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Simulation of an uncertain emission market for greenhouse gases using agent-based methods*

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Simulation of an uncertain emission market for greenhouse gases using agent-based methods

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Abstract

The paper presents the problem of a simulation of the greenhouse gases emission permits market where only low accuracy emission amounts are known. An organization of the market with uncertain emissions is proposed and trading rules for individual market participants are discussed. Simulation of the market is based on a multi-agent system. Negotiation of purchase/sale prices between the parties are introduced, where the trading parties adopt one of two options: (i) bilateral negotiations, and (ii) sealed bid reverse auctions. Results of simulation runs show trajectories of transaction prices, as well as probability distributions of learning agents' bidding prices.

Keywords: greenhouse gases, emission trading, uncertainty, market simulation, multi-agent systems, negotiations

1. Introduction

Markets for emisions of greenhouse gases (GHG) were designed to lower the costs of GHG emission abatement. In GHG markets every party is allocated an emission quota, otherwise known as permits (usually smaller than the actual emissions). At the end of the trading period, a party has either to keep its emissions within the allowed quota, or buy permits for superfluous emissions. In this context, we either use the term on trading emisions or trading emission permits.

Some existing markets, such as the EU ETS (IETA, 2005), trade only these gases, whose amount can be adequately defined. However, a market covering all anthropogenic emissions, like that covered by the Kyoto Protocol, must also include very poorly assessed emission amounts, such as those connected with agriculture or land-use change. Then the question arises, does trading such emissions actually ensure their reduction as hoped and planned for? And, should a tonne of poorly estimated and well defined emissions be priced equally?

Markets are often analyzed by either using a static or dynamic optimization or a game-theoretic approach. With full information on the parties, these approaches allow for analytical analysis of markets. Recently, agent-based models have been used to investigate market behavior dynamics using a simulation approach. Parties are represented by intelligent programming agents which negotiate and trade goods according to given market rules and the partial information they possess. This approach is much more flexible as to any assumptions made on the market information and tries to better mimic real market behaviours, including also their transients. It is applied in this paper to simulate a GHG emission permits market.

The method proposed does not assume an ideal market. Neither the equilibrium prices are assumed to be known in the trading, nor is an approach made without trading prices, Ermoliev et al. (1996) or Ermolieva et al. (2010), is considered. A more sophisticated market model is introduced, with price negotiations and price influenced agent behavior, similar to that in Bonatti et al. (1998). Each successive transaction moves the market toward an equilibrium.

In simulations, a multi-agent platform for multicommodity exchange (Kaleta et all, 2009) has been used. Each agent minimizes its own objective function, which is the cost of emission reduction plus any expenditure for the permits. The permit purchase/sale prices have an influence upon the profitability of transactions and the decision to buy/sell permits, i.e. whether it is better to reduce emissions or to buy permits. Two trading mechanisms are considered: a bilateral trade and a sealed-bid reverse auction (a tender).

Trade negotiations are the way to solve uncertainty in selling/buying prices, whichever trading mechanism is used. However, as mentioned earlier, uncertainties in emission amounts also characterize GHG markets, as well as some other markets designed for trading environmental quantities. These uncertainties have not influenced the prices in any of the previous and existent markets. In the solution that is presented in this paper emission uncertainties are taken into account in the simulation by using effective permits, as proposed in Nahorski et al. (2007). This brings the problem of trading uncertain emissions to that of usual trade with a perfectly known amount of the good being traded. The goal of this paper is to simulate a market using the rules proposed in earlier papers, and particularly in Nahorski et al. (2007) and Nahorski and Horabik (2012).

Only a few methods have been proposed to solve the trade in uncertain environmental goods, and in particular goods with very differentiated uncertainties, say 2-5% as against 80-100% uncertainty. Apart from that which described in Nahorski et al. (2007), the present authors are only aware of that described by Ermolieva et al. (this issue). Our method is simple in implementation and does not demand far reaching changes in the trading rules. Knowledge of the uncertainty parameters is necessary, but it is also needed in other methods.

2. Market with known emissions

Before presenting the method, we should like to start with a short presentation of an analytical solution of the market with fully known variables. Let us consider a market with N parties P_n, $n = 1, \dots N$, trading the emission permits. Each party has been allocated K^{P_n} permits. K^{P_n} are called the targets. Their distribution, in the form of emision permits, is supported by computer simulations and is conducted during political negotiations to reduce the harmful enviromental effect of total emissions. At the compliance time a party must not emit more than the number of permits they possess. However, it may freely sell or buy permits to achieve the target. We denote by x^{P_n} the emission of the n-th party and by E^{P_n} the traded permits. Emission must be nonnegative, $x^{P_n} \geq 0$. In this section, the number of traded permits E^{P_n} may be positive, when bought, or negative, when sold[1]. At the starting time, the total emission is greater than the number of permits

$$\sum_{n=1}^{N} x^{P_n} > \sum_{n=1}^{N} K^{P_n}, \tag{1}$$

which necessitates its reduction. It is quite common to refer to percent emission reduction, i.e. to express the targets as $K^{P_n} = (1 - \delta^{P_n})x_0^{P_n}$, where $100\delta^{P_n}$ is the percent of reduced emissions. In this paper we shall deal with absolute reductions, but any recalculations in both ways are obvious.

If the abatement cost functions $c^{P_n}(x^{P_n})$ of market participants are known to the central planner, the total cost optimization can be calculated and marginal cost λ at the equilibrium that are equal for all participants, can be found, see supplementary material. However, the cost functions are known only to the respective parties, so, consequently, prices of permits are set at the market. During trading, the n-th party is looking to minimize its cost of reducing the emission and buying/selling the permits E^{P_n}, i.e. to minimize the function

$$f^{P_n}(x^{P_n}, E^{P_n}) = c^{P_n}(x^{P_n}) + \lambda_t E^{P_n}, \tag{2}$$

subject to

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[1] In further sections E^{P_n} will be always nonnegative; the minus sign is used for negative values.

$$x^{P_n} \leq K^{P_n} + E^{P_n}, \tag{3}$$

with known targets K^{P_n}. Above, $\lambda_t \geq 0$ is the price of one unit of permits in the transaction t. Typically, λ_t is different from the unknown optimal equilibrium price λ, as trade is continuing in time. The parties simply have to live with the uncertainty in earning/loosing money during trading.

In the market considered in this paper, emission amounts are also not precisely known. The market for uncertain inventories has been already discussed in Nahorski et al. (2007), Nahorski and Horabik (2008), Bartoszczuk and Horabik (2007), Ermolieva et al. (2010; this issue). It was formulated as an optimization problem (the central planner's view), with a minimization of the total cost to achieve the common limit on emissions, subject to compliance with a fixed risk α. It focused on the equilibrium solution, and the time evolution of the prices on the market was not considered. In the real market, the parties make decisions on trading prices in a process of price negotiation. Some negotiations in simulating GHG trading were considered in Nahorski et al. (2012), but emission uncertainty were not dealt with. The organization of the market presented here follows the ideas presented in Nahorski and Horabik (2010, 2012).

In Ermoliev et al. (2000), an approach to simulating a bilateral exchange of permits was proposed. The idea is that two parties meet randomly and exchange their permits if the transaction is feasible for both parties, i.e. the marginal costs of the parties differ. Each such transaction makes the social cost function smaller, see Ermolieva et al. (2010). As the cost function is constrained from below by 0, and the sequence of cost function values is decreasing with each transaction, then it will eventually converge to a minimum. In the original paper by Ermoliev et al. (1996) it is assumed that the number of exchanged permits reaches zero (albeit not hastily) and then it is proved that the sequence converges to the global minimum with a probability of 1, as in the stochastic approximation method. The prices of the permits are not taken into account.

Prices were included in simulation by Stańczak and Bartoszczuk (2010). As before, only feasible transactions are considered. The price p of the transaction is drawn at random from the interval constrained by the marginal costs of the trading parties. The number of trading permits is also drawn randomly.

In the present paper the transaction price is set through bilateral negotiations or through auctions (tenders). No central institution is needed, except for the purpose of designing the market rules. This differs from the solution proposed in Ermolieva et al. (this issue), where a central institution helps to set prices of bilateral contracts at the point of equal marginal costs for both trading parties.

3. A market with uncertain emissions

3.1. Basic notions

We assume here that any lack of exact knowledge is expressed by the uncertainty interval

$$\hat{x} - d^l \leq x \leq \hat{x} + d^u, \tag{4}$$

where $\hat{x}$ is the reported emission (inventory) and d^l and d^u are the lower and upper spreads of the uncertainty interval of a party. For the sake of simplification, the indices P^n which identify parties have been omitted. The graphical interpretation of the derivations below can be found in Figure S1 in the supplementary material. To be absolutely certain that a party fulfills the limit K, the full uncertainty interval should be below the limit (Figure S1(a)). However, a weaker condition will be used in this paper. Following Nahorski et al. (2007) we will state that a party is *compliant with the risk α*, if its emission inventory satisfies the condition

$$\hat{x} + d^u \le K + \alpha(d^l + d^u). \tag{5}$$

This condition means that the α-th part of the uncertainty interval of the party's emission volume estimate (inventory) is allowed to lie above the limit K (Figure S1(b)). The condition (5) can be also written as

$$\hat{x} + \left[1 - \left(1 + \frac{d^l}{d^u}\right)\alpha\right] d^u \le K. \tag{6}$$

Thus, a part of the upper spread of the uncertainty interval is added to the emission estimate before compliance is checked. This can be also interpreted in such a way that an unaccounted emission, due to uncertainty, is included in the condition to reduce the risk of non-compliance. Let us introduce *the relative upper and lower spreads* of the uncertainty intervals and denote them as

$$R^l = \frac{d^l}{\hat{x}} \quad \text{and} \quad R^u = \frac{d^u}{\hat{x}}, \tag{7}$$

respectively. Denoting *the fraction of the unaccounted emission* in the emission estimate as

$$u(\alpha) = \left[1 - \left(1 + \frac{d^l}{d^u}\right)\alpha\right] R^u, \tag{8}$$

the compliance with the risk α of equation (6) can be also written as

$$\hat{x}[1 + u(\alpha)] \le K. \tag{9}$$

The value on the left hand side is called *the expanded emission*, and this value

$$\widetilde{K} = \frac{K}{1 + u(\alpha)} \tag{10}$$

is called *the corrected limit.*

3.2. Effective emissions

The above compliance-proving policy can be used to modify the rules of emission trading. The main idea, as presented in the earlier papers by Nahorski et al. (2007); Nahorski and Horabik (2008; 2010), involves transferring the uncertainty of the seller's emission volume to the buyer's emission volume together with the volume of traded emissions, and then including it in the buyer's emission balance. We use superscript ^{S} and ^{B} to distinguish between the seller and the buyer respectively in such a transaction.

Let us denote by $\hat{E}^S$ the amount of estimated seller emission allocated for trade, in tonnes. This emission is associated with the lower and upper spreads of the uncertainty intervals $\hat{E}^S R^{lS}$ and $\hat{E}^S R^{uS}$, respectively. The value

$$E_{eff} = \hat{E}^S[1 - u^S(\alpha)] \tag{11}$$

is called *the effective emission* (Nahorski et al., 2007). To interpret this notion, let us be aware that the buyer subtracts the purchased emission permits from their initial number of permits. Thus, to check the buyer's condition of compliance with the risk α, after having purchased $\hat{E}^S$ units of emissions, the following expression has to be considered

$$\hat{x}^B - \hat{E}^S + \hat{x}^B u^B(\alpha) + \hat{E}^S u^S(\alpha) = \hat{x}^B - E_{eff} + \hat{x}^B u^B(\alpha) \leq K^B. \tag{12}$$

It can also be written as

$$\hat{x}^B + \hat{x}^B u^B(\alpha) \leq K^B + E_{eff}. \tag{13}$$

Put simply, buying effective emissions is equivalent to directly increasing with their added value the buyer's compliance limit with the risk α. Consequently, the transaction helps the buyer to achieve their limit.

3.3. Basic relations in trading

The observation (13) is used below to organize a market with uncertain emissions, with the effective emissions as the trading good. The rules of trading are given for the individual participant of the market. The initial values before starting the trade are denoted by the subscript $_0$, and those after the transaction number $t \geq 1$ by the subscript $_t$.

Let us assume that the amount of $\hat{E}_t^S \geq 0$ is sold by the seller S to the buyer B. The lower e_t^{lS} and the upper e_t^{uS} uncertainty spreads related to this amount are

$$e_t^{lS} = R_0^{lS}\hat{E}_t^S = \frac{\hat{E}_t^S}{\hat{x}_0^S} d_0^{lS} \qquad \text{and} \qquad e_t^{uS} = R_0^{uS}\hat{E}_t^S = \frac{\hat{E}_t^S}{\hat{x}_0^S} d_0^{uS}, \tag{14}$$

where $d_0^{lS} = d^{lS}$, $d_0^{uS} = d^{uS}$, $R_0^{lS} = R^{lS}$, and $\hat{x}_0^S = \hat{x}^S$. Thus, after the transaction we have

$$\hat{x}_t^S = \hat{x}_{t-1}^S + \hat{E}_t^S \quad \text{and} \quad \hat{x}_t^B = \hat{x}_{t-1}^B - \hat{E}_t^S, \tag{15}$$

with $\hat{x}_0^B = \hat{x}^B$. According to the rules of interval algebra we have for the uncertainty spreads

$$d_t^{lS} = d_{t-1}^{lS} + e_t^{lS} \qquad d_t^{uS} = d_{t-1}^{uS} + e_t^{uS} \tag{16}$$

$$d_t^{lB} = d_{t-1}^{lB} + e_t^{lS} \qquad d_t^{uB} = d_{t-1}^{uB} + e_t^{uS}. \tag{17}$$

Estimated emissions of parties not involved in the transaction do not change, and $\hat{E}_t^S = 0$ is taken to stand for them.

Let us notice that the effective emissions in the transaction can be expressed as

$$E_{eff,t}^S = \hat{E}_t^S\left\{1 - \left[1 - \left(1 + \frac{e_t^{lS}}{e_t^{uS}}\right)\alpha\right]\frac{e_t^{uS}}{\hat{E}_t^S}\right\} = \hat{E}_t^S[1 - u^S(\alpha)], \tag{18}$$

where the last equality stems from (14). The quantity of effective emissions is smaller than those that are estimated, unless precise knowledge of the inventory is known or α zeros

$u^S(\alpha)$. The more uncertain the inventory is, and the smaller α, the less effective are emissions allocated to the party.

3.4. Organization of the market

Now, we shall outline a market in effective emissions, acting according to the following principles.

- When trading, the effective emissions (11) and corrected limits (10) are used.
- After the t-th transaction, the seller adjusts their accumulated estimated emission according to the rule

$$\hat{x}_t^S = \hat{x}_{t-1}^S + \frac{E_{eff,t}^S}{1 - u^S(\alpha)}. \tag{19}$$

- After the t-th transaction, the buyer adjusts their accumulated estimated emission according to the rule

$$\hat{x}_t^B = \hat{x}_{t-1}^B - \frac{E_{eff,t}^S}{1 + u^S(\alpha)}. \tag{20}$$

By adopting the above rules, a party is compliant with the risk α after transaction t if its accumulated estimated emission is not greater than its corrected limit

$$\hat{x}_t \leq \tilde{K}\,. \tag{21}$$

Evidence to substantiate this assertion is given in the supplementary material.

The bounds below show a reasonable amount of effective emissions to be traded in a transaction and reflect the requirement for lacking permits by the buyer and the possibility to spend exceeding permits by the seller, respectively. They are

$$E_{eff,t}^S \leq \min\{[1 + u^B(\alpha)](\hat{x}_{t-1}^B - \tilde{K}^B), [1 - u^S(\alpha)](\tilde{K}^S - \hat{x}_{t-1}^S)\}. \tag{22}$$

A derivation of this formula can be found in Nahorski and Horabik (2011). In the simulations, the parties generally conform to this condition, but a party may find it profitable to abate emissions and in this way increase its bounds (a seller) or decrease them (a buyer). The decision on abatement may be taken at each stage of the negotiations. The seller abates if it is profitable for them to sell any additional permits. The buyer purchases permits, if it is unprofitable for them to abate.

In conclusion, the organization of the market is as follows.

1. Before starting, all the limits are recalculated to the corrected limits $\widetilde{K}$, according to (10).
2. The parties negotiate the trading condition, taking into account the effective emissions E_{eff}, which are used in the negotiation of the selling/purchasing price. The maximum amount of effective emissions for selling is $[1 - u^S(\alpha)](\widetilde{K}^S - \hat{x}^S_{t-1})$. The maximum amount of effective emissions required by the buyer is $[1 + u^B(\alpha)](\hat{x}^B_{t-1} - \widetilde{K}^B)$.
3. Having terminated the transactions, the seller and the buyer adjust their accumulated estimated emissions according to (19) and (20), respectively.
4. To check the compliance, the current accumulated estimated emissions are compared with the corrected limits.

The trade above is in effective emissions, which is the common exchanged "good". However, to compare the prices of the effective emissions with the marginal costs of reducing the emissions, it is necessary to recalculate the prices for effective emissions to those of estimated emissions. As for the seller it holds that $E_{eff} = [1 - u^S(\alpha)]\, \hat{x}^S$, one unit of the estimated emissions $\hat{x}^S_t$ is equivalent to $1 - u^S(\alpha)$ units of the effective emissions $E_{eff,t}$. Similarly, for the buyer, one unit of the estimated emissions is equivalent to $1 + u^B(\alpha)$ units of the efficient emissions. Therefore, the following holds.

5. The price of one unit of efficient emissions $p_{eff,t}$ and one unit of estimated emissions p^S_t for the seller are related as follows

$$p_{eff,t}[1 - u^S(\alpha)] = p_t^S. \tag{23}$$

6. The price of one unit of efficient emissions $p_{eff,t}$ and one unit of estimated emissions p_t^B for the buyer are related as follows

$$p_{eff,t}[1 + u^B(\alpha)] = p_t^B. \tag{24}$$

In any successive transaction it holds that

$$\frac{p_t^S}{1 - u^S(\alpha)} = \frac{p_t^B}{1 + u^B(\alpha)}.$$

So, the smaller the uncertainty of a party inventory is, the higher is its estimated emission price when it is a seller, and the smaller is the price when it is a buyer.

4. Simulation system

4.1. Trading mechanisms

Two trading mechanisms are considered: the bilateral trade and the sealed bid reverse auction. In the bilateral trading, agents split randomly into pairs. Once this is done the paired agents negotiate independently of on another. If an offer is received that lowers an agent's cost, it is accepted; if not, it is not accepted. The next splitting occurs after this running negotiation has been terminated. This is repeated iteratively. Each negotiation may terminate in an agreement or not, depending on the negotiator's expected profits. The transaction costs are neglected.

In the sealed bid reverse auction one participant takes on the role of an auction operator, while the others assume the role of bidder. The operator is chosen randomly using the bully election algorithm (Mamum et al., 2004). To ensure equal opportunities for each participant to become the operator, a priority is chosen randomly at the beginning of each auction. All other participants may submit a bid for trading a number of permits with a specified price. The operator chooses the most profitable bid, taking into account its preference. Afterwards, a new operator is chosen and the process is repeated iteratively. The operator calls either for selling or buying emissions, depending on their requirements.

4.2. Multi-agent system

To retain asymmetric information of the trading parties an agent-oriented paradigm (Shoham 1993. Shoham and Leyton-Brown 2009, Woolridge 2009) is applied. Individual parties interact there by interchanging messages, which ensures separation of their data. Each entity (or group of entities) in such a multi-agent system is represented by a piece of software, called an agent. Agents are embedded in an environment that allows them to communicate by using a protocol, in which some of the frequent communication patterns are designed.

A multi-agent system is a system composed of two or more autonomous software agents communicating with each other and working towards their own individual ends. Such a system is designed to achieve some overarching objectives and to operate in accordance with the intentions of the system designer. These goals are not implemented directly, but rather through the individual objectives of each of the agents and their interactions. Each agent represents a single party, which is guided by its own interests. In our case, an individual agent is motivated by the desire to achieve certain gains from the exchange of permits, i.e. to reduce its costs. The overarching goal of the system is that of the central planner, i.e. to minimize the total cost of fulfilling the emission limits. To achieve their goals, agents cooperate and compete (so called coopetition, Bengtsson and Kock (2000)). The coopetition is modeled by the strategic behavior of the agents. For a general discussion of the negotiations between programmable agents see Lopes et al. (2008).

A widely used standard for the description of multi-agent systems is the ODD-protocol. Grimm et al. (2010). A detailed description of the system prepared that uses the ideas of the ODD-protocol can be found in the supplementary material.

4.3. Non-learning agents

To prepare an offer the agents use knowledge stored in their state. Two kinds of agents are considered, learning and non-learning ones. A **non-learning agent state** consists of its emission, its emission reduction cost, the bought/sold permits and their costs. Using this knowledge, an agent is able to assess whether the coming offer is profitable or not.

Negotiation in the bilateral trading arena commences with establishment of **the number of permits** offered for negotiation by a trading agent. A prototype number is drawn randomly according to a uniform distribution on an interval extending from zero to a determined empirically upper value in order to ensure the stable behavior of the market. Once this is done the minimum of the number of permits lacking to achieve the limit given by (22) and the prototype number, is computed. Finally, the minimum from the numbers submitted by both parties is taken. The number of permits is not changed during any bargaining over the price. **An offer price** of an agent is computed randomly using a given (see Section 5.3 for details) probability distribution defined by the agent's price interval, which extends from the agent's marginal cost to the last most favorable price of an accepted offer. In this way, starting prices for negotiations are formed. Then the parties try to reach the final price, step by step, by successive incrementation (by a buyer) or decrementation (by a seller) of any previous offers with a constant predetermined step value. The negotiations succeed when both parties agree upon a price. If one side has reached its limit of profitability and the other refuses to accept the actual price, the negotiation fails and no transaction is performed. An exchange of offers takes place until the end of negotiations. Only then may the agents become engaged in any successive negotiations.

A similar random mechanism is used to form an offer price in the tender. Having gathered the offers, the auction operator chooses the most favorable one, and the transaction is concluded. The parties taking part in the tender are not engaged in any other negotiations during this period.

4.4. Learning agents

Agents' actions modify the agents' state. This is used by learning agents to improve the selection of transactions and any subsequent bidding and/or negotiating, through setting and modifying the probabilities of their execution. For example, if a bid price in an auction is too low when buying, it is very likely that some other party overbids it. If it is unnecessarily high, the gain will be small. Agents use a variant of the reinforcement learning method, see Brenner (2006). More precisely, a **learning agent state** is augmented with additional variables, which store information on transactions concluded by the agent, separately for selling and buying. This is used to form and adapt probability distributions of succeeding in a transaction. The interval of possible offers is divided into ten subintervals. Having concluded a transaction, the value in an appropriate subinterval is increased. The initial distribution is uniform. Any predicted gain is calculated by multiplying the experimental probability of profiting from the potential gain in the middle of the corresponding subinterval. These expected gain distributions are used for generating successive bids, which are selected using a roulette wheel method. This way better bids are employed more frequently.

5. Simulation results

5.1. Case considered

The simulation was carried out using the case study described in Horabik (2007) and Nahorski at al. (2007). Five trading parties: the USA, the EU, Japan, CANZ (Canada, Australia, New Zealand) and the EEFSU (East European and Former Soviet Union), indexed $n = 1, \ldots, 5$, respectively, are assumed to take part in the Kyoto Protocol trade agreement. We assume that all these parties conform to the Kyoto Protocol regulations to reduce CO_2

equivalent emissions. The parties have been specified emission limits K^{P_n}, BAU (business as usual) emissions $\hat{x}_0^{P_n}$, and the cost reduction functions c^{P_n}. As set forth in Horabik (2007), the cost of reduction can be well approximated by a square function of the size of the reduced emissions

$$c^{P_n}\left(\hat{x}_0^{P_n} - \hat{x}^n\right) = \begin{cases} a^{P_n}\left(\hat{x}_0^{P_n} - \hat{x}^{P_n}\right)^2 & \text{for} \quad \hat{x}^{P_n} < \hat{x}_0^{P_n} \\ 0 & \text{for} \quad \hat{x}^{P_n} \geq \hat{x}_0^{P_n} \end{cases} . \tag{25}$$

The variable $\hat{x}^{P_n}$ stands for the current accumulated emission permits. The marginal cost of the emission permit λ^{P_n} is the derivative of the function c^{P_n}. Symmetric uncertainty distributions are considered, for which better estimates of uncertainty can be given. The data for the problem are given in the Table S1 in the supplementary material.

5.2. Learning agents

Results obtained in all simulations are quite similar. The values of final emissions, numbers of traded permits, final marginal costs and the reduction cost are almost identical. The equilibrium results obtained are also similar to the results for the centrally planned market (Horabik, 2007; Nahorski at al., 2007; Bartoszczuk and Horabik, 2007). Bigger differences can be noticed in the values of permit costs, caused by the different ways of reaching contracts in the methods under consideration.

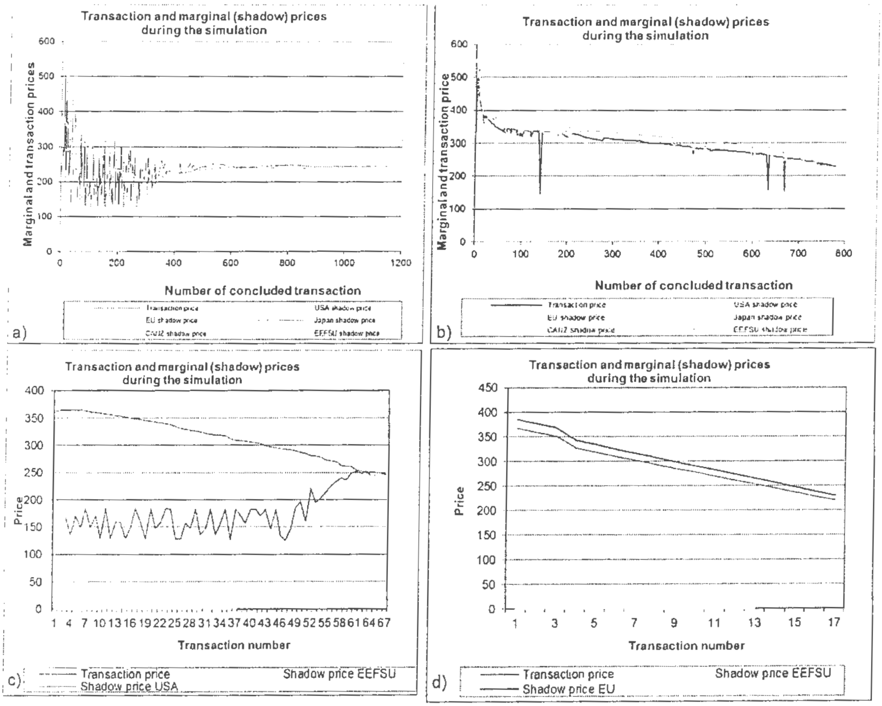

Figure 1. Trajectory of prices in consecutive contracts, in USD/MtC/y, in a single simulation, with $\alpha = 0.3$, for: a, c – bilateral negotiations, and b, d – sealed bid reverse auctions (operated only by sellers), a, b – for all market participants, c – only transactions between the USA and the EEFSU are shown, d – only transactions between the EU and the EEFSU are shown

In Figures 1a and 1b a few exemplary trajectories of transaction prices are depicted, while in Figures 1c and 1d examples of trajectories of consecutive prices in a trade between only two individual parties are shown. In both cases, the tender trade (operated only by sellers in this case) gives smoother trajectories, because selection of the best price filters out the outlying higher ones.

In the sealed bid reverse auction trade, final marginal costs do not converge precisely to the same value. This is caused by the competition among parties. Less profitable transactions are rejected and some parties are unable to win transactions that would lead them to the equilibrium point. This fact is more visible for smaller α. When ignoring uncertainty, for $\alpha = 0.5$, the final marginal costs are almost equal. In the bilateral case marginal costs tend to the equilibrium because the contracting parties are selected randomly, and if the transaction is profitable for both parties, it is concluded, even if more profitable transactions could be made, Figure 1a. In the tender case more profitable transactions are prefered by seller or buyer operators and some parties actually finish the simulation with worse results[2].

The curves depicted in Figure 2 present the experimental probability densities of bids, dependent on either call prices or final transaction prices, and expressed as a percentage of its actual marginal costs (100% corresponds to the marginal cost and 900% corresponds to 9 times the marginal cost when selling[3], while 0% corresponds to the marginal cost, and 100% corresponds to the lowest price when buying). The data are recorded from 1000 simulations. Generally, market participants buy permits at prices close to their marginal costs, with distributions similar to exponential ones. Selling is more complicated, because the EEFSU sells permits with almost equal probability for all possible bid prices. But for the final transaction prices the distribution is much closer to an exponential one. This effect is caused by the fact that high bid prices in bilateral trade are negotiated to much lower prices in any concluded transactions. Still, the EEFSU was able to sell a big share of permits at much higher prices.

[2] Only selected examples of extended simulations are included.

[3] EEFSU has most time the marginal cost 0 due to excessive permits ("hot air"). But in this case a small value *min* was introduced to keep more realistic conditions in the price calculation.

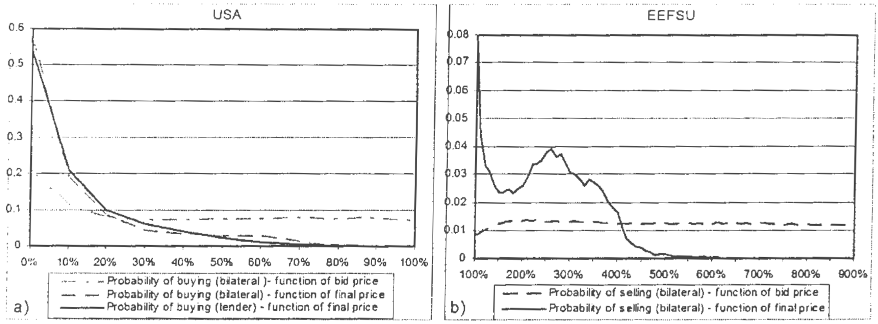

Figure 2. Probabilities of concluded transactions for selected market participants as a function of a bid or final price expressed as a marginal cost percentage: a) bilateral and tender the USA, b) bilateral trade the EEFSU.

5.3. Non-learning agents

Non-learning agents have a fixed probability distribution of bidding. After experiments with cut-normal and cut-lognormal distributions, the empirical distributions which have been gathered in learning-agent simulations, have finally been used to generate prices offered by non-learning agents. In Figure 3 the trajectories of consecutive transaction and marginal costs during single simulations are depicted, with $\alpha = 0.3$. Figure 3a presents the results for the bilateral negotiations, and Figure 3b for the sealed bid reverse auction. In both cases, the marginal costs of the parties converge to the final equal marginal cost. The transaction prices are located inside the marginal costs, so they converge as well. They gather mostly in the upper part, close to the upper marginal costs. This is particularly visible for the bilateral negotiations (Figure 3a), and in the later part of the sealed bid reverse auctions (Figure 3b). The auctions are mostly won by the EEFSU, due to its possession of the most competetive selling prices.

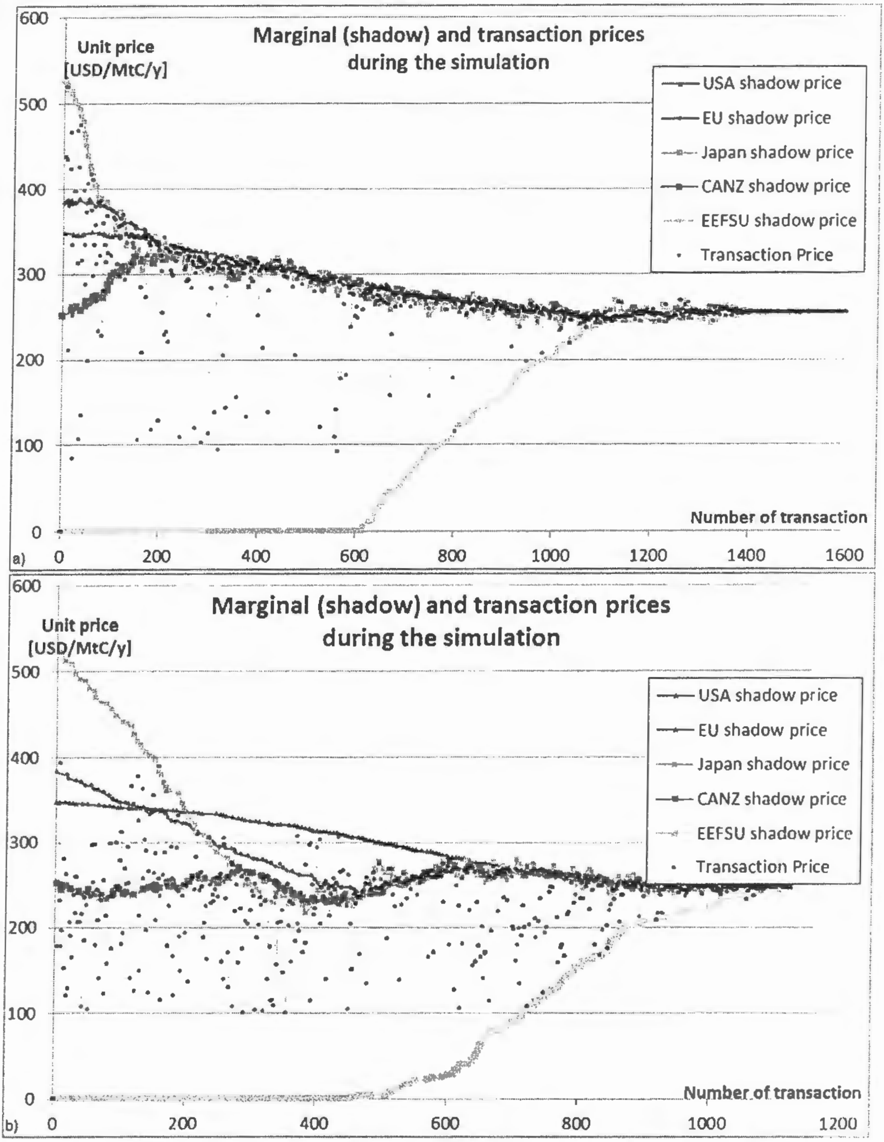

Figure 3. Trajectory of unit prices in consecutive contracts, in USD/MtC/y, in a single simulation, for $\alpha = 0.3$; a – with bilateral negotiations, b – with sealed bid reverse auction.

Analyzing the parties' behaviours, let us first consider Japan for the sealed bid reverse auction model (see Figure 3b). To begin with, Japan is prompt in its purchase of permits; it lowers its marginal cost equally quickly. When the marginal cost reaches the level of the prices of other countries, Japan begins selling the permits. The same process, on a smaller scale, can be observed for the EU and CANZ. The USA, in turn, is mostly buying the permits while conditions are favorable, and EEFSU is generally selling permits: initially its "hot air" permits, and then from reducing its emissions.

In the bilateral trade (see Figure 3a) CANZ, the EU and Japan are energetic in their purchase or sale of permits. As in the previous case, the USA is mostly buying and the EEFSU is mostly selling. The transaction prices, agreed randomly, are much higher than in the sealed bid reverse autions. Nevertheless, they tend to decrease in time. Hence, the overall profit for the seller (the EEFSU) is greater, and for the buyer (the USA) it is smaller than in the sealed bid reverse auction trade. Japan profits more in the tender trade.

Equilibrium results, averaged after 5 simulations, are presented in Table S2 in the supplementary material. In all cases the market converges to a point of (almost) equal marginal costs, which is a necessary condition of optimality. The marginal costs and traded permits are close to those obtained by central optimization of the market. There are differences between emission reduction and permit cost distributions among parties for the bilateral and tender trade, as a result of different ways of reaching the equilibrium.

The emission reduction costs are almost equal for both trade mechanisms. They rise when the uncertainty parameter α decreases. The same is for the overall costs for each party. Thus, for a smaller α, it may be profitable to invest in a decreasing uncertainty of the inventory. For a smaller α, more parties reduce their emissions and sell their permits. For $\alpha = 0.1$, every party globally sells more emissions than buys. This is due to the specific features presented by the trading of uncertain emissions, which balances the traded effective emissions but imbalances the estimated ones.

5.5. Comparison and discussion of results

Equilibrium results of all methods are similar, see Figure S2 in the supplementary material. For a parameter α larger than 0.2, values of final marginal costs are almost identical for all cases. For one that is smaller than 0.2, the results slightly differ. Larger marginal costs are for the learning and the smaller are for the non-learning agents. The results obtained indicate that the marginal cost $\lambda(\alpha)$ in the equilibrium and the total final emission $x(\alpha)$, for a given α, can be well approximated by the following linear function

$$\lambda = -3.6502x + 4418.7.$$

This suggests the possibility of anticipating the final marginal cost for a given final emission. Also, knowing the marginal cost functions and the equilibrium price, the total final emissions can be determined. These dependencies may be helpful for the market designer.

During trading, the prices tend to keep close to the buyers' marginal costs. This is partly due to the rapidly converging marginal costs of most parties, except for the EEFSU and the fact that transaction prices between the parties fall between their marginal costs. Parties are aware of this tendency as is shown in the distribution for the USA in Figure 2a. In bilateral negotiations the transactions are concluded far more often when they start with offers close to the buyer's marginal costs. The distribution for the final prices are close to the buyer's marginal costs both in bilateral negotiations and auctions. But the EEFSU often successfully finalizes its bilateral negotiations for different initial prices, Figure 2b, and gets prices accepted that are often much higher then its marginal costs. This is an effect of the monopoly of the EEFSU, which is the main seller, while other participants have to compete to buy permits. The effect of transaction prices gathering close to the buyer's marginal costs is also evident in Figure 3.

As the buyer's marginal costs decrease in time, the prevailing prices on the market also decrease. It is particularly visible for the bilateral negotiations, Figure 3a, but also in the final stage of the auctions, Figure 3b. This effect has been also observed in the real markets. The decrease is rather slow, which is in part due to the severe limitation of the traded volumes in our simulation. Greater volumes cause, however, bigger difficulty in their precise convergence to the equilibrium.

6. Conclusions

This paper concentrates on presenting the possibility of using agent-based computation tools to simulate trading of goods, which can not be quantified with satisfactory accuracy. A conservative compliance rule approach is considered, dependent on an accepted risk (probability) of not fulfilling an emission limit. The smaller the risk is, the more emissions have to be reduced. The market is designed to guarantee that the reductions are introduced, but a distinct feature is that the uncertainty of emissions influences their market prices. Those that are more uncertain are cheaper than those that are more certain. A specially designed muli-agent system was constructed to simulate trading with the two market mechanisms mentioned above. Multi-agent methods are used for market simulations, mainly because they are able to deal with complicated multi-interaction systems. The applied multi-agent approach seems to be a suitable tool for analyzing the economic phenomena of a market with uncertainties in prices, which appears in the GHG emissions market that is considered in this paper. Consequently, it was possible to observe the behavior of the market and its participants alongside a growing number of concluded transactions. The approach is also suitable for investigating the strategies of participants in the market as well as other market mechanisms than those considered in the paper, bilateral negotiations or tenders.

The results obtained are recognised as being preliminary ones, as rather simple assumptions have been taken in the simulations. Firstly, a simple negotiation of prices is assumed, in which the agents do not apply sophisticated strategies, and do not take into account any contract prices in other transactions. This is connected with the market mechanisms considered in the paper. In both the final transaction prices can remain secret.

The simulations provided phenomena which resembled real trade. A particularly interesting fact is that of grouping the transaction prices nearer the buyer's marginal costs, which has the effect of decreasing the prevailing transaction prices in time, with a corresponding rise in the number of concluded transactions. This result has been obtained for five parties (groups of states) active on the market. Five participants form a very small market in comparison with the real market's dimensions. However, any simulation of bigger markets presents a difficulty in the acquisition of the participants' emission abatement cost curves.

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