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ON THE APPLICATION OF TRIMAP TO PROBLEMS WITH MULTIPLE DECISION MAKERS

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Abstract: In this paper the potentialities of TRIMAP to provide decision support in multiobjective problems with multiple decision makers are exploited. TRIMAP is an interactive three-objective linear programming package which enables a progressive and selective learning of the nondominated solution set. The aim is to aid the opposing parties explore their own preferences and to explore the dynamic nature of the negotiation process.

Keywords: Multiple objective linear programming; interactive methods; multiple decision makers; group decision support systems; negotiation; computer graphics.

1. Introduction

Most real-world problems involve multiple, conflicting, incommensurate criteria. Mathematical models as well as the perception of the problems by the decision maker (DM) become more realistic if several criteria are considered explicitly in the models, instead of encompassing the different aspects of reality in a single criterion. The concept of optimal solution in single objective optimization gives place to the concept of nondominated solution in a multiple criteria context. A feasible solution is nondominated if no improvement in any objective function is possible without sacrificing on at least one of the other objective functions. These decision problems entail tradeoffs among the objectives, in order to get a satisfactory compromise solution from the set of nondominated solutions. Different methods to deal with multiple criteria problems exist, using different solution techniques and requiring distinct degrees of involvement of the DM. Interactive methods allow for the intervention of the DM in the solution search process, by inputing information into the procedure which in turn is used to guide the search process (thus minimizing the computational effort) in order to compute a new solution which more closely corresponds to his/her evolutionary preferences.

On the other hand, in many real-world situations decisions are seldom made by an individual. The decision making process is participated by different opposing parties (consisting of one or more individuals) who interact in order to obtain a solution which can be accepted by all members of the group. The interaction of the multiple DMs configures a negotiation or bargaining process in the search for satisfactory solution which could be a commonly accepted. Multiple criteria problems with multiple DMs may be characterized as double conflict problems: tradeoffs must be made not only among the criteria which reflect distinct aspects of the reality, but also among the DMs who have different perspectives and interests.

The great importance of multiple criteria problems with multiple DMs has been recognized by several authors, and many previous approaches to deal with these problems exist, both from purely theoretical and algorithmic perspectives. Within the classical theoretical studies we would like to mention Arrow (1951), Cross (1965) and Raiffa (1982). The works of algorithmic nature include the methodologies proposed by Wendell (1980), Isermann (1985), Kersten (1985), Kersten and Szapiro (1986), Korhonen et al. (1986), Lewandowski (1989), and Korhonen and Wallenius (1990), among others. A survey of recent developments in group decision support systems is presented in Vetschera (1990). Most authors recognize the need of carrying out more application experiments in order to assess the potentialities of the methodologies proposed, in the operational framework of decision support systems.

In this paper the interactive and user-friendly capabilities of the TRIMAP package are exploited, as a tool for providing decision support in negotiation processes based on three-objective linear programming models. The aim is to help the parties to explore their own evolutionary preferences (as more knowledge about the set of nondominated solutions is gathered in each interaction) and to make the most of the dynamic nature of the group decision process.

2. The TRIMAP Package

The TRIMAP method enables a progressive and selective learning of the set of nondominated solutions. The method combines three main procedures: weight space decomposition, introduction of constraints on the objective function space and weight space reduction. Furthermore, the introduction of constraints on the objective function values can be translated into weight space reductions. The dialogue with the DM is made mainly in terms of the objective function values in order to reduce the cognitive burden on the DM. The weight space is used in TRIMAP as a means for collecting and presenting the information to the DM.

The interactive process continues until the DM has sufficient knowledge about the set of nondominated solutions to make an informed selection of a satisfactory compromise solution. There are no irrevocable decisions as it is always possible to go backwards at a later interaction and thereby rescind an earlier decision. Given the limited capacity of human beings to process information, the interactive aspects of TRIMAP were designed to be flexible (i.e., rescindable) and simple (i.e., information demands on the DM are not too demanding). TRIMAP is dedicated to support DMs in dealing with three-objective linear programming problems. Although this limits its application to such problems, this permits the use of graphical means which are particularly suited for the dialogue with the DM. These graphical techniques enhance the DM's capacity in processing the tradeoff information by simplifying the DM-computer dialogue phase and allowing the DM to make visual comparisons.

The fundamental structure of the DM-computer interface offered by TRIMAP is a menu bar at the top of the screen which lists the titles of the available pulldown menus, grouping the actions available to the user (thus not occupying screen space and not requiring command memorization). Overlapping windows improve the availability of the information, and are used for displaying graphical and text information. Dialogue boxes are used to request further information from the user before a given command can be carried out or to convey some useful information to the user. Pictorial controls provide the user an intuitive way for specifying his/her preferences.

2.1. Three-objective linear programming

TRIMAP is designed for three-objective linear programming problems:

$$\text{"Max"} \quad C\underline{x}$$
$$\text{s. t.} \quad \underline{x} \in X \equiv \{ \underline{x} \in \mathbb{R}^n : A\underline{x} = \underline{b}, \underline{x} \geq \underline{0} \}$$

where $C = [\underline{c}_1, \underline{c}_2, \underline{c}_3]'$ is the objective function coefficient matrix, $\underline{c}_1, \underline{c}_2, \underline{c}_3$ are nx1 column vectors, A is a mxn technological matrix and $\underline{b}$ is a mx1 RHS column vector. Without loss of generality we will admit that A has rank m and that all the constraints are converted into equalities. "Max" denotes the operation of finding nondominated solutions.

The set of efficient solutions is defined by $X_E = \{ \underline{x} \in X \mid \nexists\, \underline{x}' \in X : \underline{f}(\underline{x}') \geq \underline{f}(\underline{x})\}$, where $\underline{f}(\underline{x}')\geq\underline{f}(\underline{x})$ iff $\underline{f}(\underline{x}')\geq\underline{f}(\underline{x})$ and $\underline{f}(\underline{x}')\neq\underline{f}(\underline{x})$, and $\underline{f}(\underline{x}')\geq\underline{f}(\underline{x})$ iff $f_k(\underline{x}')\geq f_k(\underline{x})$, k=1,2,3.

The criterion vector $\underline{f}(\underline{x})$ is nondominated whenever $\underline{x} \in X_E$. The concept of nondominance refers generally to the objective function space, whereas the concept of efficiency refers to the decision variable space. A satisfactory compromise solution must be an element of this set.

$\underline{x} \in X$ is an efficient solution iff it is an optimal solution to the following linear problem

$$\max \quad \lambda_1 f_1(\underline{x}) + \lambda_2 f_2(\underline{x}) + \lambda_3 f_3(\underline{x})$$
$$\text{s. t.} \quad \underline{x} \in X$$
$$\underline{\lambda} \in \Lambda \equiv \{ \underline{\lambda} \in \mathbb{R}^3 : \sum_{k=1}^{3} \lambda_k = 1 \; ; \lambda_k > 0 \,, \; k=1,2,3 \}$$

The graphical display of the set $\underline{\lambda}$ which leads to each efficient solution can be achieved through the decomposition of the weight space Λ. From the simplex tableau corresponding to an efficient basic solution to this problem the corresponding $\underline{\lambda}$ set is given by $\underline{\lambda}^T W \geq \underline{0}$, where $W = C_B B^{-1} N - C_N$ is the reduced cost matrix (one line for each objective function, where the element w_{kj} is the marginal rate of change of objective function $f_k(\underline{x})$ caused by the introduction of one unit of variable x_j into the basis). B (C_B) and N (C_N) are the submatrices of A (C) corresponding to the basic and nonbasic variables, respectively. The region comprising the set of weights corresponding to a nondominated extreme solution (region where $\{\underline{\lambda}^T W \geq 0, \underline{\lambda} \in \Lambda\}$ is consistent) is called indifference region. The

DM may then be indifferent to all combinations of weights within it, because they lead to the same nondominated solution. The boundaries between contiguous indifference regions represent the nonbasic efficient variables (those which when introduced into the basis lead to an adjacent efficient extreme point through an efficient edge). A common boundary between two indifference regions means that the corresponding efficient solutions are connected by an efficient edge. If a point $\underline{\lambda} \in \Lambda$ belongs to several indifference regions this means that these correspond to efficient solutions lying on the same face. The analysis of the weight space is thus a valuable tool in the learning of the shape of the nondominated region.

2.2. An overview of TRIMAP

The interactive process begins with the automatic computation of the nondominated solutions corresponding to the optima of the three objective functions. This is intended to provide the DM with information that allows him/her to have a global knowledge of the nondominated region. Next, a new computation phase is prepared through a dialogue phase with the DM. These two phases are repeated until the DM has decided that sufficient information regarding the options and the tradeoffs among them has been generated to enable him/her to select a satisfactory compromise solution.

The selection of the weights at each interaction may be made in a direct or indirect manner. In the indirect manner the DM selects three nondominated solutions, which are used to construct a weighted function the gradient of which is normal to a constant cost plane passing through them. In the direct manner the selection of weights is made by the DM's selection of the unfilled weight space regions, which he/she thinks it is important to evaluate.

The introduction of additional constraints on the objective function values and its translation into the weight space enables the dialogue with the DM to be conducted in terms of those values (generally, the space most familiar to DMs). TRIMAP automatically converts this information into the weight space in graphical form. Whenever the DM imposes an additional limitation on the objective function values $f_k(\underline{x}) \geq L_k$ ($L_k \in \mathbb{R}$), the auxiliary problem

$$\max \; f_k(\underline{x})$$
$$\text{s. t.} \;\; \underline{x} \in F_a \equiv \{\, \underline{x} \in X : f_k(\underline{x}) \leq L_k \,\}$$

is solved. By maximizing $f_k(\underline{x})$ over F_a alternative (basic) optimal solutions are obtained. The efficient extreme points of F_a which optimize this auxiliary problem are selected and the subregions of the weight space corresponding to each of these points are computed and displayed (these are the indifference regions defined by $\underline{\lambda}^T W \geq 0$, corresponding to each efficient alternative basis). The union of all these subregions determines the region of the weight space where the additional limitation on the objective function value is satisfied. If the DM is only interested in the nondominated solutions which satisfy $f_k(\underline{x}) \geq L_k$, then it is sufficient, from now on, to restrict the search to sets of weights within this region. If more than one limitation is imposed, then the auxiliary problem is solved for each one and the corresponding regions of the weight space are filled with different patterns, thus enabling to visualize clearly the zones where they have their intersections.

It is also possible that regions of the weight space be eliminated by imposing limitations directly on the variation of the weights.

In each interaction of TRIMAP two main graphs are presented to the DM. The first one is the weight space displaying the indifference regions corresponding to each nondominated extreme solution already known. Constraints on the variation of the weights are also presented, whether they are directly introduced into the weight space or result from additional constraints imposed on the objective function values. The second graph displays the nondominated solutions already computed projected on a plane of the objective function space. Some complementary indicators corresponding to each efficient solution are also available: the Tchebycheff (L_∞), Euclidean (L_2) and "city block" (L_1) distances to the "ideal solution" (the one that would optimize all the objective functions simultaneously, which it is not feasible when the objectives are in conflict) and the area of the indifference region (which is a measure of the robustness of the solution regarding the variation of the weights). Further details about the working of the TRIMAP method as well as the main features of its computer implementation can be found in Clímaco and Antunes (1987, 1989).

3. The Application of TRIMAP to Problems with Multiple Decison Makers

In this work we admit that the opposing parties agree on the mathematical model (decision variables, constraints and objective functions), and none of the parties has a privileged position with respect to the decision variables. There may exist external players in the decision process who can make suggestions that will get the opposing parties to agree on a solution or relax their requirements throughout the negotiation process (a mediator), and to mediate the communication between the parties and the computer by helping to interpret the graphical displays and the available commands (an analyst). However, these players (which can be the same individual) perform technical functions only, and do not possess any power to dictate a final compromise solution. Eventually negotiations may break down and a stalemate may arise.

The main goal is to propose rules for the negotiation process aimed at supporting the parties in the search for a satisfactory compromise solution, in the framework of an interactive decision process. In multiple objective programming problems the set of nondominated solutions is generally infinite and implicitly defined by the set of constraints. A previous stage of the interactive decision process where the DMs can carry out an exploration of the nondominated region in order to learn about the problem is necessary. This enables the DMs to gather knowledge about the problem which in turn may contribute to revise his/her preferences throughout the process, and accommodates the complexity of group decision problems, in which the DMs are often incoherent, they bluff and they do coalitions.

The procedure to use TRIMAP in problems with multiple DMs is as follows:

* TRIMAP computes the nondominated solutions which optimize each objective function separately. This information is displayed graphically and numerically.

* Each party p (p=1,...,P) specifies reservation r_{pk} and aspiration s_{pk} levels for each objective function k (k=1,2,3). The reservation level r_{pk} is the minimum value that DM p is willing to accept for objective k. The aspiration level s_{pk} is the minimum value for which DM p is completely satisfied regarding objective k. If the DM does not specify r_{pk}, then he/she accepts any value for objective k, and it is initialized as the minimum value previously computed for that objective function. If the DM does not specify s_{pk}, then it is initialized as r_{pk} (thus defining a threshold value below which objective k is not acceptable and above which it is fully acceptable).

* Based on these levels an acceptability function G_p^q is defined for each party p concerning a nondominated solution q as $G_p^q = (\sum_{k=1}^{K} G_{pk}^q) / K$, (K=3),

where $G_{pk}^q = \dfrac{f_k^q - r_{pk}}{s_{pk} - r_{pk}}$, for $r_{pk} \le f_k^q \le s_{pk}$ (f_k^q is the value of objective function k for solution q)

$G_{pk}^q = 0$, for $f_k^q < r_{pk}$

$G_{pk}^q = 1$, for $s_{pk} < f_k^q$

* An overall acceptability function for the group concerning a nondominated solution q may be computed as $G^q = (\sum_{p=1}^{P} v_p G_p^q) / (\sum_{p=1}^{P} v_p)$, where v_p is the voting power of DM p. This value is computed for each nondominated solution already known.

* The limitations $f_k \ge s_{pk}$ are translated into the weight space for each DM p.

* The limitations $f_k \ge \max_p s_{pk}$ are translated into the weight space.

By visual inspection of the weight space graph it is easy to conclude whether nondominated (extreme) solutions satisfying these additional requirements exist. If the corresponding regions in the weight space overlapp then a search in the intersection region may be carried out. If this is not the case, the analysis of the objective function projection graph permits to conclude whether nondominated solutions which are not extreme points satisfying those limitations are already known.

* The nondominated solution which minimizes a weighted Tchebycheff distance to the ideal solution or to $S=(s_1,s_2,s_3)$, where $s_k = \max_p s_{pk}$, k=1,2,3, is computed. The nondominated extreme points defining the face (or the edge) where this solution is located may also be computed.

* The parties may specify new reservation and/or aspiration levels at any time of the interactive process. The acceptability values are recomputed for all solutions.

* If nondominated solutions which have an acceptability function value $G_p^q \neq 0$ for all DMs exist, then the one with the higher overall acceptability is selected as the final compromise solution. Eventually compromise solutions may be selected, for which $G_p^q=0$ for some $p=1,...,P$.

* The interactive process ends when a compromise solution may be recognized. A stalemate arises when no DM is willing to relax his/her bounds on the objective function values, by specifying new reservation and/or aspiration levels in order to compute a solution with greater overall acceptability.

The procedure proposed herein is a unstructured one in the sense that there is not a pre-specified rigid sequence of steps, but several actions are available in each interaction to be used as they seem more convenient to search for new solutions which may have greater group acceptability. By making the most of the modular structure of the TRIMAP package new interfaces have been added designed to provide decision support in multiple DMs problems. These include a graph of the acceptability space, as well as means of interaction for inputing parameters in a user-friendly manner.

4. An Illustrative Example

In order to illustrate the use of TRIMAP to problems with multiple DMs let us consider the following three-objective problem.

$$\text{"Max"} \begin{bmatrix} 3 & 1 & 2 & 1 \\ 1 & -1 & 2 & 4 \\ -1 & 5 & 1 & 2 \end{bmatrix} \underline{x}, \ \text{s. t.} \begin{bmatrix} 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \\ 1 & 2 & 3 & 4 \end{bmatrix} \underline{x} \leq \begin{bmatrix} 60 \\ 60 \\ 50 \end{bmatrix}, \underline{x} \geq 0 \,, \underline{x} \in \mathbb{R}^4.$$

The program begins by computing the nondominated solutions which optimize each objective function. The results obtained are presented in Table I and displayed in fig. 1.

Solution	f_1	f_2	f_3	$\underline{x}_B$	L_∞	Area (%)
1	**66.0**	30.0	-12.0	x_1=18.0; x_3=6.0	87.0	13.43
2	51.0	**50.0**	4.0	x_1=14.0; x_4=9.0	71.0	17.01
3	15.0	-15.0	**75.0**	x_2=15.0	65.0	1.25
4	12.5	50.0	25.0	x_4=12.5	53.5	7.87

Table I - Numerical information concerning the solutions which optimize each objective function

In Table I the values in bold are the components of the ideal solution, which is displayed as a black square on the objective function graph. The area of the indifference regions are given in percent values of the total weight space area. Under the column $\underline{x}_B$ only the decision variables are shown.

Note that solution 4 is an alternative optimum of solution 2 with respect to f_2, and the nondominated edge between these extreme solutions is already known.

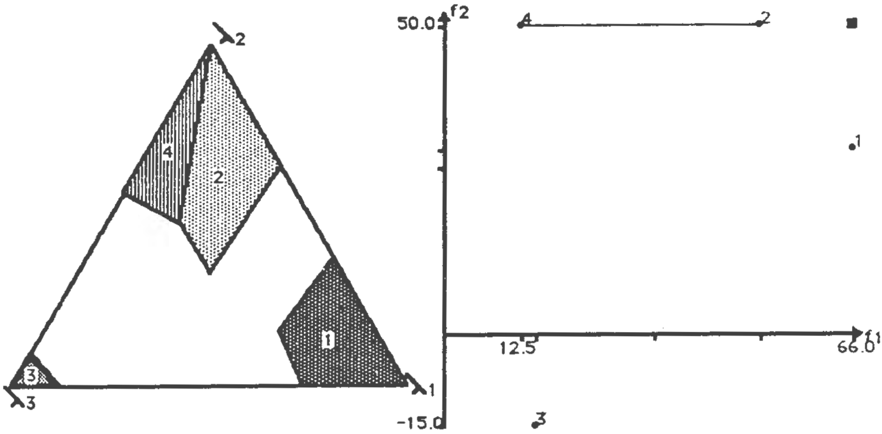

Fig. 1 - Graphs displayed after optimizing each objective function

With this information let us suppose that two hypothetical DMs establish the following reservation and aspiration levels:

DM 1: r_{11}=20, s_{11}=60; r_{12}=10, s_{12}=25; r_{13}=30, s_{13}=40;

DM 2: r_{21}=18, s_{21}=25; r_{22}=15, s_{22}=48; r_{23}=20, s_{23}=35;

The acceptability values of each solution for each DM are:

DM 1: G_1^1=0.667, G_1^2=0.592, G_1^3=0.333, G_1^4=0.333;

DM 2: G_2^1=0.485, G_2^2=0.667, G_2^3=0.333, G_2^4=0.444;

Considering that each DM has equal voting power, at this stage solution 2 is the most acceptable one: G^1=0.576, G^2=0.629, G^3=0.333, G^4=0.389.

Additional constraints on the objective function values are then introduced (DM 1: $f_1 \geq 60$, $f_2 \geq 25$, $f_3 \geq 40$; DM 2: $f_1 \geq 25$, $f_2 \geq 48$, $f_3 \geq 35$;) which are translated into the weight space where they are displayed graphically. In this manner the DMs may grasp the tradeoffs to be made among the objectives concerning his/her own preferences.

By analyzing the graphs in fig. 2, it can be concluded that no extreme solutions exist which satisfy simultaneously all the limitations on the objective function values for both DM 1 and DM 2. For DM 1 there are extreme solutions which satisfy simultaneously the limitations on f_1 and f_2 (solution 1, see fig. 1); for DM 2 there are extreme solutions which satisfy simultaneously the limitations on f_1 and f_2 (solution 2, see fig. 1) and on f_1 and f_3 (a region not yet explored).

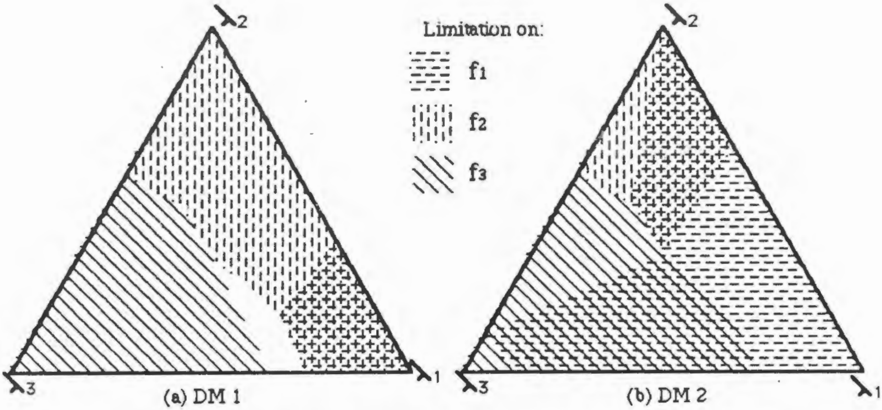

Fig. 2 - The weight spaces after imposing additional constraints on the objective function values

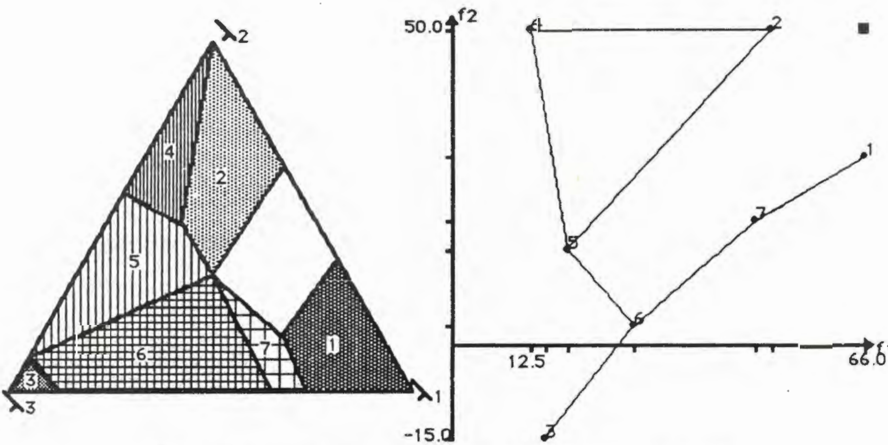

Fig. 3 - Computing new solutions by performing a selective search

At this stage the DMs may use this information to search for new nondominated solutions by using the means of interaction available in the TRIMAP package. The graphs in fig. 3 were obtained after both DMs have explored the regions in which solutions more close to their preferences are located.

Some information gathered in this process may be summarized as follows. Solutions 6 and 7 satisfy simultaneously the limitations on f_1 and f_3 derived from the aspiration levels of DM 2. There are solutions on the face 2-4-5 which satisfy simultaneously the limitations on f_1 and f_3 derived from the aspiration levels of DM 1.

The acceptability values of the new extreme solutions for each DM are:

DM 1: G_1^5=0.444, G_1^6=0.408, G_1^7=0.682; DM 2: G_2^5=0.349, G_2^6=0.667, G_2^7=0.712;

The overall acceptability values for these solutions are G^5=0.397, G^6=0.538, G^7=0.697.

In order to enable a further agreement let us suppose that the DMs are willing to establish lower aspiration levels: DM 1: s_{11}=54; s_{13}=35; and DM 2: s_{22}=45;

The most restrictive aspiration levels of both DMs may be used to introduce additional limitations on the objective function values: $f_1 \geq 54$; $f_2 \geq 45$; $f_3 \geq 35$. A region exists in which the limitations on f_1 and f_2 are satisfied simultaneously, and the search in this region leads to the situation displayed in fig. 4.

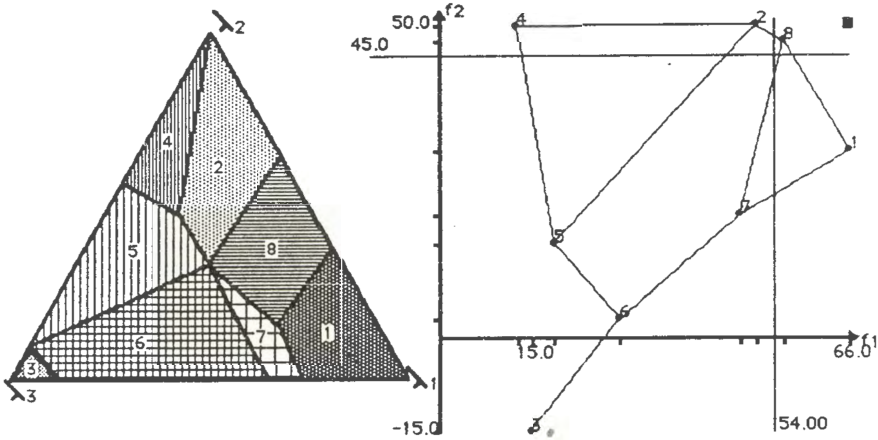

Fig. 4 - Relaxing the aspiration levels and computing new solutions

Solution	f_1	f_2	f_3	x_B	L_∞	Area (%)
5	18.3	15.0	71.7	x_2=11.7; x_4=6.7	47.7	17.03
6	29.0	3.0	73.0	x_2=13.0; x_3=8.0	47.0	23.71
7	48.5	19.5	37.0	x_1=7.5; x_2=7.0; x_3=9.5	38.0	4.62
8	55.5	47.5	2.0	x_1=14.5; x_3=2.5; x_4=7.0	73.0	15.09

Table II - Numerical information concerning the new solutions

The acceptability values are recomputed for all solutions:

G_1^1=0.667, G_1^2=0.637, G_1^3=0.333, G_1^4=0.333, G_1^5=0.444, G_1^6=0.422, G_1^7=0.824, G_1^8=0.667;

G_2^1=0.500, G_2^2=0.667, G_2^3=0.333, G_2^4=0 444, G_2^5=0.349, G_2^6=0.667, G_2^7=0.717, G_2^8=0.667;

Solution 7 is the most acceptable one:

G^1=0.583, G^2=0.652, G^3=0.333, G^4=0.389, G^5=0.397, G^6=0.544, G^7=0.770, G^8=0.667;

The acceptability graph displayed to the DMs is presented in fig. 5.

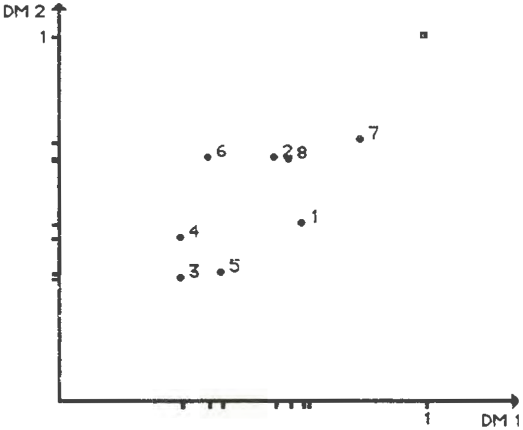

Fig. 5 - Acceptability graph (the black square is the ideal point where G_1=G_2=1)

Solutions on nondominated edges and faces may also be computed. For instance, the solution which minimizes a weighted Tchebycheff distance to the ideal solution is $\underline{f}(\underline{x})$=(33.88,26.88,44.24) and its acceptability value is $G_1^{L\infty}$=0.803, $G_2^{L\infty}$=0.799 ($G^{L\infty}$=0.801). This would be the most acceptable solution considering the reservation and aspiration levels currently specified .

Note : All figures are copies of the screens presented to users, and some minor cosmetics were added in some figures in order to comply with the space limitations. Although the available graphical and numerical information would have enabled a more detailed analysis we have limited ourselves (due to space limitations) to some comments which are illustrative of the type of help this decision support tool can provide to DMs in negotiation problems.

5. Conclusions

In this paper the interactive and user-friendly capabilities of the TRIMAP package are exploited, as a tool for providing decision support in problems with multiple DMs based on three-objective linear programming models. The aim is to help the parties to explore their own evolutionary preferences and to make the most of the dynamic nature of the interactive negotiation process. Although an extensive practical experimentation is necessary in order to evaluate the potentialities of the procedures suggested in real-world problems, it seems that TRIMAP possesses characteristics well suited to provide decision aid in problems with multiple DMs. These features include the possibility of performing a progressive and selective learning of the nondominated solution set, the user-friendly and graphical potentialities and the ability to enable a comparative study of the weight space, the objective function space and an acceptability space.

An extension of the TRIMAP package to perform sensitivity analyses in three-objective linear programming problems is exploited in Antunes and Clímaco (1992). In group decision problems, the possibility of changes in the initial model coefficients, including the relaxation of constraints, may be a contribution to reach a greater agreement among parties. The operational framework proposed may also be useful for the case of multiple criteria discrete alternative problems.

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