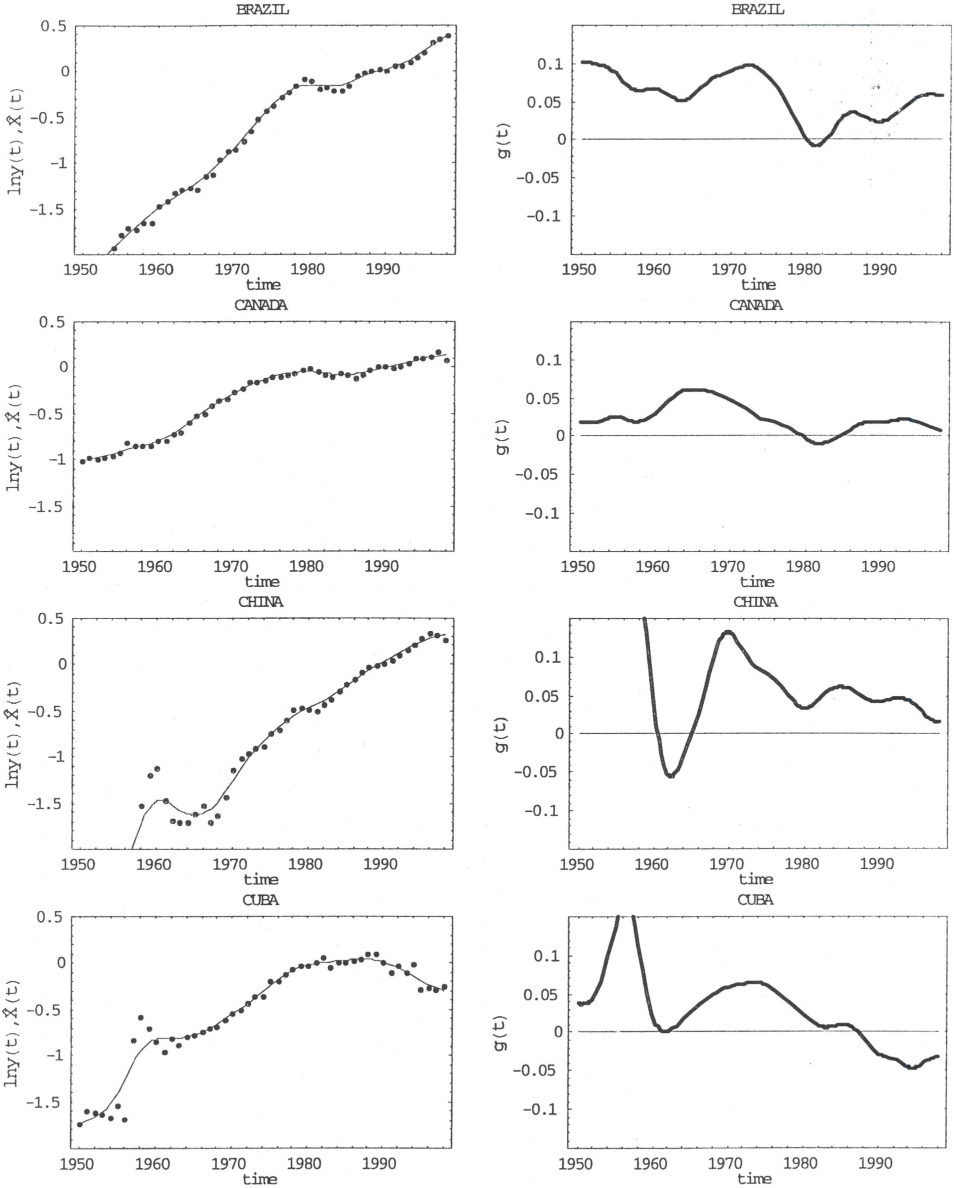
$$\hat{X}_N = \int_{t_0}^{t_N} \hat{g}(\tau)d\tau \tag{5.7}$$

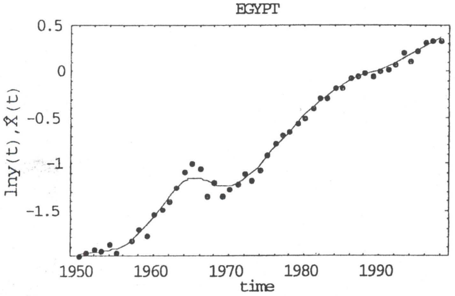
5.3 Estimating the function $g(t)$

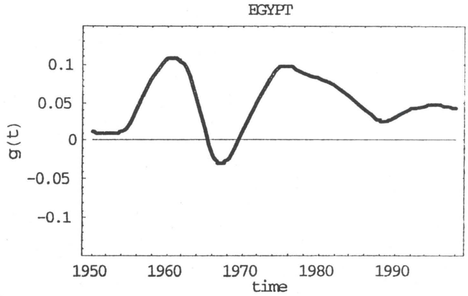
Expression (5.5) was used to estimate the function $g(t)$ for few countries from the previously mentioned data of CO_2 emission from the fossil fuels published by Marland et al. (1999). For calculating the derivatives the data were presmoothed using the smoothing splines. The results are presented in Fig. 5.1.

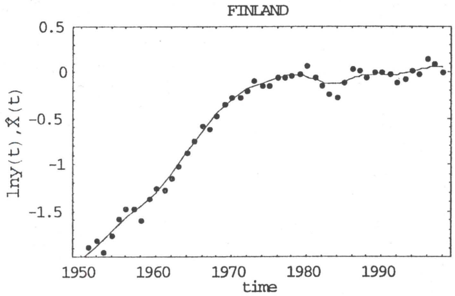
Figure 5.1. Results of smoothing and estimation of $g(t)$ for different countries.

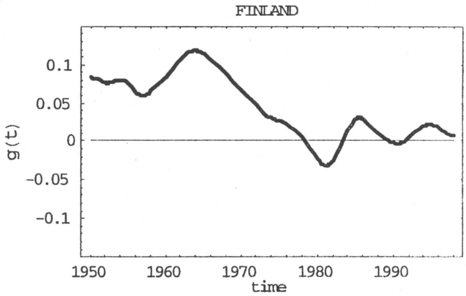



BRAZIL
lny(t), x̂(t)
g(t)
CANADA
CHINA
CUBA
time
1950
1960
1970
1980
1990
0.5
0
-0.5
-1
-1.5
0.1
0.05
-0.05
-0.1


EGYPT
lny(t), $\hat{x}$(t)
0.5
0
-0.5
-1
-1.5
1950
1960
1970
1980
1990
time


EGYPT
g(t)
0.1
0.05
0
-0.05
-0.1
1950
1960
1970
1980
1990
time


FINLAND
lny(t), $\hat{x}$(t)
0.5
0
-0.5
-1
-1.5
1950
1960
1970
1980
1990
time


FINLAND
g(t)
0.1
0.05
0
-0.05
-0.1
1950
1960
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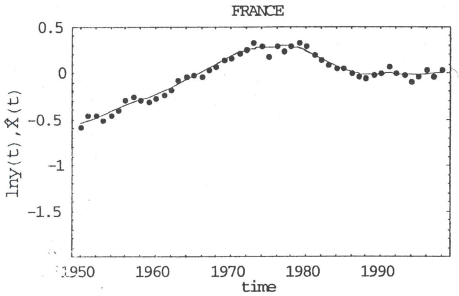
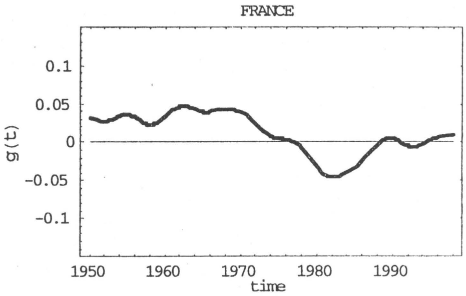
time


FRANCE
lny(t), $\hat{x}$(t)
0.5
0
-0.5
-1
-1.5
1950
1960
1970
1980
1990
time


FRANCE
g(t)
0.1
0.05
0
-0.05
-0.1
1950
1960
1970
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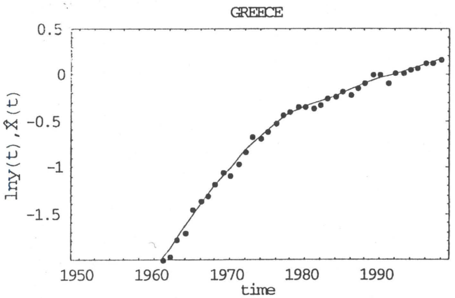
time


GREECE
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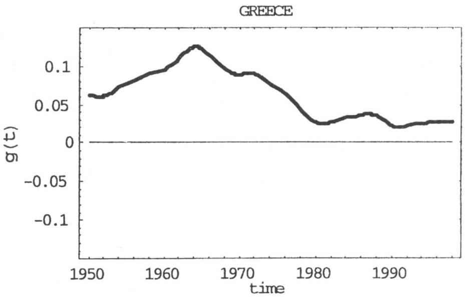
0.5
0
-0.5
-1
-1.5
1950
1960
1970
1980
1990
time


GREECE
g(t)
0.1
0.05
0
-0.05
-0.1
1950
1960
1970
1980
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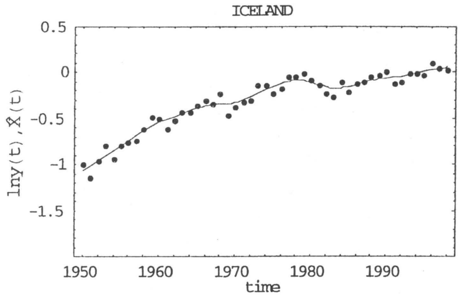
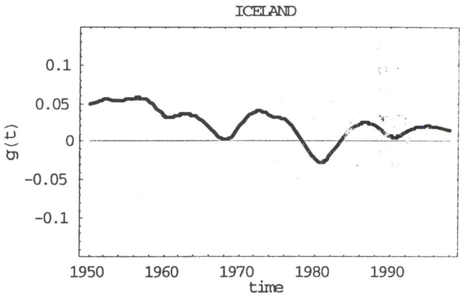
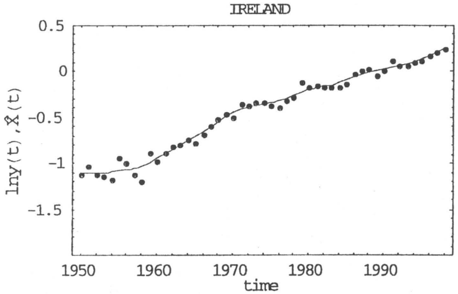
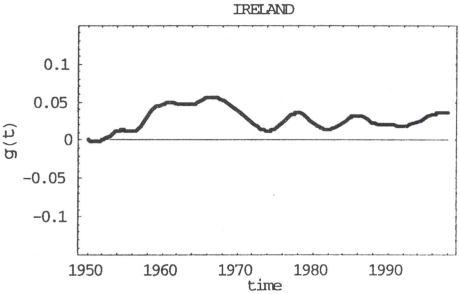
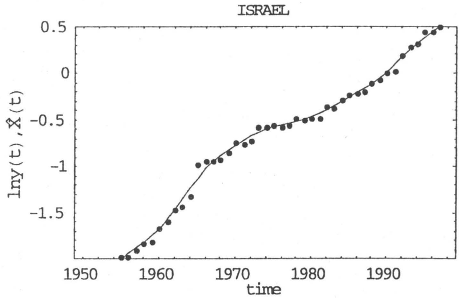
time

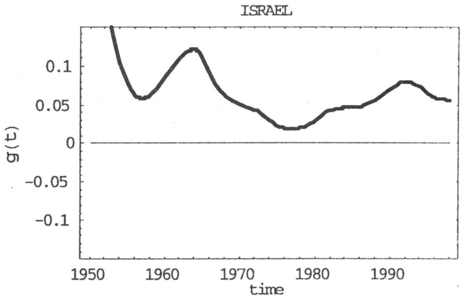

ICELAND
lny(t), X̂(t)
time

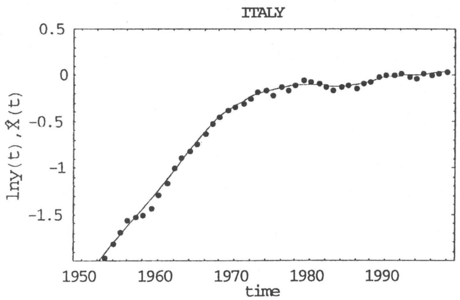

ICELAND
g(t)
time

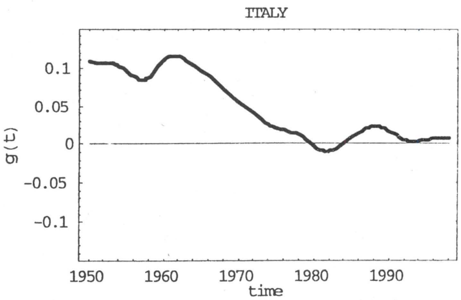

IRELAND
lny(t), X̂(t)
time


IRELAND
g(t)
time

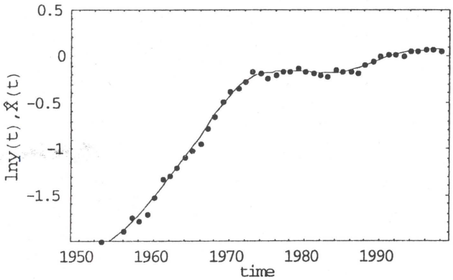

ISRAEL
lny(t), X̂(t)
time


ISRAEL
g(t)
time

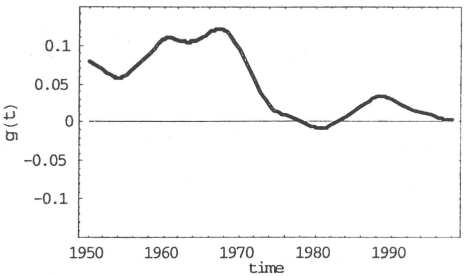

ITALY
lny(t), X̂(t)
time


ITALY
g(t)
time

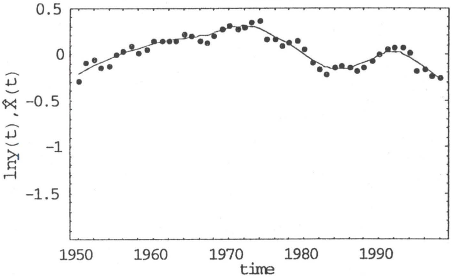
JAPAN



JAPAN



LUXEMBOURG



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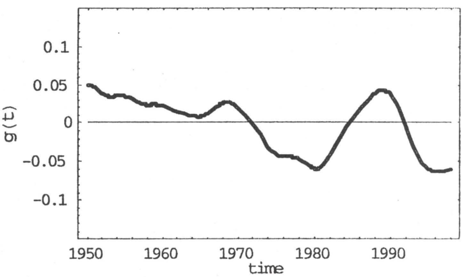
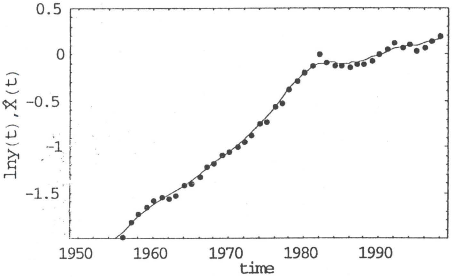


MEXICO



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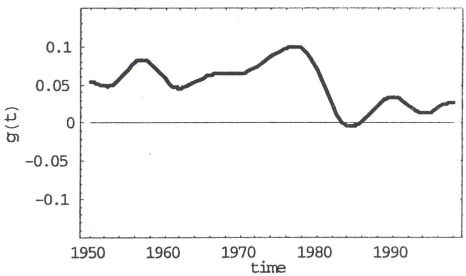
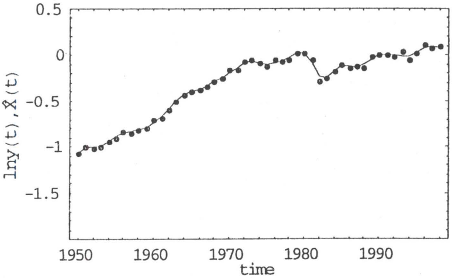


NETHERLANDS



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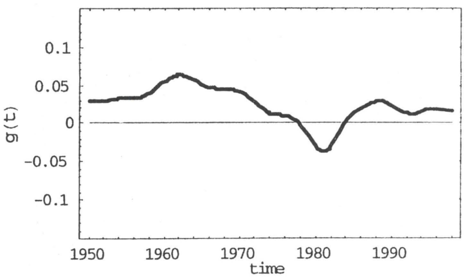

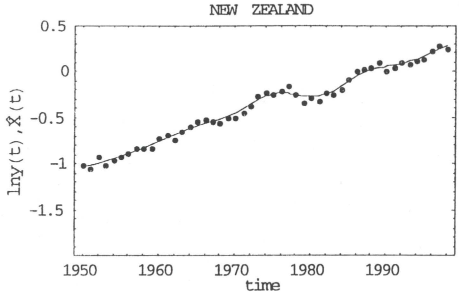
NEW ZEALAND
0.5
0
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lny(t), X̂(t)
1950
1960
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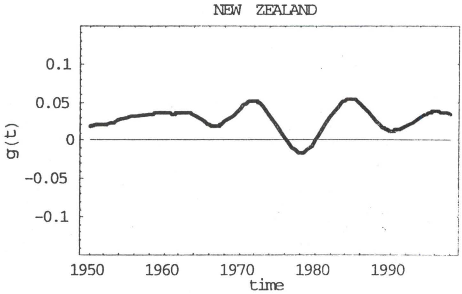
1990
time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g(t)
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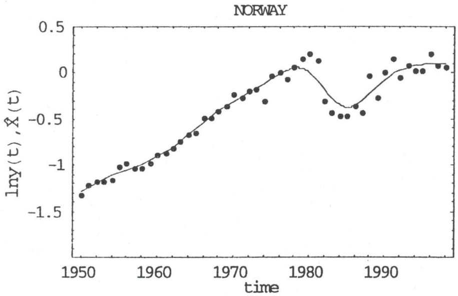
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lny(t), X̂(t)
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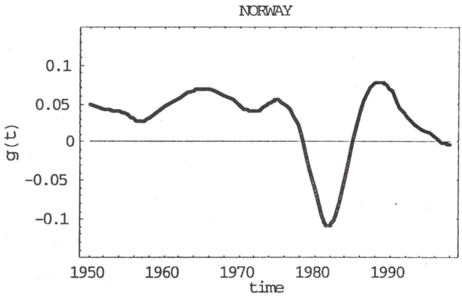
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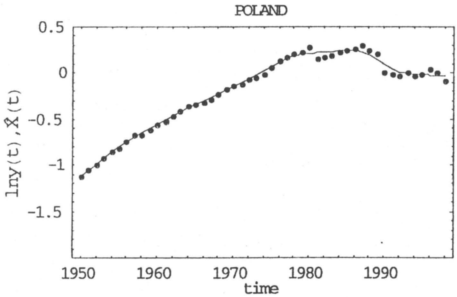
time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lny(t), X̂(t)
1950
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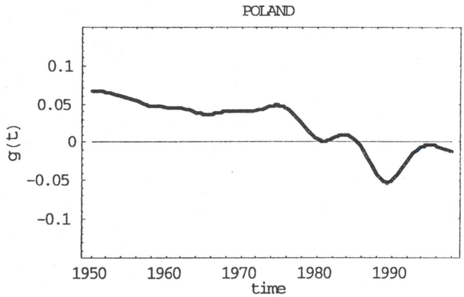
time


POLAND
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g(t)
1950
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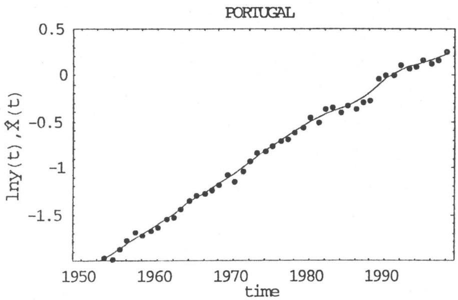
1990
time


PORTUGAL
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-0.5
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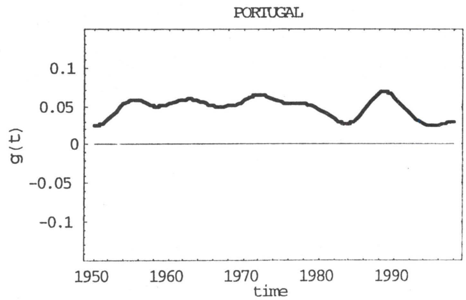
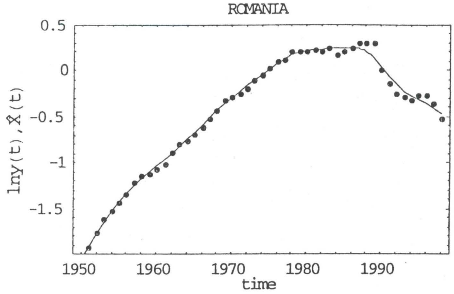
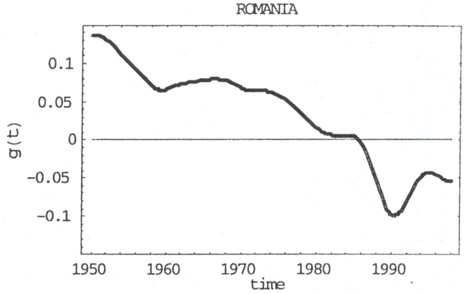
-1.5
lny(t), X̂(t)
1950
1960
1970
1980
1990
time


PORTUGAL
0.1
0.05
0
-0.05
-0.1
g(t)
1950
1960
1970
1980
1990
time


ROMANIA
0.5
0
-0.5
-1
-1.5
lny(t), x̂(t)
1950
1960
1970
1980
1990
time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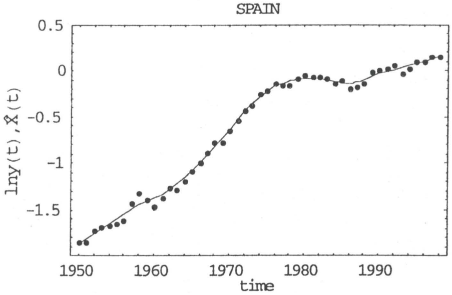
0.1
0.05
0
-0.05
-0.1
g(t)
1950
1960
1970
1980
1990
time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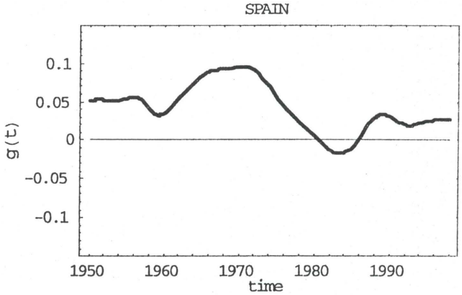
0
-0.5
-1
-1.5
lny(t), x̂(t)
1950
1960
1970
1980
1990
time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0.05
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g(t)
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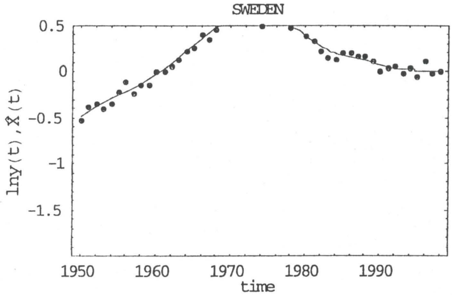
time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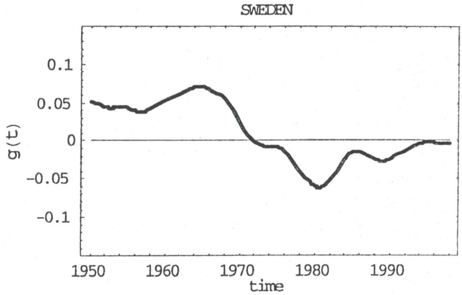
0
-0.5
-1
-1.5
lny(t), x̂(t)
1950
1960
1970
1980
1990
time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g(t)
1950
1960
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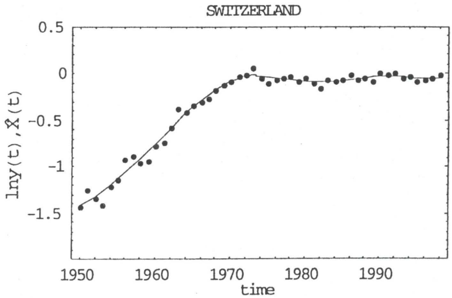
time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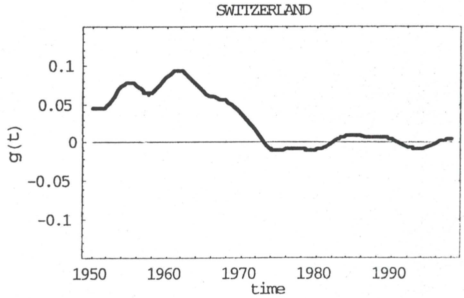
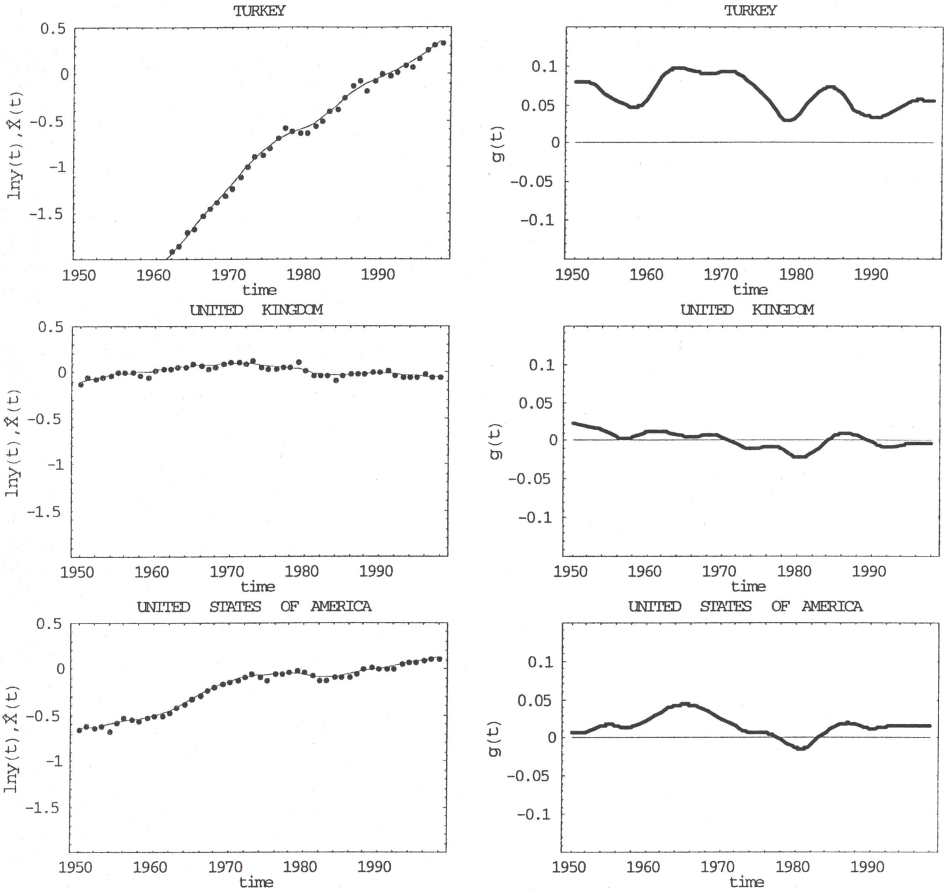
0
-0.5
-1
-1.5
lny(t), x̂(t)
1950
1960
1970
1980
1990
time


SWITZERLAND
0.1
0.05
0
-0.05
-0.1
g(t)
1950
1960
1970
1980
1990
time



5.4 Estimation of parametric models

Although the estimated function $\hat{g}(t)$ in the previous section vary in time, sometimes even quite rapidly, the question is whether they could be approximated by simpler ones, and possibly expressed analytically.

Let us start with examining few curves. Fig. 5.2 contains emission curves $y(t_i)$ and logarithmic curves $Y(t_i) = \ln \frac{y(t_i)}{y(t_0)}$ for the global emission data. Similar data for Poland are depicted on Fig. 5.3. It can be seen that the data evolve along exponential curves, at least in some intervals where the logarithmic curves are close to the straight lines. Even better indication of the exponential relation can be inferred from Fig. 5.4. It shows the dependence of $y_i = y(t_i)$ on $y_{i-1} = y(t_{i-1})$. Referring to (5.1) this dependence should have the form $y_i = (1+g_i)\, y_{i-1}$. The curves indicate that this dependence holds with g_i close to 0. Similar relation can be inferred from the logarithmic data depicted on Fig. 5.5 for which it holds $Y_i = \ln(1+g_i) + Y_{i-1}$.

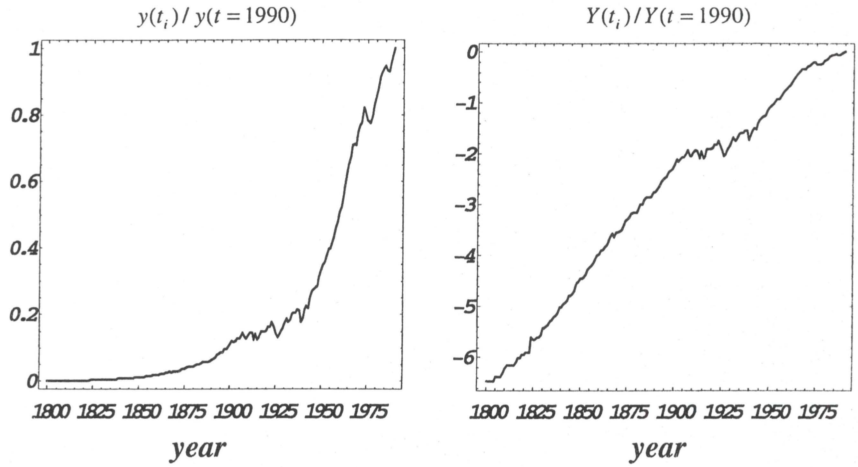


Figure 5.2. **Global data:** CO_2 total emissions (1800 – 1990) [Marland, 1990]

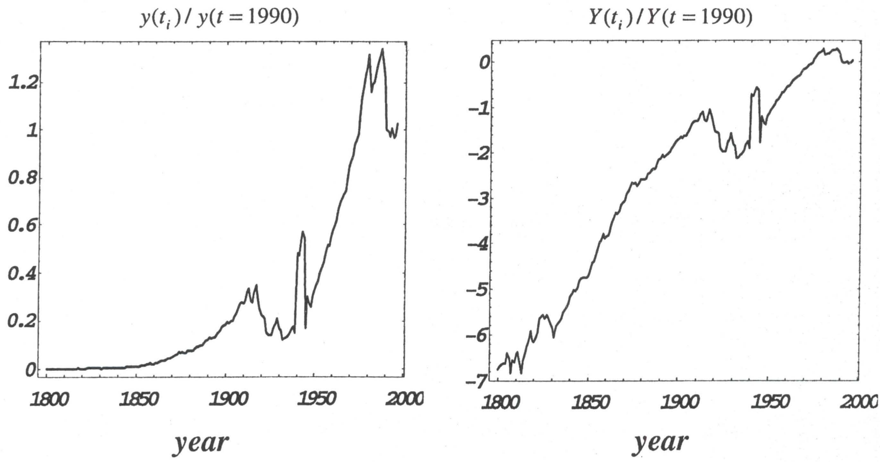
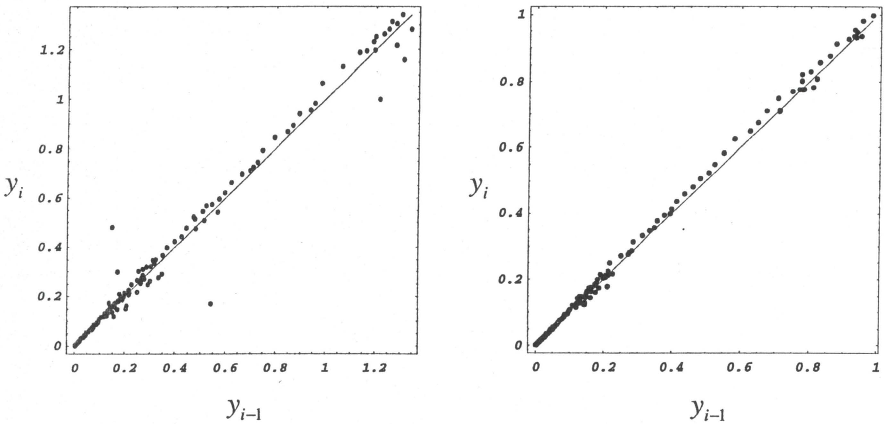


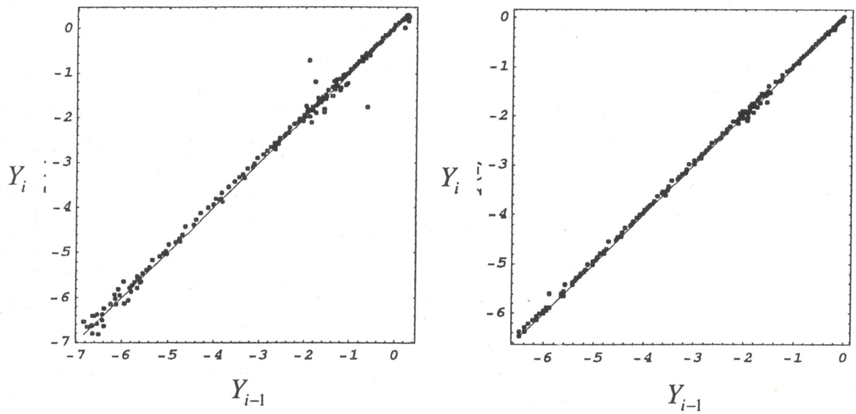
Figure 5.3. **Poland's data:** CO_2 emissions (1800 – 1998)

Phase plane



Poland's data Global data

Figure 5.4. The phase plane portraits of the data



Poland's data Global data

Figure 5.5. The phase plane portraits of the logarithmic data

These observations can be also confirmed by fitting the autoregressive models to the data. These models have the following general form

$$x_i = a_0 + \sum_k a_k x_{i-k} + \varepsilon_i$$

Exemplary results of fitting the model to the data for Argentina and Austria are given in Tables 5.2 and 5.3. The tables present also results of statistics used for model selection. They indicate that the statistically best model is of the first order and has the form

$$\hat{y}_i = 1.023 y_{i-1} \qquad \text{for Argentina}$$

$$\hat{y}_i = 1.002 y_{i-1} \qquad \text{for Austria}$$

or, respectively

$$\hat{Y}_i \cong Y_{i-1} + 0.023 \qquad \text{for Argentina}$$

$$\hat{Y}_i \cong Y_{i-1} + 0.002 \qquad \text{for Austria}$$

with the standard deviation of the parameter estimator equal to 0.005 and 0.011, respectively. Intuitively, the growth coefficient 0.023 seems to be significant for Argentina but the respective value 0.002 for Austria is insignificant

Table 5.2. Results of fitting the autoregressive models to the data for Argentina in the period 1898-1998

	$\hat{a}_i$	$\hat{s}_{a_i}$	*t(Student)*	*p-value*
a_1	**1.023**	**0.005**	**209.0**	**<10^{-2}**
a_0	0.005	0.004	1.4	0.15
a_1	**1.016**	**0.007**	**150.9**	**<10^{-2}**
a_0	0.005	0.004	1.4	0.17
a_1	**1.090**	**0.097**	**11.3**	**<10^{-2}**
a_2	-0.076	0.099	-0.8	0.44

Table 5.3. Results of fitting the autoregressive models to the data for Austria in the period 1898-1998

	$\hat{a}_i$	$\hat{s}_{a_i}$	*t(Student)*	*p-value*
a_1	**1.002**	**0.011**	**88.1**	**<10^{-2}**
a_0	0.035	0.025	1.4	0.16
a_1	**0.985**	**0.017**	**58.8**	**<10^{-2}**
a_0	0.035	0.025	1.4	0.17
a_1	**0.965**	**0.075**	**12.8**	**<10^{-2}**
a_2	0.020	0.076	0.3	0.79

The reason of the insignificance of the estimate for Austria is easy to notice. Although fitting the model to the historical data provides us with a first order constant parameter equation, we can easily notice periods where this simple dependence does not hold. This is particularly visible for the Polish data in the periods of the Great Crisis of 1930s, the 2nd World Word, and the collapse of the communist regime. Neither we can assume this simple constant parameter model in the Kyoto period when reduction of the emission is required.

However, as shown in the Appendix, these simple first order or exponential growth models describe quite well development of data in some intervals. These seem to be periods with constant conditions, mainly economic ones. One can easily distinguish on the figures periods of the 19th century industrial revolution, periods of the World Wars and Great Crisis of 1930th, period of post-war prosperity of 1950th-1960th and the energy shocks of 1970th-1980th. Also smaller ripples can be distinguished and explained, like for example in the case of the Polish transformation period.

6. Assessment of obligation conditions

These interval-wise exponential models allow us to assess possibilities of achieving the Kyoto obligations by different countries by simple extrapolation of the present trends to the future. These rough assessments may be valid until new impetus changes the emission trend. However, quite big volatility of the data in some periods, particularly those related with the decline in emission, may deteriorate the assessment quality even if the overall trend remains. On the other side, these assessments can be calculated repeatedly in consecutive years giving, hopefully, more and more accurate prevision of this possibility.

Let us assume that we are in the year t and define the year 2005 as T_5, $t_0 < t < T_5$.

6.1. Basic relations

Using relation (5.7) we get

$$\hat{X}(T_i) = \int_{t_0}^{T_i} \hat{g}(\tau)d\tau$$

Then, inserting it in the appropriate relations from the subsections 2.1-2.3 we can get the following conditions.

A. Interval uncertainties of both x_0 and $x(T_i)$

$$\int_{t_0}^{T_i} \hat{g}(\tau)d\tau \leq -\delta - (1-\alpha)\frac{\Delta_{0i}}{\hat{x}_0} \tag{6.1}$$

B. Interval uncertainty of x_0 and stochastic uncertainty of $x(T_i)$

$$\int_{t_0}^{T_i} \hat{g}(\tau)d\tau \leq (1-\alpha)\frac{\Delta_0}{\hat{x}_0} - \delta - q_{1-\alpha}\frac{\sigma_{\hat{x}}(T_i)}{x_0} \tag{6.2}$$

C. Stochastic uncertainty of both x_0 and $x(T_i)$

$$\int_{t_0}^{T_i} \hat{g}(\tau)\mathrm{d}\tau \le -\delta - q_{1-\alpha}\frac{\sigma_{\hat{x}}}{\hat{x}_0} \tag{6.3}$$

6.2. Assessment for 2005

In the year t, $t_0 \le t \le T_i$, the past values of the function $\hat{g}(\tau)$ are known and the future can be extrapolated using the model. This leads to the following relations.

A. Interval uncertainties of both x_0 and $x(T_i)$

From (6.1) we have

$$\int_{t}^{T_i} \hat{g}(\tau)d\tau \le -\delta - (1-\alpha)\frac{\Delta_{0i}}{\hat{x}_0} - \int_{t_0}^{t} \hat{g}(\tau)d\tau$$

Then inserting $i = 5$ and $\delta = 0$ we get

$$\int_{t}^{T_5} \hat{g}(\tau)d\tau \le -(1-\alpha)\frac{\Delta_{0i}}{x_0} - \int_{t_0}^{t} \hat{g}(\tau)d\tau \tag{6.4}$$

The above expression allows us to assess in the year t the possibility of fulfilment of the obligations in the year 2005 provided we assume the function $\hat{g}(\tau)$ for $t < \tau \le T_5$ and the value of $\frac{\sigma_{\hat{x}}(T_5)}{x_0}$.

B. Interval uncertainty of x_0 and stochastic uncertainty of $x(T_i)$

Similarly, using (6.2) we get the condition

$$\int_{t}^{T_5} \hat{g}(\tau)d\tau \le -(1-\alpha)\frac{\Delta_0}{\hat{x}_0} - q_{1-\alpha}\frac{\sigma_{\hat{x}}(T_5)}{x_0} - \int_{t_0}^{t} \hat{g}(\tau)d\tau \tag{6.5}$$

for assessing in the year t the possibility of fulfilment of the obligations in the year 2005.

C. Stochastic uncertainty of both x_0 and $x(T_i)$

In analogy to two earlier cases we have

$$\int_{t}^{T_5} \hat{g}(\tau)d\tau \le -q_{1-\alpha}\frac{\sigma_{\hat{x}}}{\hat{x}_0} - \int_{t_0}^{t} \hat{g}(\tau)d\tau \tag{6.6}$$

6.3. Assessment for 2008 - 2012

Similarly, we can assess the possibility of fulfilment of the obligations in the years 2008-2012.

A. Interval uncertainties of both x_0 and $x(T_i)$

The condition to assess the possibilities of fulfilment of the obligations

$$\int_{t}^{T_i} \hat{g}(\tau)\,d\tau \le -\delta-(1-\alpha)\frac{\Delta_{0i}}{\hat{x}_0}-\int_{t_0}^{t}\hat{g}(\tau)\,d\tau \tag{6.7}$$

B. Interval uncertainty of x_0 and stochastic uncertainty of $x(T_i)$

The condition to assess the possibilities of fulfilment of the obligations

$$\int_{t}^{T_i} \hat{g}(\tau)\,d\tau \le -(1-\alpha)\frac{\Delta_0}{\hat{x}_0}-\delta-q_{1-\alpha}\frac{\sigma_{\hat{x}}(T_i)}{x_0}-\int_{t_0}^{t}\hat{g}(\tau)\,d\tau \tag{6.8}$$

C. Stochastic uncertainty of both x_0 and $x(T_i)$

The condition to assess the possibilities of fulfilment of the obligations

$$\int_{t}^{T_i} \hat{g}(\tau)\,d\tau \le -\delta-q_{1-\alpha}\frac{\sigma_{\hat{x}}}{\hat{x}_0}-\int_{t_0}^{t}\hat{g}(\tau)\,d\tau \tag{6.9}$$

6.4. Fulfilment time

We can be interested to predict, under present knowledge of data, when a country is able to fulfil the obligations. Let us call the time T_v when this is achieved *the fulfilment time.* At time t the basic equations for the verification time T_v for the three approaches discussed earlier are as follows.

A. Interval uncertainties of both x_0 and $x(T_i)$

$$\int_{t}^{T_v} \hat{g}(\tau)d\tau = -\delta-(1-\alpha)\frac{\Delta_{0i}}{\hat{x}_0}-\hat{X}(t)=d_A(t) \tag{6.10}$$

B. Interval uncertainty of x_0 and stochastic uncertainty of $x(T_i)$

$$\int_{t}^{T_v} \hat{g}(\tau)\,d\tau = -(1-\alpha)\frac{\Delta_0}{\hat{x}_0}-\delta-q_{1-\alpha}\frac{\sigma_{\hat{x}}(t)}{x_0}-\hat{X}(t)=d_B(t) \tag{6.11}$$

C. Stochastic uncertainty of both x_0 and $x(T_i)$

$$\int_{t}^{T_v} \hat{g}(\tau)\,d\tau = -\delta-q_{1-\alpha}\frac{\sigma_{\hat{x}}}{\hat{x}_0}-\hat{X}(t)=d_C(t) \tag{6.12}$$

We consider two simple cases.

i. Constant $\hat{g}(\tau)$

From the historical data we see that the values of $\hat{g}(\tau) = \frac{d\hat{X}(\tau)}{d\tau}$ tend to be approximately constant in time intervals. If the obligations are not fulfilled yet, then for the positive $\hat{g}$ in the future they will be never met. Therefore we assume that $\hat{g}$ is negative in the future.

Thus, if the future values $\hat{g}(\tau)$ are assumed to be constant in time, i.e. $\hat{g}(t) = \hat{g} < 0$, then the fulfilment time T_v calculated in the time t can be expressed for the three approaches A, B, C as

$$T_v = t + \frac{d_W(t)}{\hat{g}}, \qquad W = A, B, C \tag{6.13}$$

Notice that by the assumption $d_W(t) < 0$.

ii. Linear $\hat{g}(\tau)$

If the future values of $\hat{g}(\tau)$ are assumed to change linearly in time, for example as $\hat{g}(\tau) = b\tau + a$, then the fulfilment time T_v calculated in the time t for the three approaches A, B, C can be found as a solution of the following quadratic equation

$$\frac{1}{2}bT_v^2 + aT_v - \frac{1}{2}bt^2 - at + d_W = 0, \qquad W = A, B, C$$

and is the least value satisfying the condition $T_v \geq t$.

The above equation can be transformed to the following form

$$\frac{1}{2}b(T_v - t)^2 + (bt + a)(T_v - t) - d_W(t) = 0$$

We observe that $bt + a = \hat{g}(t)$ and denote $T_v - t = \theta$. Thus, we have to solve the equation

$$\frac{1}{2}b\theta^2 + \hat{g}(t)\theta - d_w(t) = 0$$

The positive solution of this equation is

$$\theta = \frac{\sqrt{\hat{g}^2(t) + 2bd_W(t)} - \hat{g}(t)}{b} = \frac{2d_W(t)}{\hat{g}(t)} \frac{1}{1 + \sqrt{1 + \frac{2bd_W(t)}{\hat{g}^2(t)}}}$$

so the solution of the original equation is

$$T_v = t + \frac{d_W(t)}{\hat{g}(t)} \frac{1}{\frac{1}{2}\left(1 + \sqrt{1 + \frac{2bd_W(t)}{\hat{g}^2(t)}}\right)} = t + \frac{d_W(t)}{\hat{g}(t)}\zeta(t) \tag{6.14}$$

The above formula generalises that of (6.13). If $\frac{2bd_W(t)}{\hat{g}^2(t)} \cong 0$, then (6.14) is close to (6.13).

Because we assumed that $d_W(t) < 0$, then if $b < 0$ we get $\zeta(t) < 1$ and T_v is shorter than that calculated from (6.13). If $b > 0$, then T_v is longer.

iii. Higher order $\hat{g}(\tau)$

Let us notice that we can get even better approximations than the above using polynomials of higher order. If we denote $\hat{G}(t) = \int \hat{g}(t)dt$, then the solution must satisfy the equation

$$\hat{G}(T_v) - \hat{G}(t) - d_W(t) = 0$$

Expanding $\hat{G}(T_v)$ in the Taylor series around t we get

$$\hat{g}(t)(T_v - t) + \frac{1}{2}\hat{g}'(t)(T_v - t)^2 + \frac{1}{6}\hat{g}''(t)(T_v - t)^3 + \ldots - d_W(t) = 0$$

Taking k first terms of this expansion we get the kth order algebraic equation. Its solution will give us the kth order approximation of the solution of a chosen equation (6.10)-(6.12).

7. Conclusions

In the paper the problem of verification of the Kyoto obligations is discussed. The present knowledge makes obvious to us that verification of the obligations cannot be done when uncertainty of the reported values is not taken into account. This paper addresses this problem and proposition of a solution is given.

The main idea of our proposition concentrates in replacing the reduction rate by a linear combination of the reduction and uncertainty. The exact proportions proposed are related with the risk that the real emission has not satisfied the obligations.

Although this idea is elaborated in many details, some of them are still open to be decided yet. One is the definition of uncertainty measure. One possibility, used mainly up to know, is to aggregate uncertainties of all the partial emissions. An alternative way is proposed in the paper. Namely, the uncertainty is related to the empirical distribution of the error of smoothing the observations. Both of these approaches have defects. The methodology of the uncertainty aggregation is still not generally recognised and, moreover, its applications may involve big uncertainty estimates. On the other side, the empirical approach proposed in the paper heavily relies on the assumption of the smoothness of the emission process. Volatility of observations is, however, related not only to the observation errors but also to such factors as changing weather conditions and economic situation of the country. Still, the calculations performed for the fossil fuels indicate that the empirical approach gives reasonable estimates, comparable to the aggregate ones. This leads us to believe that these estimates may be relevant as estimates of uncertainty, if only observations do not include systematic errors. Whether the situation is similar for the full carbon accounting is not known yet.

Other unsolved problem is in understanding the reduction level. We propose to replace it by a linear combination of the reduction and uncertainty. Thus, we can leave the reduction level unchanged and in fact require overshooting the obligation target by a part of the uncertainty level. Alternatively, we can agree to make the reduction level smaller and allow for adding a part of the uncertainty level in order to get the combination equal to the original reduction. Our suggestion goes towards the latter solution. That is, we propose to make the reduction level smaller by a "reasonable" level of uncertainty. This way, countries that have smaller uncertainty than the "reasonable" one, could even end with the reported value of emission not reaching the Kyoto obligation target. On the other way, countries with big uncertainty in the reported values will have to overshoot the reduction level.

Finally, acceptation of the idea of verification proposed in the paper makes it necessary to change the emission trading rules. When bargaining the price, the buyer should combine the reduction of emission with uncertainty of its reporting, because both of them will count in the final verification of the Kyoto obligations. The paper contains a proposition of solving this problem.

Apart of these static state considerations we look at the dynamics of the process, in order to gather knowledge helpful in verification and prediction. This is done in two steps. In the first one the smoothing splines are used to estimate the standard deviation of the observation errors. In the second one, a model describing evolution of the observation is proposed. The model is of the form of a simple exponential growth function with constant coefficients in time periods, jumping from one period to another. This model can be useful to roughly predict the emission in the commitment period providing there is no jump in the growth coefficient in the meantime, see the figures in the Appendix. This last assumption is crucial, as prediction of the jumping times and levels seem to be a rather difficult task.

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APPENDIX

POLAND 1870 - 1998

Estimated parameters and model verification statistics for the logarithmic regression models.

YEARS 1870-1914

	Estimate	SE	TStat	PValue
1	-29.9226	0.535618	-55.8655	0.
t	0.0179755	0.000283089	63.4976	0.

E.Var. 0.000608259

YEARS 1918-1938

	Estimate	SE	TStat	PValue
1	25.2475	5.74289	4.3963	0.000310319
t	-0.0109162	0.00297866	-3.66481	0.00164665

E.Var. 0.00683176

YEARS 1947-1978

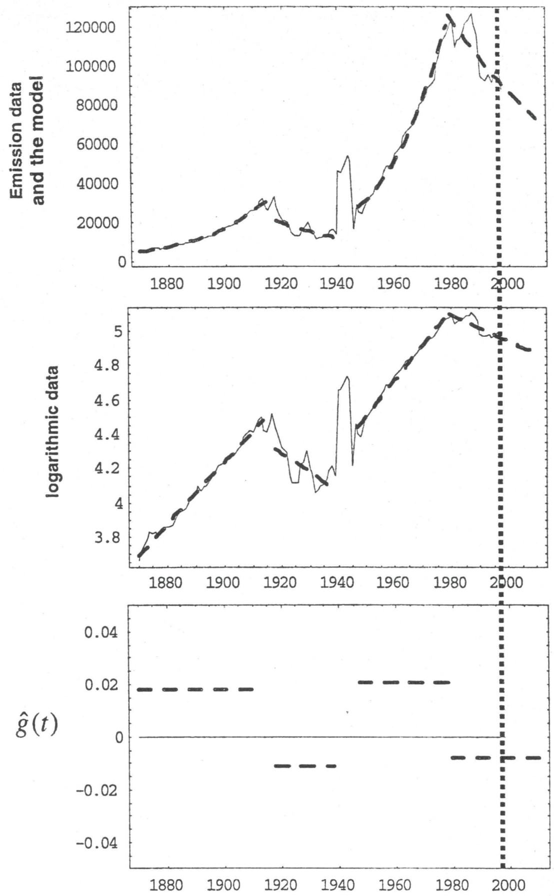
	Estimate	SE	TStat	PValue
1	-35.7734	0.935288	-38.2486	0.
t	0.0206553	0.000476575	43.3411	0.

E.Var. 0.000619593

YEARS 1979-1998

	Estimate	SE	TStat	PValue
1	20.1063	2.60431	7.72036	4.05251×10^{-7}
t	-0.00758245	0.00130968	-5.78953	0.0000173922

E.Var. 0.00114065



AUSTRALIA 1870 - 1998

Estimated parameters and model verification statistics for the logarithmic regression models.

YEARS 1870-1914

	Estimate	SE	TStat	PValue
1	-65.4169	1.4499	-45.1182	0.
t	0.0361896	0.000766314	47.2255	0.

E.Var. 0.00445713

YEARS 1915-1934

	Estimate	SE	TStat	PValue
1	-4.22192	4.06755	-1.03795	0.313035
t	0.00418793	0.00211355	1.98147	0.0630237

E.Var. 0.00297062

YEARS 1935-1979

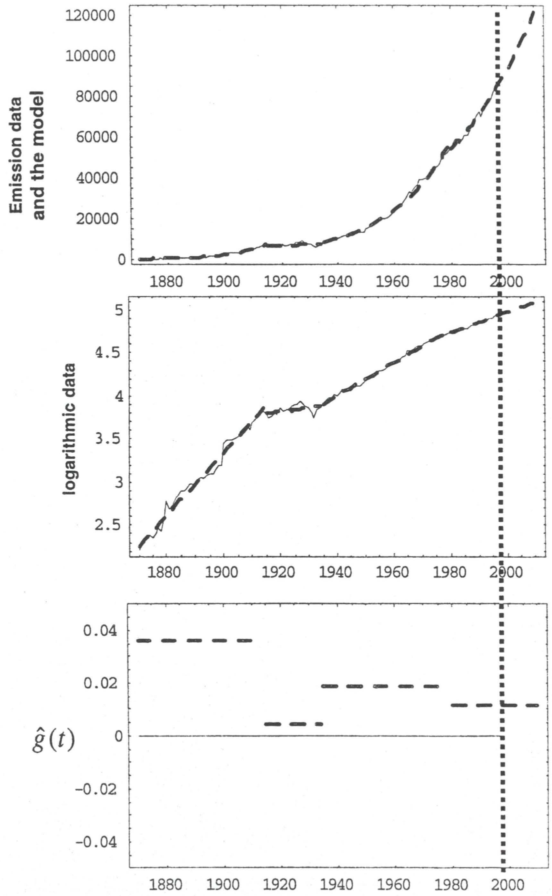
	Estimate	SE	TStat	PValue
1	-32.8581	0.487671	-67.3777	0.
t	0.0190004	0.000249188	76.2495	0.

E.Var. 0.000471297

YEARS 1980-1998

	Estimate	SE	TStat	PValue
1	-18.5516	1.00012	-18.5494	1.02063×10^{-12}
t	0.0117601	0.000502822	23.3882	2.28706×10^{-14}

E.Var. 0.000144113



AUSTRIA 1870 - 1998

Estimated parameters and model verification statistics for the logarithmic regression models.

YEARS 1870 -1914

	Estimate	SE	TStat	PValue
1	-34.0125	2.05615	-16.5418	0.
t	0.0199313	0.00108673	18.3406	0.

E.Var. 0.00896373

YEARS 1920 -1939

	Estimate	SE	TStat	PValue
1	37.439	9.09654	4.11574	0.000649088
t	-0.0175225	0.00471443	-3.71679	0.00157852

E.Var. 0.0147802

YEARS 1945 -1974

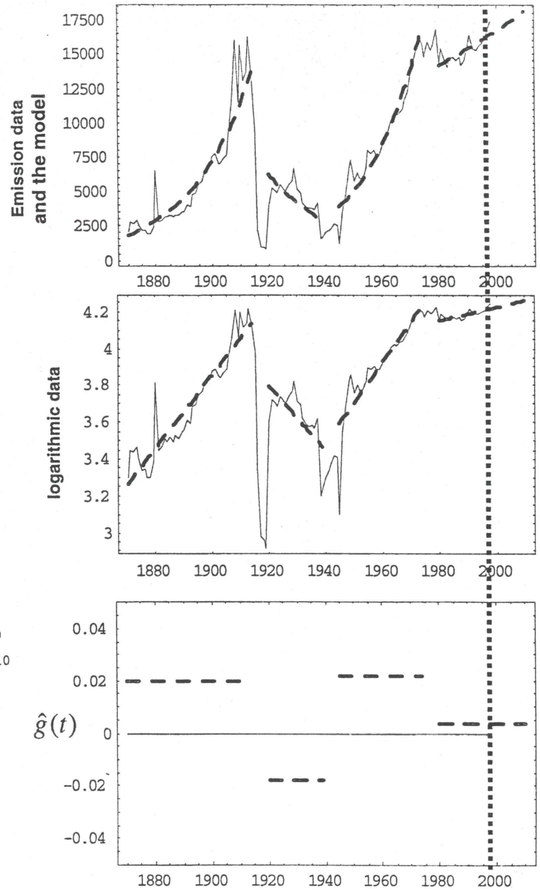
	Estimate	SE	TStat	PValue
1	-39.0009	4.59309	-8.49121	3.12151×10^{-9}
t	0.0219002	0.00234399	9.34312	4.21976×10^{-10}

E.Var. 0.0123484

YEARS 1984 -1998

	Estimate	SE	TStat	PValue
1	-5.33736	1.76231	-3.02861	0.0096906
t	0.00478445	0.000885136	5.40532	0.000120043

E.Var. 0.000219371



UNITED STATES OF AMERICA
1870 - 1998

Estimated parameters and model verification statistics for the logarithmic regression models.

YEARS 1870–1914

	Estimate	SE	TStat	PValue
1	-45.7851	0.770813	-59.3985	0.
t	0.0268672	0.000407397	65.9485	0.

E.Var. 0.00125973

YEARS 1915–1940

	Estimate	SE	TStat	PValue
1	5.08135	2.70856	1.87603	0.0728651
t	0.000290048	0.00140521	0.206409	0.838212

E.Var. 0.00288787

YEARS 1941–1979

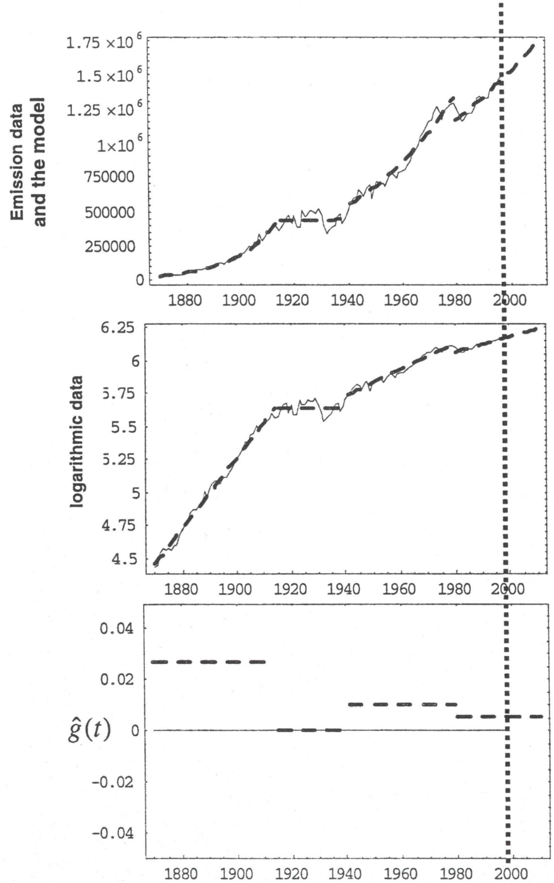
	Estimate	SE	TStat	PValue
1	-13.6495	0.690558	-19.7659	0.
t	0.00999051	0.00035232	28.3563	0.

E.Var. 0.000613199

YEARS 1980–1998

	Estimate	SE	TStat	PValue
1	-5.38114	1.11315	-4.83417	0.000155248
t	0.00578105	0.000559649	10.3298	9.6028×10^{-9}

E.Var. 0.000178528



CANADA 1870 – 1998

Estimated parameters and model verification statistics for the logarithmic regression models.

YEARS 1870-1914

	Estimate	SE	TStat	PValue
1	-70.7008	2.42254	-29.1845	0.
t	0.0392146	0.00128038	30.6272	0.

E.Var. 0.0124429

YEARS 1918-1939

	Estimate	SE	TStat	PValue
1	2.2835	3.14261	0.726627	0.475874
t	0.00107509	0.00162955	0.659746	0.516943

E.Var. 0.00235139

YEARS 1945-1979

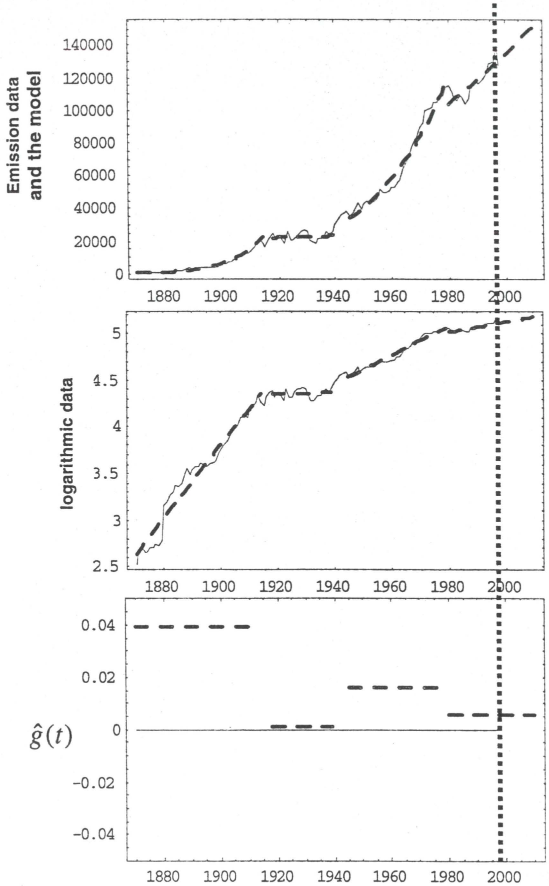
	Estimate	SE	TStat	PValue
1	-25.9854	0.976209	-26.6187	0.
t	0.0156893	0.000497551	31.5331	0.

E.Var. 0.00088378

YEARS 1980-1998

	Estimate	SE	TStat	PValue
1	-5.7115	1.61099	-3.54534	0.00248709
t	0.0054192	0.000809947	6.69081	3.79577×10^{-6}

E.Var. 0.000373928



FRANCE (INCLUDING MONACO) 1870-1998

Estimated parameters and model verification statistics for the logarithmic regression models.

YEARS 1870-1914

	Estimate	SE	TStat	PValue
1	-15.4856	0.67664	-22.886	0.
t	0.0105217	0.000357624	29.4211	0.

E.Var. 0.000970722

YEARS 1918-1929

	Estimate	SE	TStat	PValue
1	-54.8631	6.14916	-8.92204	4.47441×10^{-6}
t	0.0309564	0.00319685	9.6834	2.13301×10^{-6}

E.Var. 0.00146144

YEARS 1945-1979

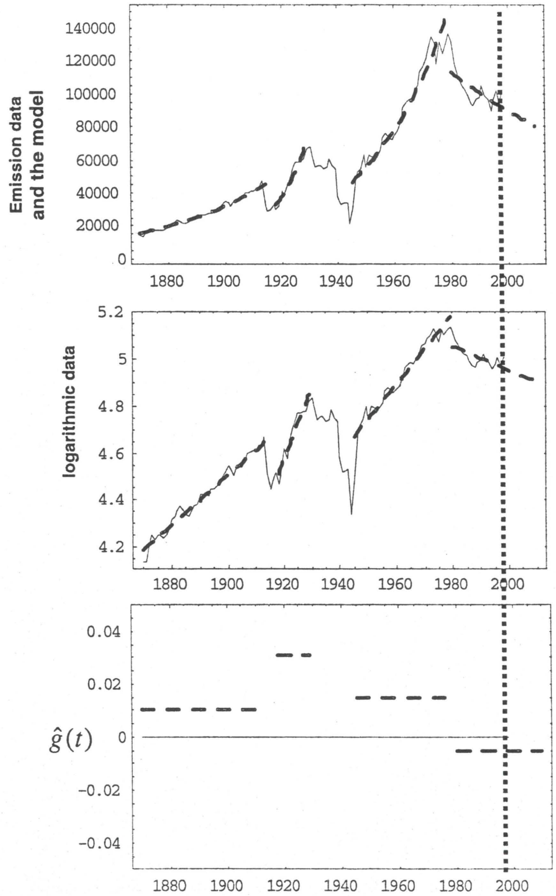
	Estimate	SE	TStat	PValue
1	-24.442	1.39318	-17.544	0.
t	0.0149665	0.000710073	21.0774	0.

E.Var. 0.00180001

YEARS 1980-1998

	Estimate	SE	TStat	PValue
1	14.9833	2.53428	5.91227	0.0000170544
t	-0.00501543	0.00127414	-3.93632	0.00106445

E.Var. 0.00092536



JAPAN 1900 - 1998

Estimated parameters and model verification statistics for the logarithmic regression models.

YEARS 1900 -1940

	Estimate	SE	TStat	PValue
1	-32.496	1.38183	-23.5166	0.
t	0.0191317	0.00071969	26.5833	0.

E.Var. 0.00297305

YEARS 1945-1974

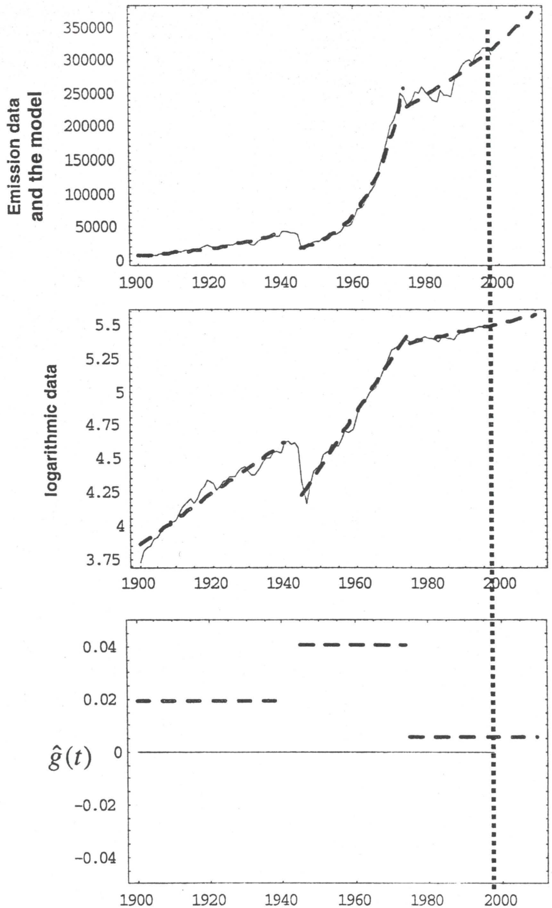
	Estimate	SE	TStat	PValue
1	-75.012	1.85253	-40.4916	0.
t	0.0407414	0.000945402	43.0943	0.

E.Var. 0.00200878

YEARS 1975-1998

	Estimate	SE	TStat	PValue
1	-6.45422	1.21319	-5.32004	0.0000243589
t	0.00598255	0.000610713	9.796	1.7541×10^{-9}

E.Var. 0.000428916



NETHERLANDS 1980 - 1998

Estimated parameters and model verification statistics for the logarithmic regression models.

YEARS 1880-1914

	Estimate	SE	TStat	PValue
1	-23.8167	0.875393	-27.2069	0.
t	0.0144343	0.000461455	31.2799	0.

E.Var. 0.000760199

YEARS 1920-1939

	Estimate	SE	TStat	PValue
1	-20.8725	3.81226	-5.4751	0.0000335984
t	0.0128839	0.00197577	6.52098	3.94722×10^{-6}

E.Var. 0.00259593

YEARS 1945-1974

	Estimate	SE	TStat	PValue
1	-40.5533	3.273	-12.3903	6.97553×10^{-13}
t	0.0228893	0.00167031	13.7036	6.12843×10^{-14}

E.Var. 0.00627036

YEARS 1980-1998

	Estimate	SE	TStat	PValue
1	-7.60261	2.98052	-2.55077	0.0206771
t	0.00612994	0.00149849	4.09073	0.00076184

E.Var. 0.00127993