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Long-term financial planning by local government. A new method implementing multi-criteria optimization

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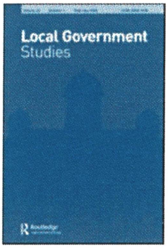

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Abstract
New, computer supported method is developed for decision making regarding long-term financial planning by local government (LG). It includes the model describing financial flows and stock of the LG budget, enables formulation of long-term plans and their changes and helps formulate polices for safe long-term investment, debt and operating budget. Fiscal rules and budget liquidity are formulated as the model constraints. The method mirrors long-term thinking process and links long-term financial planning, performance measures and budgeting for results and outcomes. It generates, with the utilization of the model, alternative budget projections based on H. Simon concept of multi-stage formation of goals (aspirations), enables modification of goals as search progresses and includes learning in the decision making process. A reference point multi-criteria optimization technique is applied. Respective criteria are simultaneously maximized (cumulated investment), and minimized (debt service costs). Numerical solutions are presented based on Polish LG data.

Keywords: local government budget projection, long-term financial planning, multi-criteria optimization, satisficing behavior theory.

1. Introduction

New method of budget projection is developed implementing multi-goal optimization and Herbert Simon theory. It supports long-term financial planning and adds time dimension to budgeting and liquidity analysis. The method, and the formulated budget financial flows model, offers new approach to investigation of fiscal rules impact on budget projection.

Long-term financial planning (l-tfp) presents revenue and expenditure projections, debt position and important financial policies in long-term. It identifies resources constraints so that strategic initiatives including service preferences can be realistically prioritized and the long-term financial implications of current decisions examined to address potential financial imbalances. A financial plan generates strategies that are made operational in consecutive budgets and helps a local government (LG) remain effective over long-term. LG budget reflects important priorities, determines how much investment and debt should be selected to facilitate infrastructure and services development.[1] L-tfp can discipline the fiscal process by establishing how much investment and debt a government can afford

[1] In many countries LG sector is the largest public investor and its contribution to the public sector debt is small. In 2013 LG's share of debt in GDP was below 9% (Bitner, Cichocki 2014).

over a several years period maintaining budget's liquidity. Safe debt policy is the key (Kavanagh 2007; Miranda, Picur 2006, GFOA 2013). Rubin (2014: 1-2), writes "Budget ,…, implies balance between revenues and expenditures,…, supported by safe borrowing. Borrowing is part of budget choices,.., but one has to have a plan how to pay the loans back and ensure budget liquidity when the debt will be paid off in future years". Detroit city failed to maintain liquidity. Presently, the challenges of Detroit are enormous, all the city does contributes to direct recovery, including public health, connectivity across the city, housing and green infrastructure.[2] L-tfp process integrates budgeting and strategic planning and helps financial managers identify emerging problems – difficulties and opportunities - before they pass by. It enables the budget planners to recognize implications of present decisions and integrate policy choices into the budgeting process.[3]

Budget planning requires also a decision-making process which assures decisions made in proper order and envisage long-term consequences of capital planning and borrowing (Kavanagh 2007: 60, 68; Miranda, Picur 2006: 37, the U.S. and Canadian examples). The benefits from l-tfp include: stimulation of long-term thinking about the budget process; clarification of strategic intent and advance recognition of potential problems; incorporation of financial perspective into organizational planning, and imposition of discipline in budgeting procedure (Kavanagh 2007, Vogt et al. 2009, Miranda, Picur 2006, GFOA Recommendation 2013 and 2010a).

There are many techniques available for revenue and operational expenditure forecasting - qualitative judgmental, incremental techniques and quantitative statistical methods (Meier, Brudney 1997, Schroeder 1996). The ability to understand historical data, the expertise of the forecaster, bringing multiple perspectives - the forecast objective, and building specialized economic models to be applied in forecasting is central to forecast quality.

Lindblom (1959) suggested that budgets change incrementally, a small amount per year. It is a feasible short-run strategy for the operating budget programs when economic environment is stable. In

[2] The City government has long been in a bit of a triage mode. For instance open natural spaces in the city are often viewed as blight.

[3] Rubin, in Kavanagh 2007, "As a long-time observer of public budgeting, I have wondered what the next wave of reform would be, and how well it would address the problems that local budgeters actually face. Now I know. The answer is long-term financial planning."

times of financial crises the operating budgets change drastically from year to year. Capital budgets in general do not follow an incrementalism logic, they exhibit dynamic, sometimes drastic changes.; when new facilities are constructed, the capital and current expenditures might grow fast even in stable times. Many capital projects are not repetitive, depend on economic growth, funds availability, and often call for reevaluation of budget priorities. Baumgartner, Jones (2002), and Jones, Baumgartner (2005) put forward budgets' dynamics analysis and explained that budgets result from policy choices that assume prior's year proportionally increased values plus random adjustments, including shifts within budget structure (punctuations, resulting from changes of governments or financial crises, Wildavsky 1964). Breunig 2006:1072 offered a new understanding of budget punctuation as a stochastic process and emphasized that budgeting is the most institutionalized process among all forms of policy making.

Capital expenditures, often funded by debt, can produce long-term benefits (improve social welfare). However, funding a capital project in a given year commits future resources for facility maintenance. The multi-period nature of a capital improvement plan (CIP) often makes it difficult to define program start and end points (Miranda, Picur, 2000: 3). The CIP and l-tfp have impact on both future and current budgets (Kavanagh, 2007: 8, Cichocki, 2013: 40-41, 232). Operating expenditure, funded by current operating revenues, generally produce benefits in the current period. In several EU countries, including new member states which joined the EU after 2004, the interest on long-term debt (part of service costs) and the facility maintenance costs are classified as current operating expenditure. Therefore, long-term capital investment and debt proceeds impact the operating budget over many years. Higher capital expenditures and debt (including short-term capital borrowing) increase operating expenditures. There are arguments that projecting separately operating and capital budgets does not influence welfare of a voter. Poterba 1995 argues that the presence or absence of a capital budget does not affect the level of non-capital spending. This is contrary to many authors (Cichocki 2013, GFOA Recommendations 2010b, 2007a, Casey, Mucha 2008, Kavanagh 2007: 43-44, Vogt 2004: 111, Tigue 1996) and practice in hundreds of European and the U.S cities. Poterba considers a two-period median voter model (neglects effects longer than two years), assumes the interest rate on borrowing equal zero and depreciation of capital (purchased in periods 1 and 2) only at the end of period 2. He assumes a concave utility function dependent on current and capital expenditure and, in part of analysis, independence of the non-capital spending marginal utility on capital stock. The preference for capital expenditures always changes over time, and the utility *does* depend on capital stock which depreciates every year. In practice, and in our model, debt outstanding, debt service and budget liquidity depend on interest rates and the projection period is several years, which can be extended. In the developed method the dm preferences result in solutions which include results of various utility function implementations. Kavanagh, 2007: 51-77 describing examples of long-term planning process (San Clemente, California, Coral Springs, Florida; and Edmonton, Alberta, Canada) shows that the l-tfp process consists of "lanes" which comprise, implemented simultaneously, the city's strategic planning, capital budgeting, operating budgeting and business planning processes - an implementation plan describing how its long-term financial and service strategies will be implemented. Jacobs 2008 emphasizes the importance of simultaneous considerations of operating and capital budgets, but projected separately.

We were motivated to undertake the analysis aimed at balancing planned investments and safe debt, which ensure budget liquidity each year, and reducing, to a maximum extent, a gap between the currently financed projects and the projects identified as necessary over a long-term. The developed method helps solve this problem. It generates projections, which satisfy the above objectives, and additionally satisfy legal regulations (fiscal rules). The existing projection methods do not include multi-goal considerations, do not allow for the maximum gap reduction, and do not involve a decision maker (who understands local environment and needs) in the projection process. The method is a practical complement to a variety of existing projection methods, offers new approach to the projection's generation process - implements optimization theory, enables simultaneous consideration of several goals and many substitutable real-life conditions. It accounts for decision maker's preferences in selection of prognoses and ensures optimal selection of goals. The method allows for budget projections changes, additions of goals and projection's conditions.

Statistical techniques, for instance regression-based methods, can be used to validate predictions. However, linear regression has the major drawback of neglecting the points located far from the regression line[4]. Regression techniques do not simultaneously ensure budget liquidity, alternative goals selection and legal restraints' satisfaction.

The developed multi-criteria optimization method generates financial projections and helps analyze budgetary decision. It offers a local government decision maker (dm) a tool for projections' construction, and update of policy goals. It combines optimization results and decision maker (human) judgment to come to a "satisficing" solution introduced by Simon (1959). Based on alternative budget projections, the dm builds consensus for financial policy decisions, highlights situations for which a decision is needed today to avoid problems tomorrow (reevaluate capital improvement plan, when infrastructure needs in future years exceed available revenues and violate liquidity).

The method imbeds the original model of budget financial flows and multi goal approach into a computer-based interactive procedure supporting construction and analysis of financial projections made by the dm. The budget's financial flows model (Cichocki 2013: 101) bases on general framework of LGs' finance and debt management (Bitner et al. 2013; Cichocki 2013; GFOA Recommendations 2013, 2010, 2007; Hallerberg et al. 2009; Kavanagh 2007; Casey, Mucha 2008; Miranda, Picur 2006; Rossi, Dafflon 2002; Cichocki, Leithe 1999; Leonard 1996; Josef 1994). It mirrors interrelations between financial variables – LG budget flows (new debt, debt service, investment, operating expenditure and surplus), and stock (fixed assets, indebtedness). Decision variables are looked for which maximize planned cumulative investment over a given planning period and simultaneously minimize the total cost of servicing debt after that time. They are considered goals (criteria) in the decision making process.[5] The decision variables include investment co-financed by the EU funds, separately by LG budget and debt, and credits and bonds issued during the planning period.

[4] Baumgartner and Jones (2002), and Padgett (1980: 354, 359) emphasized methodological limitations of linear regression techniques widely used in analysis of budgetary dynamics. See Citi 2013 and the Anscomb's quartet – an example of four entirely different data sets for which variance, correlation coefficients and the regression line are identical.

[5] The developed method can include various goals reflecting functions of government (COFOG, ESA 2010, ch. 23, 541-543).

The new computerized optimization procedure, with the budget flows model included, implements Simon's satisficing behavior (Simon 1947, 1959) and the multi-criteria optimization (Keeney and Raiffa 1976) theories. It includes learning in the process of decision making. The multi-criteria solution method utilizes the reference point methodology (Wierzbicki 1986, Kruś 2008), implements the Simon's theory (the concept of formation the decision maker aspirations in a multi-stage, adaptive way) and leads to Pareto solutions. Decision makers begin with available revenues and basic operating expenditures projected based on historical trends and GDP and inflation growth, continue with a consideration of goals (desired outcomes) formulated as needed cumulated investment. Then, they solve the model and analyze solutions, deciding what values of the model decision variables best fit the desired goals over a several year period. The goals are adjusted during the optimization process as the dm learns about consequences of previous decisions (goals' selection) - the decision variables and criteria values. The model is solved several times for various goals. The procedure can be implemented for various initial values, different revenue projections and basic operating expenditures structure.

2. Mathematical model of local government budget

2.1. Model description

The LG budget financial flows model encompasses various categories of flows: operating, capital and total revenues and expenditures, new debt, debt proceeds, and stock: indebtedness and fixed assets - defined each year of an investment period. Cichocki 2013 described time dependent relations between financial flows and stocks specified as the model constraints. They include budget liquidity and fiscal rules regarding limiting operating expenditures and debt service costs to prohibit budget deficits and excessive debt of LGs[6]. Some flows are decision variables, some are values computed based on these variables, other are exogenous in the model. We look for a maximum planned investment cumulated over a given planning period $[t_1, T_N]$, which can be financed from the LG budget, the EU funds, and from debt. The model enables, each year, determination of safe

[6] Other rules may regard outstanding debt, debt related to operational or total revenue, and deficit (Kavanagh 2007:148-149, Cichocki, Leithe 1999).

investment (and its financing structure) and debt - bond covenants and medium and long-term credits, when the model constraints are satisfied. The debt can be issued in consecutive years, until T_N, and is structured in a way that the cumulative costs of the debt service after T_N (until the debt is repaid), are minimized. The debt service includes repayment of debt principals, interest on the outstanding debt and guarantees given by a LG to other institutions. It includes the initial debt, issued prior to t_1 (results from past contracts) and future debt (issued over t_1, T_N). The debts are safe, conform to legal fiscal regulations and each year ensure budget liquidity and the balance of operational accounts.

The financial flows model is consistent with the Polish and EU legal regulations, and can accommodate other regulations. Investment (the decision variable), contributes to the value of fixed assets (GFCF – gross fixed capital formation), which facilitate better services, but increases operating expenditures - costs associated with maintenance of new assets. Debt can increase investment, but must be repaid, and generates new operating expenditures (interest). The implementation of the model and analysis of its solution helps budgeters make informed decisions regarding long-term investment and debt planning.

2.2. Model assumptions

The model is solved under several assumptions. The operating revenues and basic operating expenditures projections (excluding debt service and GFCF maintenance costs) base on historical trends updated by inflation, GDP and local growth rates. Ceilings for the EU funds are projected based on historical data.[7] Interest rates for bonds, medium and long-term credits, revenues from sales of property and capital grants are assumed known.

The division of the budget into operating (current) and capital (investment) budgets is assumed. It is related to the 'golden rule of public finance: current expenditure should be financed from current revenue (regular and cyclical); inflows from capital, including debt and non-regular revenue should exclusively serve investment financing (Cichocki 2013: 145, 179-180; Rossi, Dafflon 2010; GFOA Recommendations 2007, 2010; Kavanagh, 2007: 161-165; Vogt 2004; Dafflon 2002).

[7] The EU funds can be used for financing investment only when a LG provides its own share from the budget or/and from debt. The maximum share of the EU co-financing equals 85% of the total individual project's value.

2.3. Definition of the model variables

The model variables are defined for a projection period, $[t_1, T_N]$, and are used for calculations at time instants $t_1,.., T_N, T_{N+1}, T_M$, where t_1 denotes an initial, and T_N the last period of investment activity and debt issuance; T_M is the last period of debt repayment; M > N are number of years (integers). Selection of T_M results from debt structure - bond covenants and loan repayment terms. The model starting point is a year t_0 – end of the year preceding the budget year t_1. The model decision variables at year t include: investment expenditure (Inv^{EU}_t – co-financed by the EU funds, Inv^b_t – financed by a LG budget and debt); medium term credits C^{1EU}_t, C^{1b}_t, long-term credits C^{2EU}_t, C^{2b}_t; medium term bonds B^{EU}_t, B^b_t, where the superscript 'EU' denotes financing investment with the EU funds, the superscript 'b' - financing with the LG budget, without EU funds; the credit's superscript '1' denotes medium term, and '2' long-term credits. The debt issued at time t is the summation of credits and bonds at t. Other categories of credits and bonds can be added.

2. 4. Formulation of the multi-objective optimization problem

We look for a financial projection resulting from the maximization of investment cumulated over a period $[t_1, T_N]$, where $t_1 = t_0+1$, and, simultaneously, the minimization of the total costs associated with debt service over $[T_{N+1}, T_M]$. Investments are implemented and debt is issued only over $[t_1, T_N]$. All debts are repaid until T_M. Mathematical formulation is as follows.

Given initial values of the model variables and parameters at the initial period t_0, the time period $(t_1,\dots,T_N, T_{N+1},\dots,T_M)$, and separate select projections over $[t_1, T_M]$, find, for every $t \in [t_1,T_N]$, such values of investment expenditures, Inv^{UE}_t and Inv^b_t, credits and bonds, C^{1EU}_t, C^{1b}_t, C^{2EU}_t, C^{2b}_t, B^{EU}_t, B^b_t, used for financing investment, which **maximize** y_1 - **cumulative investment** over $[t_1, T_N]$, ***and simultaneously*** **minimize**, over $t \in [t_{N+1},T_M]$, y_2 - **the total cost of servicing debt** issued during $[t_1, T_N]$

$$y_1=\{\sum_{t=t_1}^{T_N} Inv_t\} \text{ \textbf{\textit{is maximized,}}} \tag{1}$$

where $Inv_t = Inv^{EU}_t + Inv^b_t$, $Inv^{EU}_t \geq 0$, $Inv^b_t \geq 0$, (1a)

and simultaneously

$$y_2=\{\sum_{t=T_N+1}^{T_M} RD_t+intD_t\}\ \textbf{\textit{is minimized}}, \quad (2)$$

where debt repayment RD_t includes repayment of the cumulated new debt and the old debt D_0, (issued prior to t_1) outstanding at time t. Repayment schedule of D_0, over $t_1,..,T_M$, is given - results from commitments made prior to time t_1.[8] Repayment of new debt includes repayment of credits C^{1EU}, C^{1b}, C^{2EU}, C^{2b}, repurchase of bonds B^{EU}, B^b, issued starting t_1. The variables' names are acronyms, for instance RD denotes repayment of debt.

New debt issued at $t \in [t_1,T_N]$

$$ND_t=C^{1EU}_t+C^{1b}_t+C^{2EU}_t+C^{2b}_t+B^{EU}_t+B^b_t, \quad (2a)$$

where $C^{1EU}_t \geq 0, C^{1b}_t \geq 0, C^{2EU}_t \geq 0, C^{2b}_t \geq 0, B^{EU}_t \geq 0, B^b_t \geq 0$ (2b)

The indebtedness (debt) D_t at t

$$D_t=ND'_t+D_{0t} \quad (3)$$

The cumulated new debt outstanding at the year $t = t_1, t_2,\ldots,T_N$, equals

$$ND'_t=\sum_{k=0}^{t-t_1} \delta ND_{t-k} \quad (3a)$$

where

$$\delta ND_t=ND_t-RND_t, \quad (3b)$$

is a change in the new debt ND_t issued at t, RND_t is the repayment of the cumulative new debt at t.

A change in debt outstanding at t, δD_t, equals a change in the cumulative new $\delta ND'_t$ and old debt δD_{0t}

$$\delta D_t=D_t-D_{t-1}=\delta ND'_t+\delta D_{0t}, \quad (3c)$$

where D_t and D_{t-1} are the total indebtedness at the end of year t and t-1.

The debt repayment RD_t at t includes the repayments of the old debt, RD_{0t}, and the new debt RND_t, cumulated over $[t_1,t\text{-}1]$, which include the four year credits, RC^1_t, the ten year credits, RC^2_t, and five year bonds, RB_t issued starting t_1 (the bond repurchase takes place once in five years)[9]

$$RD_t=RD_{0t}+RND_t=RD_{0t}+RC^1_t+RC^2_t+RB_t. \quad (4)$$

[8] D_0 is the outstanding initial (old) debt at time t_0.For some excessive D_0, the model solution may not exist.

[9] In the model, more bond's and credit's categories and any bond maturity can be assumed.

The repayment of credit issued at t, starts the next year, at $t+1$ in equal parts.

The interest cost incurred at period t, $intD_t$ on the outstanding debt D_t is the summation of interest on the initial debt, $intD_{0t}$ and cumulative new debt $intND'_t$ outstanding at t, and is calculated separately for each debt category.

$$intD_t = intND'_t + intD_{0t} \qquad (5)$$

We assume that each year the debts' repayments and bonds' repurchases take place at the same time as the new debt issuance, then, the interest at time t is calculated based on the indebtedness at the end of year t-1and the year t.[10] For example interest incurred on the medium-term credits C^{1EU} equals

$$int(\sum_{j=t_1}^{t} \delta C^{1EU}{}_j)_t = \tfrac{1}{2} ic_{1t} (\sum_{j=t_1}^{t-1} \delta C^{1EU}{}_{t-1} + \sum_{j=t_1}^{t} \delta C^{1EU}{}_t), \qquad (5a)$$

where ic_{1t} is the interest rate paid at t on the credits $C^1{}_t$, and $\sum_{j=t_1}^{t-1} \delta C^{1EU}{}_t$ is the credit $C^{1EU}{}_t$ outstanding at t. The interest (5a) is summed up with the interest on credits C^{1b}, C^2 and the bonds B, with interest rates ic_{2t}, and ib_t, and serves for calculation of interest on the new debt at t and the old debt.

The model's objectives (1) and (2) must be satisfied under constraints - conditions which result from principles of financial management and fiscal rules. Three major constraints for each $t \in [t_1, T_N]$ are introduced. The first one, (6), ensures budget liquidity each year. All budget inflows minus operating and capital expenditures must be nonnegative - the revenues, expenditure and debt levels each future year $t_1,..,T_N$ must ensure balanced LG budget

$$OpS_t + \delta D_t + CapRev_t + OthRev_t + Cab'_{t-1} - Inv_t \geq 0. \qquad (6)$$

OpS_t denotes operating surplus funds - operating revenues $OpRev_t$ minus operating expenditures $OpExp_t$ at t, $OpS_t = OpRev_t - OpExp_t$. The larger the OpS_t, the more funds available for investment's financing at t. The operating expenditures include interest costs of the debt outstanding and the fixed assets maintenance costs.

Capital revenues, $CapRev_t$, consist of three major parts: the EU funds (calculated in the model over $[t_1,T_N]$), special capital grants and revenues from sales of property (determined exogenously).

[10] Such simplification was assumed in the U.S. law (2009-2010 Wisconsin States Annotations, 6703, p. 2).

Other budget *net* inflows, $OthRev_t$ (revenues minus expenditures) include inflows from privatization, capital shares owned by the LG, and other *net* inflows not associated with debt.

The current accounts balance at t, Cab'_t, includes two separate balances: the budget revenue and expenditure balance, and the debt account balance (Cab_{t-1}), which equals the debt balance from the previous year, Cab_{t-1}, plus debt receipts minus debt principals repayment at t, and the revenues and expenditures balance from the previous year, BB_{t-1},

$$Cab'_t=Cab_{t-1}+ND_t-RD_t+BB_{t-1}, \text{when } BB_{t-1}<0. \quad (6a)$$

When a deficit occurs in the previous year, $BB_{t-1} < 0$, then, the current budget account in year t must be financed by additional new debt; when $BB_{t-1} > 0$, then $BB_{t-1} = 0$. The current accounts balance at time t, Cab'_t, must be non-negative for all $t=t_1,..,T_N$

$$Cab'_t\geq 0. \quad (6b)$$

The values of OpS_t and δD_t can assume negative values, the model decision variables C^1_t, C^2_t, B_t, Inv_t - only nonnegative values. Capital revenues, the operating surplus, *net* debt proceeds at t, other inflows and funds from previous year, when available, are used to finance investment.

The second major constraint (7), ensures operating expenditures, which in a given year do not exceed operating revenues enlarged by surpluses on the account balances from the previous year

$$OpRev_t-OpExp_t+Cab'_{t-1}\geq 0, t=t_1,..,T_N. \quad (7)$$

The operating revenues are specified for all sources of revenue (from PIT, CIT, VAT taxes, property tax, fees and charges, intergovernmental transfers and incidental revenues, for example earmarked operating grants). It is a modification of the golden rule (includes surplus funds from previous year).

The total operating expenditures $OpExp_t$ consist of basic expenditures, $BOpExp_t$, and the interest and fixed assets maintenance costs which grow with new debt and investment

$$OpExp_t=BOpExp_t+intD_t+\Phi_t\cdot GFCF_{t-1}. \quad (7a)$$

$GFCF_{t-1}$ denotes *gross* fixed capital formation (fixed assets) at time t-1, and Φ_t is a ratio of the GFCF maintenance costs at t.

The third major constraint (8), is a fiscal rule, imposed to restrain LG's from excessive debt issuance. It can assume various forms. We implement the rule, used currently in Poland, imposed at time t on the total debt service costs related to the total revenue.[11] These costs include all credits repayments and bond repurchases, interest on the outstanding debt charged every period t, and payable guarantees extended by a LG. They cannot exceed a limit, depending on the past LG's performance: the average value, over three years preceding the year t, of the operating surplus enlarged by the revenue from sales of property, in relation to total revenue.[12]

$$[(RD_t+intD_t)/Rev_t]\leq 1/3\sum_{i=1}^{3}\ [(OpRev_{t-i}-OpExp_{t-i}+SalGFCF_{t-i})/Rev_{t-i}] \qquad (8)$$

The costs of debt service are calculated for each credit and each bond issue separately, on the debt outstanding at time t-1, plus new debt taken at time t, minus debt repayment at t. The interest costs are computed on the total debt – the old debt and the cumulative new debt outstanding at time t. The value of the left hand side of (8), and of the limit for the total debt service (the right hand side of (8)), are calculated from the model. When, in any year $t_1,\dots,T_N$, either the operating revenues are lower than projected, or the basic operating expenditure higher than projected, then, the upper limit for the costs of debt service is lower, and less debt can be issued in future years. Then, less EU funds can be acquired and lower investment implemented. The constraints (6), (6b), (7) and (2b), (6a) must be satisfied over $[t_1, T_N]$, the constraint (8), over the whole period $[t_1, T_M]$, otherwise, the LG's council cannot approve the budget and the obligatory financial plan for the three following years.

New investments contribute to fixed assets' value but they generate additional operating costs associated with maintenance of new facilities. The interest cost on new debt also increase the operating costs (decrease the operating surplus). Revenue and operating expenditure projections are described by Kavanagh 2007, ch. 6, Cichocki 2013: 59-70, Schreder1996.

[11] Public Finance Law (Lpf), 2009 with amendments: art. 242, constraint (6), and art. 243, constraint (8). These constraints can be replaced with any other rule.
[12] To calculate the costs of debt service limit at t_1,t_2, one has to know the data for t_1-1, t_1-2, and t_1-3.

3. The multi-criteria optimization solution method

A LG's manager considers two conflicting criteria: y_1 - the cumulative investment over $[t_1, T_N]$, and y_2 - the total costs of servicing debt over $[T_{N+1},T_M]$. She looks for decision variables satisfying the model constraints (2b)-(8) that maximize the criterion y_1 and, simultaneously, minimize the criterion y_2. An increase in investment, facilitates usage of debt for projects' financing, and contributes to the increase of debt service costs. A decision how much to invest in the future is crucial for local development. LG investment needs can be associated with aspirations of a decision maker, who wants to satisfy a large portion of the needs in a given period. One can consider several investment scenarios, which depend on satisfaction of a given degree of investment needs and on future LG's revenue (tax potential).

The multi-criteria optimization problem is defined in two spaces: the first space of decision variables, $\boldsymbol{x}= (Inv^{EU}_t, Inv^b_t, C^{1EU}_t, C^{1b}_t, C^{2EU}_t, C^{2b}_t, B^{EU}_t, B^b_t)$, and the second space of criteria $\mathbf{y} = (y_1, y_2)$. The model constraints define a set X_0 of decision variables' admissible values. The optimization problem and the budget model relations define a set $\boldsymbol{Y}_0$ of the criteria's attainable values.

Consistently with the theory of multi-criteria optimization (Keeney and Raiffa 1976; Wierzbicki 1986), we look for an outcome (criteria and decision variables values) which is Pareto optimal in the set $\boldsymbol{Y}_0$ and satisfies the dm preferences. An outcome is Pareto optimal if there is no other outcome dominating it in the set $\boldsymbol{Y}_0$ (no achievable greater cumulative investments and lower total debt service costs). There may exist a set of Pareto optimal outcomes. The Pareto optimal points in $\boldsymbol{Y}_0$ and the corresponding decision variables in X_0 are not known, but they are uncovered in the optimization solution procedure. Generation of achievable Pareto optimal outcomes is carried out with an aid of the reference point method, which utilizes the order approximation achievement functions (Wierzbicki 1986). Outcomes are derived by solving the below optimization problem:

$$\max_{x \in X_0} [s(y(x), y^*)] \quad (9)$$

where:

$\boldsymbol{X}_0$ - a set of admissible decisions defined by the model relations,

$y^* = (y_1^*, y_2^*)$ - a reference (aspiration) point assumed in the space $\mathbf{R}^2$ of the criteria y_1 and y_2,

$s(y, y^*)$ - an order approximating achievement function.

The assumed achievement function is a version of the function (34) in (Wierzbicki 1986). The optimization problem (9) is solved by a specially constructed computer-based system which uses the solver embedded in the MSExcel environment. It is solved for a given reference point y^* assumed by the dm. The solution includes a Pareto optimal outcome, all corresponding decision variables, and other variables of the model calculated with the use of the decision variables.

The selection of the outcome, most preferred by the dm, bases on the H. Simon's satisficing behaviour theory (Simon 1959) and, supported by a computer based system, interactively, in a number of iterations finds a solution that satisfies the dm aspirations. The following procedure is implemented:

1. The dm sets initial data of the budget flows model and exogenous variables.

2. The computer-based system (cbs) solves two initial optimization problems with respect to the decision variables *x*, subject to the model constraints, setting limits for the outcome:

2.a.: maximizes y_1, (1),

2.b.: minimizes y_2, (2).

The model solutions: the decision variables, obtained values of the criteria and other model variables are saved in a solution data base.

3. The iterative solution procedure starts. The iteration number is set at i=1.

3a. The dm assumes a reference point (an aspiration level), a vector defined by the criteria y_1, and the y_2 values.

3b. Using the reference point method the cbs solves the problem (9), with respect to the decision variables $\mathbf{x}$, subject to the model constraints and additional constraints of the reference point method.

The reference point and the model solutions are saved in the data base.

4. The dm analyzes the current solutions and the outcome; compares it with the previous solutions and outcomes. In the 1st iteration the dm compares with the solutions obtained in point 2.

5. The dm decides whether she is satisfied with the current solution. If yes, the procedure ends, if no, the number of iteration increases, $i = i+1$, and the dm returns to point 3a.

Points 3–5 of the procedure are repeated in a sequence of iterations. The computer-based system solves the optimization problems of the points 2 and 3b, and stores the solutions in the data base. The points 1, 3a, 4, 5, are executed by the dm, who assumes exogenous variables and initial data of the model, assumes aspiration levels, analyses the solutions derived by the cbs and makes the final decision ending the procedure. Full sovereignty of the dm is assumed.

4. Solution presentation

The developed optimization procedure and the model are solved using a Polish LG's financial report and the 2014–2023 investment period. The financial projections of operating revenues and basic operating expenditures over 2014-2023 are taken from long-term financial projections of the LG. Projections of variables over 2024-2033 when the debts mature, are made by the authors. The obligatory share of the LG's budget in investments co-financed by the EU funds is fixed at 20% and an upper limit for the EU funds used for investment financing over 2014-2023 is assumed. These values can be changed. The issue time and value of credits and bonds result from the optimization model solution.

4.1. Illustration of the optimization procedure

The procedure starts from determination of upper and lower limits for the value of investment cumulated over 2014-2023 and the debt service costs over 2024-2033 (elements of outcome), which depend on the LG initial indebtedness and the model constraints. The dm, solving the model with the criterion (1) finds that the maximum cumulated investment equal 226, and the corresponding debt service costs equal 73.9 (figure 1, point 1a). Solving the model only with the criterion (2), she finds minimum debt service costs of 3 and the corresponding investment of 103.5 (point 1b).

In figure 1 select solutions of the multi-criteria optimization problem (9) are presented. Reference points used in the computational procedure are represented by rhombic points, the vector valued outcomes of the procedure - the sum of investment over 2014-2023 and the sum of the debt service costs over 2024-2033 - by bullets, denoted A, B, C, D and E. The optimization problem's outcomes results from the problem solution obtained in consecutive iterations. They belong to the Pareto frontier of the admissible set Y_0^1 (figure 2). In table 1 the reference points and select model solution over 2014-2023 are presented: the cumulated over 2014-2023 investment, the EU financed investment, the new debt issued during 2014-2023, the debt used for the EU financed investment, the operating surplus; and the maximum debt outstanding and fixed assets (GFCF) - all calculated for the initial debt D_0. In the table and in the text we present only numbers, which are in millions PLN.

Suppose the dm tries an ambitious goal of reducing 50% gap between the currently financed projects and the projects identified as necessary over 2014-2023, and in the first iteration assumes the reference point with investment equal 250 and 76 of debt service cost (the 250 and 76 values are elements of the reference point vector). From solution of the optimization problem she obtains the Pareto outcome E (investment equal 226, debt service – 73.9), figure 1, point 1a.

The dm analyzes the obtained values of the criteria and the decision variables in consecutive years. The Pareto-optimal solution E of the model yields very low investment in 2014 and 2015, which strongly decreases in 2018 (figure 3). In addition, no EU funds for financing investment in 2015-2016 are utilized (figure 4.). The solution, although ensures the investment cumulated over 2014-2023 larger than in other solutions, is not acceptable in practice. Some investment projects will have to be continued in 2014 and 2015, and the funds provided by the LG's budget, without any EU funds, might be not sufficient to continue these investments. The debt service costs cumulated over $[T_{N+1}, T_M]$ is very high, and in 2026-2028 the debt service reaches the limit (figure 7). There is a risk that high debt issued over 2014-2023 will generate high debt service costs over 2024-2033, which, although satisfy the model constraints, might not guarantee budget liquidity when a disturbance occurs.

Figure 1

Table 1

In the second iteration the dm tries a very conservative reference point, with investments planned for 2014-2023 of 160 (32% of the investments needed over 2014-2013). The obtained debt service costs, equal 3, result only from the debt issued prior to 2014. The solution procedure yields outcome A – 146 in investment and 4 of debt service costs over 2024-2033. The GFCF in 2024 equals 124.9 and the total debt issued during 2014-2023 is 25.2. The dm is not satisfied with the decreasing investment, especially during 2020-2023 (figure 3), and the GFCF value in 2024. She looks for a new outcome between points A and E. She could increase investment and debt incrementally and safely remain in the admissible set Y_0^1. However, since the debt service costs are well below the limit and the needs for the GFCF in 2024 and investment are higher than in A, in the third iteration the dm assumes the reference point of 235 in investment, and obtains the Pareto solution D, with the total investment of 217.1 and debt of 69.1. Similarly to the solution E no EU funds are used in 2015-2016, and investments in 2014 and 2015 are very low. She still wants to maximize investments but has to lower her aspiration - selects the reference point between A and D - 216 of cumulated investment (43% of investment needed over 2014-2023) and 47.5 of debt service. She obtains the Pareto outcome C. The cumulative investments are 205, the GFCF is 174.9 in 2024, the total debt service over 2024-2033 equals 50. The decision variables of the solution C are acceptable, they do not have deficiencies of the solutions D and E (figures 1, 2, 3, 6). The dm can be satisfied and stops the procedure in point C. The investments presented in figures 3, 4, and the debt (figure 8) will be implemented in consecutive years.

Figure 2

If the dm was more debt averse, she could select the reference point of 215 in investment and 27 in debt service. Then, the outcome B yields 195 cumulated investment and 30 of debt service costs (fixed assets in 2024 equal 167). Solutions for point B are presented in figures 3-10. The dm can select either point C, or point B. The choice of the outcome C yields lower cumulated operating surplus because the debt service costs and fixed assets maintenance costs are higher than in point B (the GFCF and investments are higher).

The set of possible outcomes which will satisfy the dm is decreasing during the process thanks to the knowledge of attainable Pareto outcomes and the solutions obtained in previous iterations. The Pareto outcomes yield the model decision variables which satisfy all model constrains, including fiscal rules.

The solutions of Pareto outcomes E and D of the model are more risky than the solutions A, B and C. Especially the outcome E, located on the edge of the admissible set Y_0^1 and the Pareto frontier is very risky. A slight change in exogenous projections of revenues might shift the solution outside the admissible set. Then, the constraint (8) will be not satisfied and no new debt can be issued. Select investments which had been started will not be continued because of insufficient funds.

The dm is not fully aware of her aspirations when she starts analysis of the multicriteria optimization problem - she does not know attainable outcomes. Points 1.a., 1.b. determine limits for these outcomes, for which she analyzes solutions in consecutive years and defines a first approximation of her aspiration. The aspirations are adapted as optimization proceeds in consecutive iterations. The number of the derived outcomes increases. Having analyzed several sets of solutions derived by the computer-based system including the Pareto frontier of the outcomes' set, she makes the final decision regarding ending of the procedure, and accepting the outcome which fits best her adapted aspirations.

When the initial indebtedness is large, the set of attainable outcomes Y_0^1 (cumulated investment and debt service costs) reduces to Y_0^2 (figure 2), or when it is very large, one cannot invest in future years because all funds will be used for the debt service.

4.2. Analysis of decision variables

In figures 3-10 projections of select model solutions are compared. They are associated with the Pareto outcomes A, B, C and E (table 1), which for the initial debt D_0=15 satisfy the model constraints (6)-(8), maximize the criterion (1) - investment expenditures cumulated over $[t_1, T_N]$, and simultaneously minimize the criterion (2) - debt service costs cumulated over $[T_{N+1}, T_M]$.

Figures 3 and 4

The share of investment expenditures in the total LG's investment expenditures, and the share of the EU co-financed investment in the investment expenditures over the planning period t_1=2014, t_N=2023 are volatile, and the EU financed investment in 2015 and 2016 equal zero for the Pareto point E. The cumulated investment is higher than for the point C (figures 3, 4).

The outstanding old debt issued prior to t_1 and the new debt, for Pareto points B and C are presented in figure 5. The old debt is repaid in 2025, the new debt in 2032. The total debt for the outcomes B and C is similar until 2019, then the debt for the outcome C grows because a larger debt service after 2023 is allowed (figure 6). In point E the outstanding debt grows very fast starting 2021 as a result of decreasing operating surplus (figure 9).

Figure 5

Figure 6

The fiscal rule of the Polish law on public finance requires that the total debt service, including repayment of debt principals, in relation to the total revenue (left hand side of the constraint (8)) is below the statutory limit (right hand side of (8)). In point E, the debt service costs grow over 2023-2028, equal the limit in 2026-2028, and sharply decreases in 2029. The debt service to revenue for points B and E is close over 2014-2020 and 2029-2033 (figure 7).

Figure 7

The fixed assets (GFCF), starting 2021, grow faster for the outcome E than for the outcomes B, C, and A. The maintenance costs of assets are the highest for E.

Figure 8

The share of the operating surplus in total revenue until 2020 is higher for the Pareto outcome E than for B and C, but starting 2021 it falls sharply, together with the operating surplus (figure 9), because of the rising operating expenditures - debt service costs and the GFCF maintenance costs. The implemented in Poland fiscal rule hinders the LG's falling operating surplus (figure 10). The debt service to revenue indicator and its limit in figure 6 conform to the law (artcle 243, pfl) - debt service costs associated with debt co-financing projects together with the EU funds is excluded. The total debt service with "exclusions" to revenue is below the statutory limits for all Pareto outcomes A, B, C, D and E. The debt service without "exclusions", for the outcome E exceeds the limit determined by the condition (8). The exclusion rule hides the rising debt service costs and drastically falling operating surplus (figure 9). A local government observes pictures 6 and 10, while in reality the operating surplus for the output E is very low in 2022 and 2023 (figure 9). The government might not intervene before 2021, when it's needed.

Figures 9 and 10

5. Summary

A new method supporting local government decisions regarding long-term budget projection is developed. The method utilizes a mathematical model reflecting interrelations between budget financial flows and stock over time. Budget liquidity and fiscal rules included as constraints in the LG budget optimization model are satisfied over long-term.

Implementation of the method supports decisions made by LGs' managers – helps determine decision variables, affordable levels of investment, EU funds, and safe medium and long term debt, totally and each year of the projection period. Upper (and lower) limits for cumulated investment and debt service costs - elements of the dm goals are determined. Investments must be planned very carefully because fixed assets maintenance costs grow very fast and budget operating funds may fall dramatically. When the goals are too ambitious, the debt used for investment financing might grow too fast and the debt service may drastically restrain investing in future years. The method enables selection of various goals updated during the projection process and generation of the variables and criteria values (outcomes) associated with these goals ("aspirations", resulting from rational valuation of alternatives [13]). A dm can analyze consequences of her decisions regarding investment, fixed assets formation, debt until its maturity and budget balance. It can be repeated the same year and in future years for a crawling horizon.

[13] Explicit assignment of weights to the model criteria could exclude some important Pareto optimal solutions.

The iterative procedure bases on Simon satisficing behavior theory and implements the reference point solution method of multi-criteria optimization. Outcomes are Pareto optimal. The dm looks for a preferred outcome in an iterative learning process - adapts her aspirations using information on outcomes and decision variables resulting from previous solutions of the model. The procedure includes analysis of scenarios as discussed in Kavanagh 2007: 157-159, for long-term analysis of infrastructure needs (Edmonton city), by Vogt 2004 and Cichocki 2013. The computer-based procedure derives an outcome closest to the reference point representing the dm aspirations, and calculates respective decision variables.

The method accommodates approaches presented in literature - incremental and punctuated budget projections (thanks to assuming alternative goals), budget's dynamics analysis (Lindblom 1959, Baumgartner and Jones 2002, Breunig 2006, Citi 2013), and one criterion optimization model with constraints (Cichocki 2013)[14]. A local dm is introduced who may minimize inappropriate interventions of fiscal rules imposed by the central government as discussed by Steward 2014.

The presented method with the dm involvement and her learning about implications of today actions is new in literature. Looking for the preferred projection scenario with adaptation of aspirations (alternative goals assumption), as well as the utilization of the LG budget flows model and implementation of multi-criteria optimization solution technique are novelty in the budget planning literature. The extensions of the presented budget projection method include: simultaneous consideration of several goals; implementation of the Simon's concept of multi-stage formation of aspirations related to goals, implementation of multi-criteria optimization and learning in the projection selection process, considering explicit debt structure, extension of the analysis period until debt's maturity and selection of the satisficing solution by the dm.

The fiscal rules implemented in the model come from the Polish law, however, country specific rules can be incorporated in the model, and the method can be used in many EU countries

[14] A model was formulated to maximize, over several years, total funds for financing investment (from budget and debt), subject to constraints. Each period, upper limits for safe debt and investment were determined.

with various goals assumed. The method can also be implemented by the central government to analyze alternative fiscal rules' impact on LGs' debt, deficit and investment[15].

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[15] The rules of the Polish law hinder, or allow for, growing debt and decreasing operating surplus (figures 5, 6 and Cichocki 2013).

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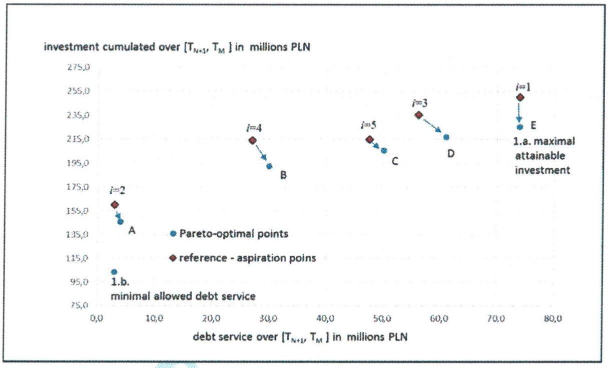
Captions

Table 1. Pareto optimal outcomes, reference points and select model solutions

Results of the procedure										
	Reference point		Pareto-optimal outcome			*Select characteristics of model solutions*				
Step 1 and itera-tions	Sum of invest-ment over 2014-2023	Total debt service over 2024-2033		y_1 Sum of invest-ment over 2014-2023	y_2 Total debt service over 2024-2033	Sum of EU invest-ment over 2014-2023	Sum of new debt over 2014-2023 (new debt: EU invest-ment)	Sum of operating surplus: 2014-2023	Maximum outstan-ding debt	GFCF in 2024
1.a				226.0	73.9					
1.b				103.5	3.0					
i=1	250.0	76.0	E	226.0	73.9	78.0	80.0 (15.6)	52.2	64.6 in 2023	196.4
i=2	160.0	3.0	A	146.0	4.0	62.6	25.2 (12.5)	57.5	27.1 in 2016	124.9
i=3	235.0	56.1	D	217.1	61.0	79.5	69.1 (15.9)	53.2	53.2 in 2023	189.3
i=4	215.0	27.0	B	195.0	30.0	94.1	45.5 (18.8)	51.9	30.3 in 2022	167.0
i=5	216.0	47.5	C	205.0	50.0	101.1	63,4 (20.2)	49.3	44.5 in 2023	174.9

Figure 1. Select Pareto optimal outcomes and reference points



Source: all figures are done by authors.

Figure 2. Feasible sets and Pareto optimal frontiers for different initial conditions

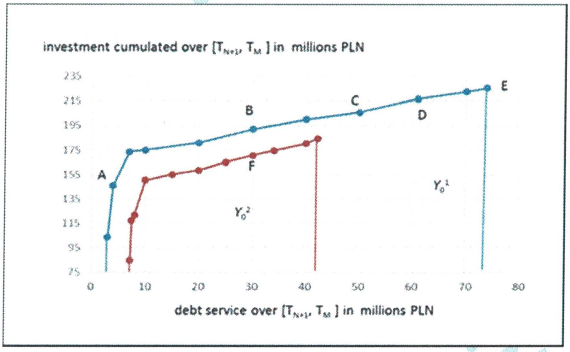


Figure 3. Investment expenditures to total expenditures

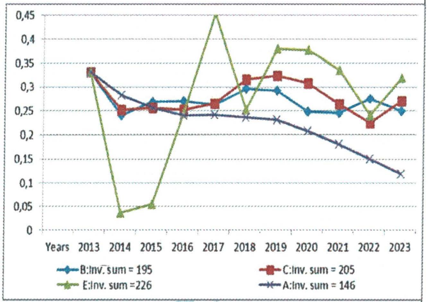


Figure 4. Investment expenditures co-financed with the EU funds to total investment

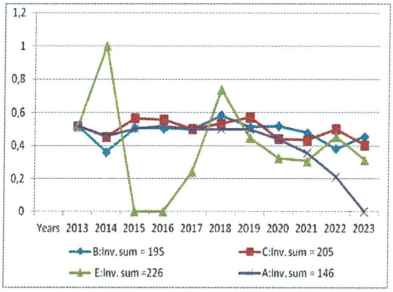


Figure 5. Outstanding old and new debts

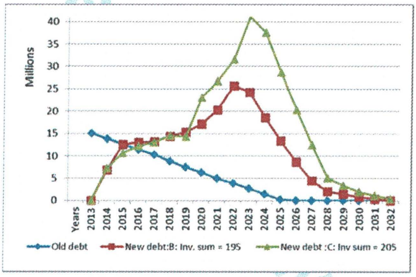


Figure 6. Total outstanding debt

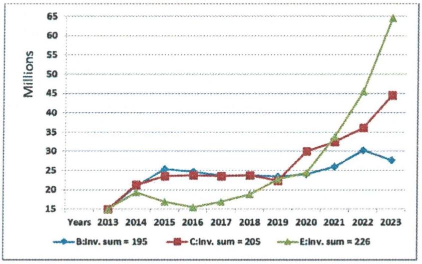


Figure 7. Debt service to total revenue

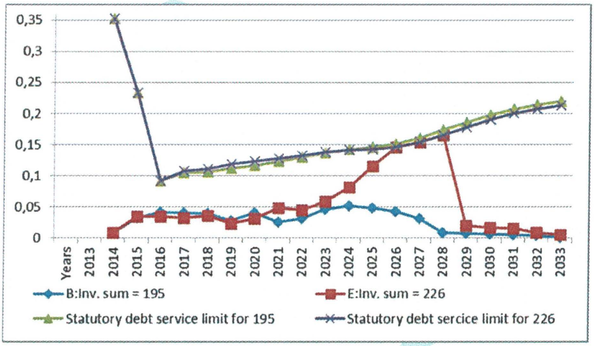


Figure 8. Gross Fixed Capital Formation

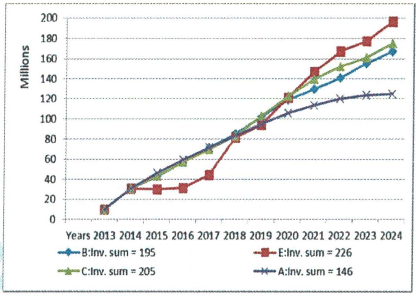


Figure 9. Operating surplus to total revenue

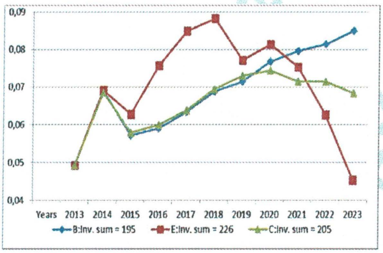


Figure 10. Operating surplus with exclusions to total revenue