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modification of the sequential
bilateral trading scheme**

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Convergence of environmental market to the equilibrium: Modification of the sequential bilateral trading scheme

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Abstract

A modified version of the sequential bilateral trading scheme is proposed - it includes selection of the best partner. Market dynamics is analysed. Simulation results performed on the carbon market show that the modified procedure converges to the equilibrium point much faster than the original scheme. Moreover, the trajectories of permits traded are characterized with smaller variance. Generally, market can be described as a more 'compact'.

1 Introduction

The sequential bilateral trading scheme has been proposed by Ermoliev *et al.* [1] to analyse the process of price formation at the market of tradeable emission permits. Under the designed system, sequential bilateral transactions proved to converge to the market equilibrium, where a minimum total costs of pollution control are reached. It offers a feasible tool to analyse dynamics of a market where transactions are made at changing non-equilibrium

prices.

The procedure has been successfully applied in [3] to the simulation of carbon market in the context of the Kyoto protocol. Special attention has been given to the problem of imperfect monitoring (reporting) of greenhouse gas emission.

In [6] Klaassen et al. make use of experimental economics to examine dynamics of global carbon permit trade. Three experiments have been performed: sequential bilateral trading, single bid auction and Walrasian repetitive auction. All the experiments showed that the market price converges to the equilibrium price. Moreover, all the settings managed to capture significant part of the potential cost savings stemming from emission trading. However, in the case of sequential bilateral trading final distribution of gains was considerably different than under perfect equilibrium.

The role of bilateral transactions in the process of reaching market cost-efficiency was also highlighted in earlier works [5].

In the present paper we analyse some convergence properties of the sequential bilateral trading procedure. In [1] the designed system assumed that at each step two partners were picked at random. They were to agree on price and quantity in bilateral negotiations. We relax this condition in such a way that we choose larger group of parties i.e. we allow more than two parties to meet and negotiate. One of them is appointed to play the leading role - it chooses among the group its best partner to trade with.

The idea behind this modification was to capture one more feature of the market - the fact that participants try to make at least limited survey of the market prior to taking their own decision.

The modification we introduce does not influence the fundamental rules which have led the sequential bilateral scheme to the least cost solution. After

the best counterpart has been appointed a transaction itself is performed bilaterally following established rules. Also, our altered version should not be misled with multilateral trades in deposition permits as outlined in [1]. The purpose of this exercise was to explore on how trade dynamics can change after introducing a kind of 'enhanced knowledge on market'.

The paper is structured as follows. Section 2 recalls the idea of sequential bilateral trade. Section 3 describes in details rules governing selection of the best partner. Results of numerical simulation are presented in Section 4 and Section 5 provides conclusions.

2 Sequential bilateral trading scheme

Below we present how the system of sequential bilateral trades is organised. For details see [1], where also mathematical proof of convergence is given in Appendix 1.

Considered is the case of pollutants (treated as) uniformly dispersed. It means that the location of the source in relation to receptor does not matter.

The following notation will be of use:

$i = (1, 2, \dots, N)$ – emission sources;

x_i – emissions level;

$f_i(x_i)$ – cost of keeping (reducing) emission at a certain level;

q_i – constraint on emission level specified for each emitter (initial allocation of permits).

The task is to minimize total costs of emission reduction and to meet prescribed emission constraints:

$$\min_{x_i} \sum_i f_i(x_i) \tag{1}$$

$$\text{s.t. } x_i \leq q_i \text{ for all } i = 1, \dots, N$$

The instrument suitable to achieve emission constraints q_i jointly in decentralized manner is tradeable emission. Then the problem is transformed to [7]:

$$\begin{aligned} & \min_{x_i} \sum_i f_i(x_i) & (2) \\ & \text{s.t. } \sum_i (x_i - q_i) = 0 \end{aligned}$$

In order to accomplish the task cost effectively on a basis of bilateral transactions, starting from disequilibrium, the abovementioned sequential bilateral trading scheme is employed.

At each step two sources with differing marginal cost meet at random. The idea behind is that an emitter with relatively high marginal cost of emission reduction is interested in buying permits, and conversely, the low cost source is willing to sell its permits. When transaction is made the seller reduces its emission with the same quantity as the purchaser increases its emission. However, permit purchaser decreases its total costs more than seller increases its own total costs. What follows is that aggregate total costs for all participants will be diminished after any single agreement. Next, another couple of parties is picked and the process is repeated. It will go on as long as there are two or more parties with differing marginal costs.

Assuming there are no transaction costs and no strategical behaviour among participants the process is proved to converge to the equilibrium with total costs of emission reduction minimized. The marginal cost will be equalized among parties and it will reflect market permit price.

3 Modification of the process - Selection of the best counterpart

A modification we introduce is aimed to limit randomness involved in selecting bilateral counterparts. In the original system two parties are picked at each step. In our setup more than two parties are picked each time. One out of the group, so called acting player, chooses his best partner to trade with. Then, the exchange of permits is made bilaterally between them according to the original system, described in Section 2.

To explain how we determine selection of the best partner, let us consider the illustrative simplified case of picking randomly three parties: A, B, C . Emission source A is appointed (also randomly) as the acting player.

Assume functions of emission reduction cost $f_i(x_i) > 0$ are continuously differentiable, convex and decreasing. Hence marginal costs $f'_i(x_i) \leq 0$ are negative and increasing for $i = 1, \dots, N$. Emission levels $x_i^k = (x_1^k, \dots, x_N^k)$ at each step k are positive. Amount of permits traded at step k is denoted as $\Delta^k > 0$.

Consider the first scenario:

$$|f'_A(x_A^k)| < |f'_B(x_B^k)| < |f'_C(x_C^k)|$$

Since source A has the lowest (negative) marginal costs it will be definitely seller of permits. Now it has to choose between potential buyers: sources B and C . Generally, higher the difference of marginal costs, better the perspective for profitable transaction for both partners. It seems reasonable to assume that party A will be rather interested in exchanging permits with this party which offers wider gap of marginal costs. The source C will be chosen over the source B , i.e.

$$x_A^{k+1} = x_A^k - \Delta^k, \quad x_C^{k+1} = x_C^k + \Delta^k$$

When the acting player A can become both seller or buyer:

$$|f'_B(x_B^k)| < |f'_A(x_A^k)| < |f'_C(x_C^k)|$$

it will compare the difference in marginal costs:

$$\begin{aligned} x_A^{k+1} &= x_A^k - \Delta^k, \quad x_C^{k+1} = x_C^k + \Delta^k \\ \text{if } |f'_C(x_C^k)| - |f'_A(x_A^k)| &> |f'_A(x_A^k)| - |f'_B(x_B^k)|, \\ x_A^{k+1} &= x_A^k + \Delta^k, \quad x_B^{k+1} = x_B^k - \Delta^k \text{ otherwise.} \end{aligned}$$

Further, we have to tackle the problem of adequate amount of traded permits Δ^k . In general, for any two sources i and j with different marginal cost ($f'_i(x_i^k) \neq f'_j(x_j^k)$) the precondition for a profitable transaction to be made is to fulfil the inequality (see [1]):

$$f_i(x_i^{k+1}) + f_j(x_j^{k+1}) < f_i(x_i^k) + f_j(x_j^k) \quad (3)$$

Aggregate costs for both sources i and j after transaction has to be lower than their aggregate cost before transaction. However, when Δ^k is too large the condition may not hold and a smaller Δ has to be tried (for a proposition of adequate procedure see [2]).

In our case of more than two sources negotiating, we adopt the following rule. Let us return to our example with three sources A, B, C . When the leading party A is unable to trade with source B , then it will check source C . Provided the formula (3) holds, an agreement will be made with source C . Unless exchange of permits is unfeasible with any party in the group, the Δ^k is decreased accordingly.

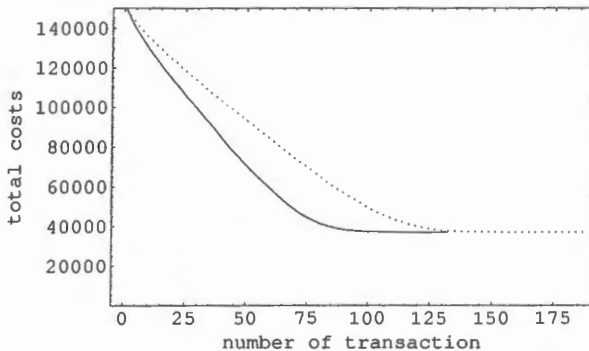
4 Simulation results

The new procedure has been examined with data on carbon reduction in perspective of the Kyoto protocol. Major industrialized Kyoto participants are grouped into five regions: United States, OECD Europe, Japan, Canada/Australia/New Zealand and finally Eastern Europe/Former Soviet Union. Data on abatement cost functions and the Kyoto endowments come from [4].

Two kinds of simulations have been done: following the original scheme from [1] and following the presented procedure with (limited) selection of a partner. In the modified version, we pick three parties at each step.

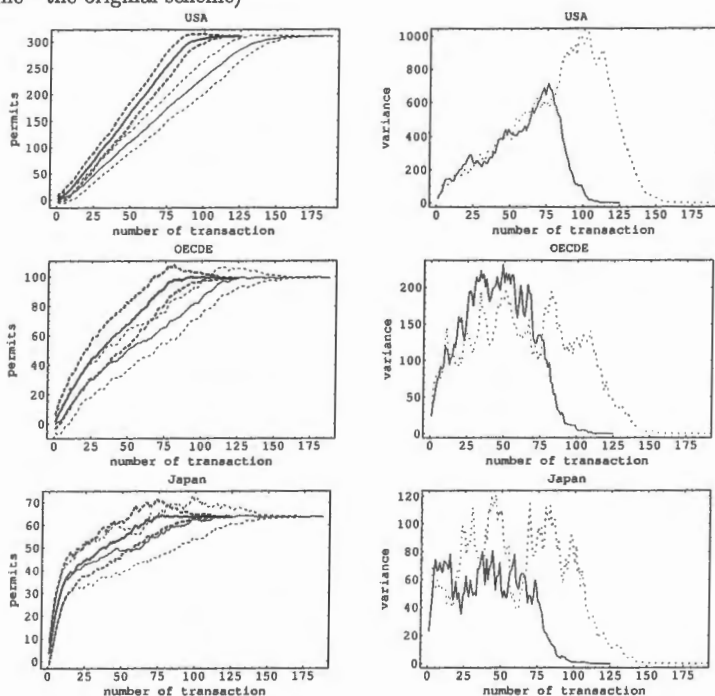
Figure 1 compares in both cases total abatement costs $\sum_{i=1}^N f_i(x_i)$ at each k bilateral agreement. To get more meaningful results the simulation has been run 50 times and the figure reflects average total costs for each transaction. The figure clearly demonstrates that the modified procedure (solid line) reaches the equilibrium level much faster than the original procedure (dotted line) does.

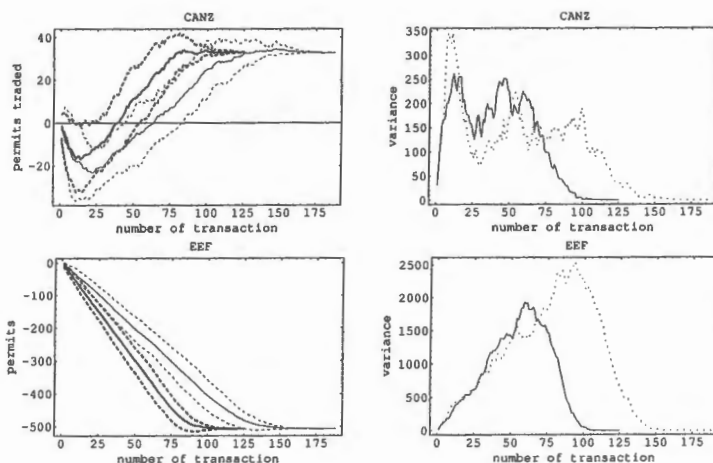
Figure 1: Total costs (MUS\$) in subsequent transactions: the modified version (solid line) and the original scheme (dotted line)



The starting point for the procedure is that each region commits to his emission constraint and has no single permit bought/sold. Consecutive bilateral transactions allow him to find his optimal amount of permits. We compare how the amount of permits held by each participant evolves over time before it reaches the optimal level.

Figure 2: Left panel: Average number of permits in MtC/year (solid lines) and its standard deviation (dashed lines) as the trade goes on in 50 simulations. Bolded lines refer to the modified algorithm; normal lines depict the original setup. Right panel: Variance of permits traded (MtC/year) as a function of number of transaction (Solid line - the modified version; dotted line - the original scheme)





Left-hand-side plots in Figure 2 show average number of permits in 50 simulations. Obviously in the modified version (bolded lines) the final amount of permits is found sooner than in the original procedure. However, we also note that standard deviation from the average generally tend to be lower for particular agreements. To get a clear picture right-hand-side plots depict also variance of permits traded in the proceeding transactions. For most of the regions variance tend to be lower for the altered procedure with selected partners. Permits traded tend to be more accumulated around the mean value. This leads to a conclusion that the market is more 'compact', e.i. market is less likely to take extreme values.

5 Concluding remarks

We have examined an extension to a sequential bilateral trading scheme which allowed to limit randomness involved in appointing bilateral partners.

A party can choose the partner that will constitute the highest difference of marginal abatement costs, thus creating better potential for gains in particular transaction.

Numerical exercise was performed for three parties picked randomly at each step. It showed that the considered procedure result in faster convergence. Moreover, permits traded are more accumulated around mean value. Further enlargement of the group picked at random would, most probably, approve those results.

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the 1990s, the number of people in the UK who are aged 65 and over has increased from 10.5 million to 13.5 million, and the number of people aged 75 and over has increased from 4.5 million to 6.5 million (Office for National Statistics 2000).

There is a growing awareness of the need to address the needs of older people, and the need to ensure that the health care system is able to meet the needs of older people. The Department of Health (2000) has published a strategy for older people, which sets out the government's commitment to older people and the need to ensure that the health care system is able to meet the needs of older people.

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- Older people should be able to live independently and actively in their own homes.
- Older people should be able to access the services they need to live independently and actively in their own homes.
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