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Genetic Algorithm and Neural Network Methods for Inverse Problems of Coupled Models using Topological Derivative

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Chapter 1

Introduction

In the paper we compare two methods based on genetic algorithm and neural network for finding the location of small holes in the domain, in which the coupled boundary value problem is defined. The initial domain consists of two components, linear and nonlinear, connected by the transmission conditions defined at the interface boundary. Both methods: genetic algorithm and neural network calculate the location of one, two or three holes located somewhere in the linear part of the domain based on input data coming from the exterior part of the domain.

We consider a coupled model described by the domain bounded in $\mathbb{R}^2$ and decomposed into two subdomains Ω and ω in such way, that the interior part ω is surrounded by the exterior sub-domain Ω. In the interior subdomain the physical phenomena are described by the linear partial differential equation and in the exterior subdomain the processes are governed by nonlinear partial differential equation subject to some external function. Here, the nonlinear boundary value problem is coupled through transmission conditions with the linear boundary value problem. As an example of such system one can consider a gravity flow around an elastic obstacle. Such situation have numerous physical interpretations, for example the water flow around submarine or gas flow inside the jet engine. For real life models the coupling conditions are still a subject of research [**?**].

Our goal in this paper is to compare two methods. First method is a combination of genetic algorithm and information given by the topological derivative. In this method the location of small hales in the interior domain is approximated by the genetic algorithm which uses the probability density in random selection for the initial population of single holes, pairs of triples, and also to supplement the population in consecutive generations. The probability density is evaluated based on the values of the topological derivative calculated in the interior subdomain ω

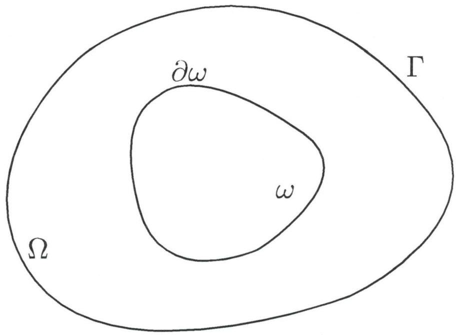
for a given shape functional defined in the exterior subdomain Ω. Second method applies an artificial neural network which calculates the locations of one, two or three holes in ω the linear component of the domain. The information comes from Ω the exterior part of the domain and is represented as a Fourier series expansion of a solution of the nonlinear partial differential equation and calculated at the interface between two subdomains.

1.1 Problem Formulation

Let $D, \omega \in \mathbb{R}^2$ with the smooth boundaries $\partial\omega$, $\Gamma = \partial D$, $D = \Omega \cup \omega$, where $\Omega = D \setminus \overline{\omega}$, such that $\partial\Omega = \Gamma \cup \partial\omega$.

$$\begin{cases} -\Delta U(x) = F(x, U(x)), & x \in D, \\ U(x) = 0, & x \in \Gamma, \end{cases}$$

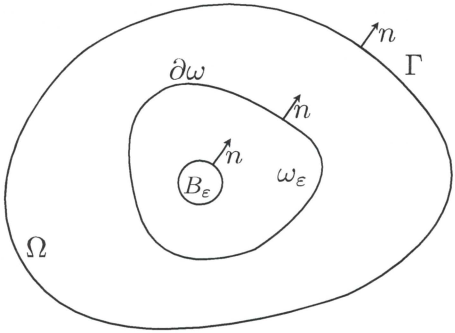
$$F(x, U(x)) = \begin{cases} -U^3(x) + f(x), & x \in \Omega, \\ 0, & x \in \omega. \end{cases}$$



Now we introduce a small perturbation in the domain ω by creating a small hole B_ε at the point $\mathcal{O}$. We denote $\omega_\varepsilon = \omega \setminus \overline{B}_\varepsilon$, $\partial\omega_\varepsilon = \partial\omega \cup \partial B_\varepsilon$

$$\begin{cases} -\Delta U_\varepsilon(x) = F(x, U_\varepsilon(x)), & x \in D \setminus \overline{B}_\varepsilon, \\ U_\varepsilon(x) = 0, & x \in \Gamma, \\ \partial_n U_\varepsilon(x) = 0, & x \in \partial B_\varepsilon \end{cases}$$

$$F(x, U_\varepsilon(x)) = \begin{cases} -U_\varepsilon^3(x) + f(x), & x \in \Omega, \\ 0, & x \in \omega_\varepsilon. \end{cases}$$



Let $\mathcal{A}_\varepsilon : \varphi \in H^{1/2}(\partial\omega) \longrightarrow \partial_n U_\varepsilon \in H^{-1/2}(\partial\omega)$: We can rewrite the condition on the boundary $\partial\omega$ using the Steklov-Poincaré operator $\mathcal{A}_\varepsilon$:

$$\begin{cases} -\Delta U_\varepsilon(x) = F(x, U_\varepsilon(x)), & x \in D \setminus \overline{B}_\varepsilon, \\ U_\varepsilon(x) = 0, & x \in \Gamma, \\ U_\varepsilon(x) = \varphi(x),\ \partial_n U_\varepsilon(x) = \mathcal{A}_\varepsilon(U_\varepsilon(x)), & x \in \partial\omega, \\ \partial_n U_\varepsilon(x) = 0, & x \in \partial B_\varepsilon \end{cases}$$

Let us then consider both linear and nonlinear problems separately.

In the domain Ω we have the following non-linear problem:

$$\begin{cases} -\Delta v_\varepsilon(x) + v_\varepsilon^3(x) = f(x) & x \in \Omega, \\ v_\varepsilon(x) = 0, & x \in \Gamma, \\ \partial_n v_\varepsilon(x) = \mathcal{A}_\varepsilon(v_\varepsilon(x)), & x \in \partial\omega. \end{cases}$$

In the domain ω_ε, for $\varphi \in H^{1/2}(\partial\omega)$ such that $\mathcal{A}_\varepsilon(\varphi) = \partial_n u_\varepsilon$:

$$\begin{cases} -\Delta u_\varepsilon(x) = 0, & x \in \omega_\varepsilon \\ u_\varepsilon(x) = \varphi(x), & x \in \partial\omega \\ \partial_n u_\varepsilon(x) = 0, & x \in \partial B_\varepsilon \end{cases}$$

1.1.1 Topological Derivative

Let us consider the following shape functional

$$J(v_\varepsilon) = \frac{1}{2}\int_\Omega (v_\varepsilon - z_d)^2 dx,$$

with v_ε the solution to the semi-linear problem and z_d a fixed target funtion defined in the domain Ω. Let us introduce the adjoint state in order to symplify the form of topological derivative

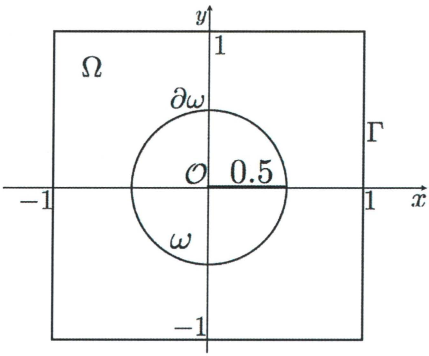
$$\begin{cases} -\Delta p + 3v^2 p &= (v - z_d), & \text{in} \ \ \Omega, \\ -\Delta p &= 0, & \text{in} \ \ \omega \\ p &= 0, & \text{on} \ \ \Gamma, \end{cases}$$

where v is solution to the semi-linear problem for $\varepsilon = 0$.

Theorem 1.1.1 *The form of topological derivative is the following*

$$\mathcal{T}_\Omega(\mathcal{O}) = -\langle \mathcal{B}(v), p \rangle = 2\pi \nabla v(\mathcal{O}) \cdot \nabla p(\mathcal{O}).$$

1.2 Numerical Approach



$$\text{In } \omega: \begin{cases} -\Delta u(x) &= 0, & x \in \omega \\ u(x) &= v(x), & x \in \partial\omega \end{cases}$$

$$\text{In } \Omega: \begin{cases} -\Delta v(x) + v^3(x) &= f(x) & x \in \Omega, \\ v(x) &= 0, & x \in \Gamma, \\ \partial_n v(x) &= \mathcal{A}(v(x)), & x \in \partial\omega. \end{cases}$$

In order to solve numerically the coupled problem, we introduce a characteristic function χ and we consider the following problem:

$$\begin{cases} -\Delta w(x) + \chi(\Omega) w^3(x) &= \chi(\Omega) f(x) & x \in D, \\ w(x) &= 0 & x \in \Gamma, \end{cases} \text{ with } \chi(\Omega) = \begin{cases} 1 & x \in \Omega, \\ 0 & x \in \omega. \end{cases}$$

1.3 Inverse Problem

The inverse problem that we consider here is to find the location of some inclusions of hollow voids inside the interior domain ω based on the information coming from exterior subdomain and such location that minimizes the value of the objective functional

$$J(v_\varepsilon) = \frac{1}{2} \int_\Omega (v_\varepsilon - z_d)^2 dx.$$

To this end we apply two methods:

1. Method based on Genetic Algorithm
2. Method based on Neural Network

1.3.1 Method based on Genetic Algorithm

1. Density of probability $\mathcal{P}_k = \dfrac{\tilde{s}_k}{\frac{1}{3}\sum_{i=1}^{M} \text{area}(t_{i*}) \sum_{j=1}^{3} \tilde{s}_{t_{ij}}}, \quad k = 1, \ldots, N$
   where $\tilde{s}_k$ contains the information given by topological derivative, $\tilde{s}_{t_{ij}}$ are the vertices of the triangles, M is the number of triangles and N is the number of nodes.

2. Genetic algorithm
   - initial population - vector of inclusions
     - not necessarily at nodes of triangles
     - in the area where density probability is the highest
   - evaluation - fitness value evaluated based on cost function J
   - crossover
     - selecting dominating elements
     - crossover with every subordinate element
   - mutation - perturbation of each element
   - new generation - $\frac{2}{3}S$ of the best elements after mutation, $\frac{1}{3}S$ individuals are again drawn randomly using appropriate probabilities,

1.3.2 Method based on Neural Network

1. Inverse mapping $g(a_0, a_1, b_1, a_2, b_2, a_3, b_3) = (x, y)$
   - $[a_0, a_1, b_1, a_2, b_2, a_3, b_3]$ are coefficients in the Fourier series expantion of the solution v of the problem (1) taken at the boundary $\partial\omega$
   - $[x, y]$ are the coordinates of a center of a hollow void, its location in ω
2. Topology of neural network
   - input vector - 7 coefficints
   - one hidden layer
   - output - 2 neurons for one hollow void, 4 neurons for 2 hollow voids
   - sigmoidal activation function for each of the layer
3. Learning set $L = \{P, T\}$
   - Paterns $P = \{p_1, \ldots, p_n\}$, $p_i = [a_0, a_1, b_1, a_2, b_2, a_3, b_3]$
   - Target $T = \{t_1, \ldots, t_n\}$, $t_i = [x, y]$

1.3.3 Numerical results

Case of one hollow void

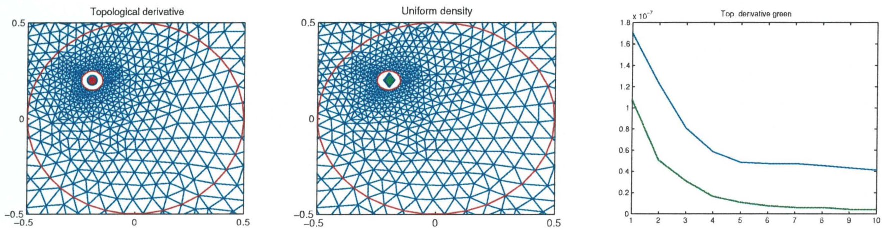


Figure 1.1: Method based on Genetic Algorithm

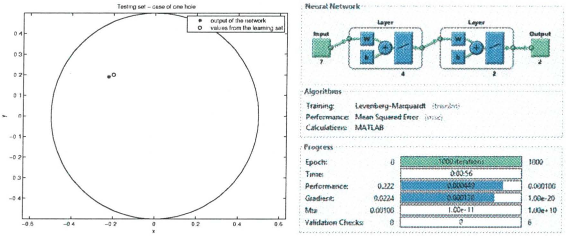


Figure 1.2: Method based on Neural Network

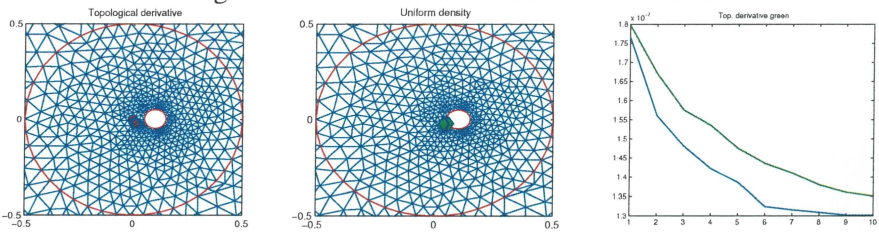


Figure 1.3: Method based on Genetic Algorithm

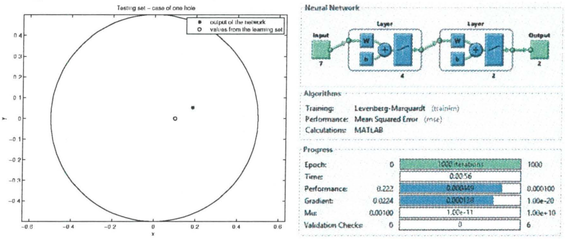


Figure 1.4: Method based on Neural Network

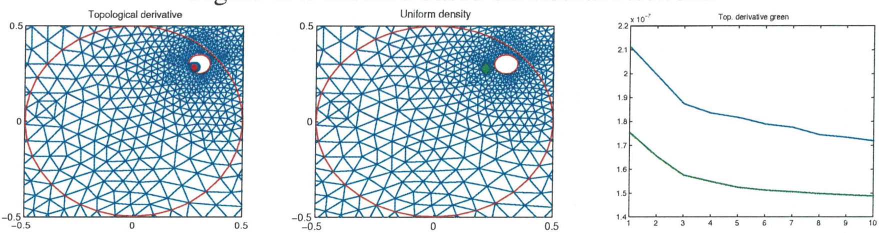


Figure 1.5: Method based on Genetic Algorithm

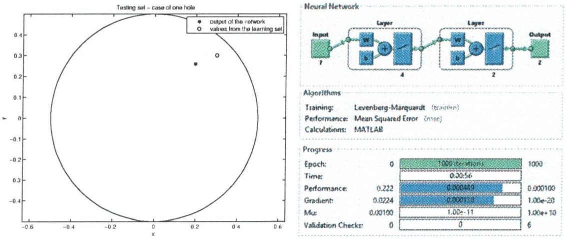


Figure 1.6: Method based on Neural Network

Case of two hollow voids

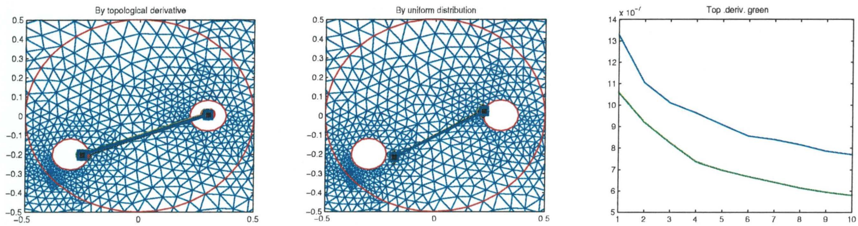


Figure 1.7: Method based on Genetic Algorithm

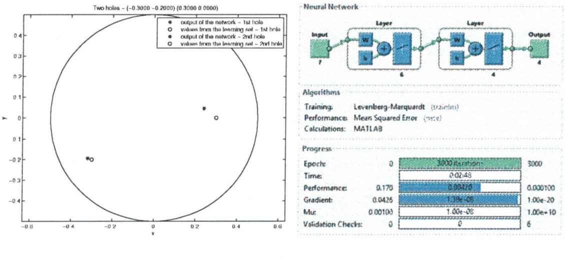


Figure 1.8: Method based on Neural Network

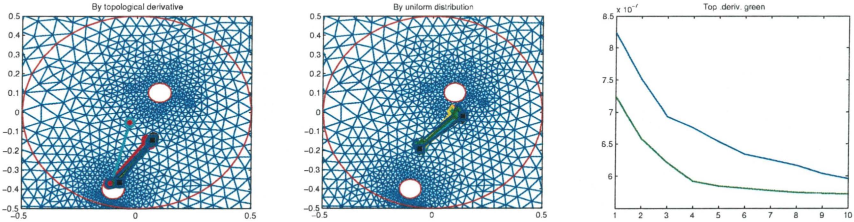


Figure 1.9: Method based on Genetic Algorithm

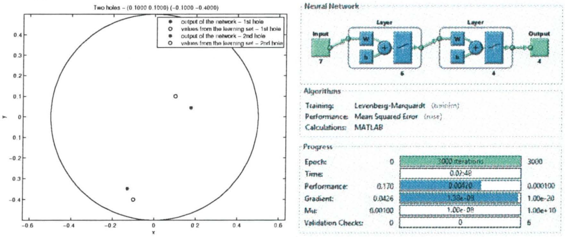


Figure 1.10: Method based on Neural Network

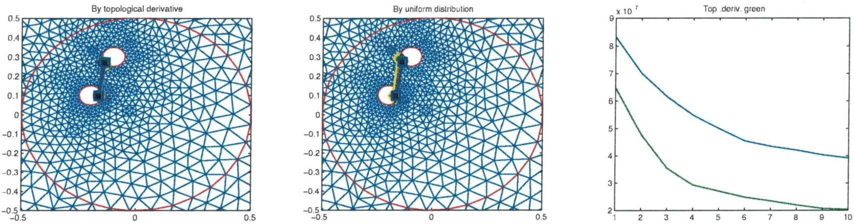


Figure 1.11: Method based on Genetic Algorithm

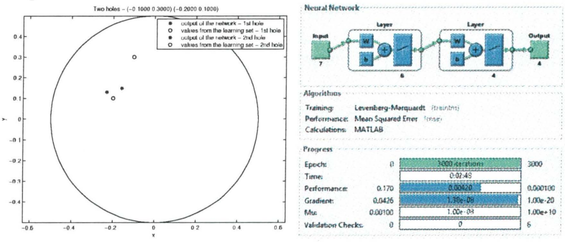


Figure 1.12: Method based on Neural Network

1.4 Conclusions

1. Methods based on Genetic Algorithm gives slightly better results if the density of probability concern the values of the topological derivative.

2. In both methods the error is comparable, no matter the number of inclusions

3. Method based on Neural Network gives the results that can be compared with GA with uniform density of probability

4. For both methods, result depends on the location of inclusion - better results for inclusion near the boundary, worts results for inclusions located far from the boundary

Bibliography

[1] P. Fulmanski, A. Lauraine, J.-F. Scheid, J. Sokołowski, A level set method in shape and topology optimization for variational inequalities, Int. J. Appl. Math. Comput. Sci., Vol.17, No 3, p.413-430 (2007).

[2] S. Garreau, Ph. Guillaume, M. Masmoudi, The topological asymptotic for PDE systems: the elasticity case, SIAM J. Control Optim., vol. 39, No 6, pp. 1756-1778 (2001).

[3] S. M. Giusti, A. A. Novotny, C. Padra, Topological sensitivity analysis of inclusion in two-dimensional linear elasticity, Engineering Analysis With Boundary Elements, vol. 32, no. 11, pp. 926-935 (2008).

[4] M. Grzanek, A. Nowakowski, J. Sokolowski, Topological derivatives of eigenvalues and neural networks in identification of imperfections, Journal of Physics: Conference Series 135 (2008) 012046.

[5] M. Grzanek, A. Laurain, K. Szulc, Numerical algorithms for an inverse problem in shape optimization, Journal of Physics: Conference Series 135 (2008) 012047.

[6] M. Ignatowa, I. Kukavica, I. Lasiecka, A. Tuffaha, On well-posedness for a free boundary fluid-structure model, Journal of Mathematical Physics, 53 (2013).

[7] M. Iguernane, S.A. Nazarov, J.-R. Roche, J. Sokolowski, K. Szulc, Topological derivatives for semilinear elliptic equations, Int. J. Appl. Math. Comput. Sci., Vol.19, No.2, p.191-205 (2009).

[8] S.A. Nazarov, J. Sokołowski, Asymptotic analysis of shape functionals, Journal de Mathématiques pures et appliquées, Vol. 82, pp.125-196 (2003).

[9] J. Sokołowski, A. Żochowski, On the topological derivative in shape optimization, SIAM Journal on Control and Optimization. Vol. 37, No. 4, pp.1251–1272 (1999).

[10] J. Sokołowski, A. Żochowski, Optimality conditions for simultaneous topology and shape optimization, SIAM Journal on Control and Optimization, Vol. 42, No. 4, pp.1198–1221 (2003).

[11] J. Sokolowski, A. Zochowski, Modelling of topological derivatives for contact problems, Numerische Mathematik, Vol. 102, No. 1, pp.145 - 179 (2005).

[12] J. Sokolowski, A. Zochowski, Asymptotic analysis and topological derivatives for shape and topology optimization of elasticity problems in two spatial dimensions, Engineering Analysis with Boundary Elements. Vol. 32, pp.533-544 (2008).

[13] J. Sokołowski, J.-P. Zolésio, Introduction to shape optimization. Shape sensitivity analysis, Springer-Verlag, New York (1992).

[14] K. Szulc, A. Żochowski, Application of topological derivative to accelerate genetic algorithm in shape optimization of coupled models. Structural and Multidisciplinary Optimization: Volume 51, Issue 1 (2015), Page 183-192.

[15] H. White, Connectionist Nonparametric Regression: Multilayer Feedforward Networks Can Learn Arbitrary Mappings, Neural Networks, vol. 3 (1990), pp. 535-549.