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# SUPPORT SYSTEMS FOR DECISION AND NEGOTIATION PROCESSES

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# A DSS FOR CONSENSUS REACHING USING FUZZY LINGUISTIC MAJORITY AND CLUSTERING OF PREFERENCES

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**Abstract:** An interactive DSS for consensus reaching is presented. Experts provide their testimonies as fuzzy preference relations. The consensus reaching process is supervised by a moderator ("super - expert"). A degree of consensus, based on the concept of a fuzzy majority given as a fuzzy linguistic quantifier is employed. Algorithms of cluster analysis are used to find groups of experts having similar preference relations.

**Keywords:** group DM, consensus, consensus degree, DSS, fuzzy majority, fuzzy logic, fuzzy linguistic quantifier, fuzzy preferences, cluster analysis.

## 1. INTRODUCTION

Decision making in real world often proceeds in a multiperson setting, and group DM tools may be helpful. A complicated character of group DM, and its related consensus formation (e.g., different actors with diverse and conflicting value systems) suggests that a computerized support, in the form of a decision support system (DSS), may be of use. And, indeed, DSS's for group DM are widely advocated, developed, and employed (DeSanctis and Gallupe, 1985; Gray, 1987; Huber, 1984).

In this paper we present briefly such a DSS. Its roots are, first, an implemented DSS for consensus reaching which constitutes a "shell" of the present system (Fedrizzi, Kacprzyk and Zadrożny, 1989), second, Kacprzyk's (1987), and Kacprzyk and Fedrizzi's (1986, 1988, 1989) works on new degrees of consensus based on a fuzzy majority, and, third, Owsiński's (1990), and Owsiński and Zadrożny's (1988, 1990) works on the aggregation and clustering of preference relations.

Basically, we have $n$ individuals (experts, decision makers, ...) who present their testimonies concerning a set of options $S = \{s_1, \ldots, s_n\}$ as *individual fuzzy preference relations* ($FPR$'s), and a "super - expert", called a *moderator* who is responsible for running a consensus reaching session. The individual $FPR$'s may initially differ to a large extent (the group is far from consensus), and the moderator – via an exchange of information, rational argument, bargaining, etc. – tries to persuade the individuals to change their $FPR$'s. If the individuals are rationally committed to consensus, such a change usually occurs, and they get closer to consensus. This is repeated until the group gets sufficiently close to consensus, i.e. until the individual $FPR$'s become sufficiently similar, or until we reach some time limit.

In many practical cases the traditional meaning of *consensus* as a *full* and *unanimous agreement* is not adequate, and a new "soft" concept of consensus due to Kacprzyk (1987), and Kacprzyk and Fedrizzi (1986, 1988, 1989) is useful. Its idea is that the consensus should be meant *to a degree* ($\in [0,1]$)as, e.g., the degree to which "*most* of the *important* (competent) individuals agree in their preferences as to *almost all* of the *relevant* options"; notice that *fuzzy majorities* (*most* and *almost all*) and degrees of importance and relevance are used. The moderator has therefore an effective means for measuring the degree of consensus in a group (which is not trivial in cases with a realistic number of individuals and options). However, to more efficiently run the session, he or she may need tools for finding groups of individuals who are sufficiently close in their preferences; it may often be more effective and efficient to work then (in the sense of trying to attain closer preference relations) within such a group. Cluster analytic methods due to Owsiński (1990), and Owsiński and Zadrożny (1988, 1990) may be very helpful here. Finally, the session should be supported by an interactive, user-friendly DSS for which some "shell" is available (Fedrizzi, Kacprzyk and Zadrożny, 1989). This work is a synergetic combination of these ideas and approaches.

## 2. FUZZY LOGIC WITH LINGUISTIC QUANTIFIERS

A fuzzy majority is defined as a *fuzzy linguistic quantifier* (most, almost all, ...). Zadeh's (1983) fuzzy-logic-based calculus of linguistically quantified propositions is then employed.

A *linguistically quantified proposition* is generally written "$Qy$'s are $F$", e.g. "most ($Q$) experts ($y$'s) are convinced ($F$)". Importance $B$ may be added yielding "$QBy$'s are $F$", e.g., "most ($Q$) of the important ($B$) experts ($y$'s) are convinced ($F$)".

Truth($Qy$'s are $F$) or truth($QBy$'s are $F$) is then sought; truth($y_i$ is $F$), $\forall y_i \in Y=\{y_1,\ldots,y_p\}$. In Zadeh's (1983) calculus, first, property $F$ is a fuzzy set in $Y$, a linguistic quantifier $Q$ is a fuzzy set in $[0,1]$ as, e.g.,

$$\mu_{\text{"most"}}(x) = \begin{cases} 1 & \text{for } x \geq 0.8 \\ 2x - 0.6 & \text{for } 0.3 < x < 0.8 \\ 0 & \text{for } x \leq 0.3 \end{cases} \tag{1}$$

Then

$$\text{truth}(Qy\text{'s are } F) = \mu_Q(\textstyle\sum\text{Count}(F)/\sum\text{Count}(Y)) = \mu_Q(\frac{1}{p}\sum_{i=1}^{p}\mu_F(y_i)) \tag{2}$$

where $\wedge$ = *minimum* (generally, a $t$-norm), and $\sum\text{Count}(F) = \sum_{i=1}^{p}\mu_F(y_i)$ is a (nonfuzzy) cardinality of fuzzy set $F$, and

$$\begin{aligned} \text{truth}(QBy\text{'s are } F) &= \mu_Q(\textstyle\sum\text{Count}(B \text{ and } F)/\sum\text{Count}(B)) = \\ &= \mu_Q(\sum_{i=1}^{p}(\mu_B(y_i) \wedge \mu_F(y_i))/\sum_{i=1}^{p}\mu_B(y_i)) \end{aligned} \tag{3}$$

## 3. A FUZZY-MAJORITY-BASED DEGREE OF CONSENSUS

The new *degree of consensus* proposed by Kacprzyk (1987) and advanced by Fedrizzi and Kacprzyk (1988) and Kacprzyk and Fedrizzi (1986, 1988, 1989) is meant to overcome some "rigidness" of conventional degrees of consensus in which (full) consensus (= 1) occurs only when "*all* the individuals agree as to *all* the issues" which may often be counter-intuitive and unnecessarily strict. The new degree can be equal to 1, which stands for full consensus, when, say, "*most* (of the important) individuals agree as to *almost all* (of the relevant) issues".

We start with individual fuzzy preference relations. If $S = \{s_1, \ldots, s_n\}$ is a set of options and $I = \{1, ..., m\}$ is a set of individuals, then an *individual fuzzy preference relation* of individual $k$, $R_k$ is given by its membership function $\mu_{R_k} : S \times S \to [0,1]$ such that

$$\mu_{R_k}(s_i, s_j) = \begin{cases} 1 & \text{if } s_i \text{ is definitely preferred over } s_j \\ c \in (0.5, 1) & \text{if } s_i \text{ is slightly preferred over } s_j \\ 0.5 & \text{if there is indifference} \\ d \in (0, 0.5) & \text{if } s_j \text{ is slightly preferred over } s_i \\ 0 & \text{if } s_j \text{ is definitely preferred over } s_i \end{cases} \tag{4}$$

If card$S$ is small enough, $R_k$ may be represented by a matrix $[r^k_{ij}] = [\mu_{R_k}(s_i, s_j)]$.

The degree of consensus is derived in three steps: (1) for each pair of individuals we derive a degree of agreement as to their preferences between all the pair of options, (2) we aggregate these degrees to obtain a degree of agreement of each pair of individuals as to their preferences between $Q1$ (e.g., most, almost all,...) pairs of relevant options, and (3) we pool these degrees to obtain a degree of agreement of $Q2$ (e.g., most, almost all, ...) important pairs of individuals as to their preferences between $Q1$ pairs of relevant options. This is meant to be the degree of consensus sought, called the *degree of* $Q1/Q2/B/I$*- consensus.*

We start with the degree of strict agreement between individuals $k1$ and $k2$ as to their preferences between $s_i$ and $s_j$

$$v_{ij}(k1, k2) = \begin{cases} 1 & \text{if } r^{k1}_{ij} = r^{k2}_{ij} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

We assume that the relevance of a pair of options, $b^B_{ij}$, and importance of a pair of individuals, $b^I_{k1,k2}$, is the mean values of the respective individual relevances (importances), i.e. $b^B_{ij} = (\mu_B(s_i) + \mu_B(s_j))/2$, and similarly $b^I_{k1,k2}$.

Then, consecutively:

$$v_B(k1, k2) = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (v_{ij}(k1, k2) \wedge b^B_{ij}) / \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} b^B_{ij} \tag{6}$$

$$v_{Q1,B}(k1, k2) = \mu_{Q1}(v_B(k1, k2)) \tag{7}$$

$$v_{Q1,B,I} = \sum_{k1=1}^{m-1} \sum_{k2=k1+1}^{m} (v_B(k1, k2) \wedge b^I_{k1,k2}) / \sum_{i=k1}^{m-1} \sum_{k2=k1+1}^{m} b^I_{k1,k2} \tag{8}$$

$$con_B(Q1, Q2) = \mu_{Q2}(v_{Q1,B,I}) \tag{9}$$

which is meant to be the *degree of* $Q1/Q2/B/I$*-consensus*, i.e. the degree of agreement of $Q1$ pairs of important individuals as to their preferences between $Q2$ pairs of relevant options.

One can also introduce a degree of defeat into (5) and obtain other degrees of consensus (cf. Kacprzyk, 1987; Kacprzyk and Fedrizzi, 1986, 1988, 1989).

## 4. STRUCTURING THE SET OF INDIVIDUAL *FPR*'s VIA CLUSTER ANALYSIS

The former section provided the moderator with a tool for measuring the degree of consensus (closeness of the individual $FPR$'s). It turns out, however, that of much value to the moderator is to know which ***groups of individuals*** are close (or distant) in their $FPR$'s, that is to find the ***structure*** of the set of individuals with respect to the similarity of their $FPR$'s.

We use here Owsiński's (1990) (cf. also Owsiński and Zadrożny, 1988, 1990) cluster analytic method with the maximization of an objective function, i.e.

$$C(P,r) = r\,\overline{C}_D(P) + (1-r)\,\underline{C}_S(P) \rightarrow \max_P \tag{10}$$

where $P$ is the partition of the set of $FPR$'s into the sets $A_q$, $\overline{C}_D(P)$ is a measure of dissimilarity (distances) between the $A_q$' s in the whole partition $P$, $\underline{C}_S(P)$ is a measure of similarity (closeness) of relations belonging to the $A_q$' s for the whole $P$, and $r$ is a parameter (weight).

The solution of (10) is difficult, and under some mild assumptions (cf. Owsiński, 1990), a suboptimal solution may be obtained by the following iterative algorithm:

**Step 1** Set $t = 1$ and $P^1 = I$, i.e. assume all the individual $FPR$'s considered as separate clusters;

**Step 2** Find a pair of clusters $A_{q^*}$ and $A_{q^{**}}$ such that

$$[\delta\underline{C}_S(P^t,q,q')]/[\delta\underline{C}_S(P^t,q,q') + \delta\overline{C}_D(P^t,q,q')] \rightarrow \max_{q,q'} \tag{11}$$

where: $\delta\underline{C}_S(P^t,q,q')$ and $\delta\overline{C}_D(P^t,q,q')$ are the increments of the measures $\underline{C}_S(P^t,q,q')$ and $\overline{C}_D(P^t,q,q')$ while aggregating the clusters $A_q$ and $A_{q'}$ belonging to $P^t$, respectively; notice that in $\overline{C}_D$ and $\underline{C}_S$ in (11) we have explicitly indicated the pair of clusters $A_{q^*}$ and $A_{q^{**}}$ belonging to the partition obtained in iteration $t$, $P^t$;

**Step 3** If no increase in the value of the right - hand side of (11) occurs, then STOP and accept the current partition $P^t$ as the (sub)optimal solution. Otherwise, combine (by the set-theoretic union) the pair of clusters $A_{q^*}$ and $A_{q^{**}}$ found in Step 2, and obtain $P^{t+1}$; set $t \leftarrow t+1$, and proceed to Step 2.

The method sketched above determines some clusters of individuals whose individual $FPR$'s are close enough. The moderator obtains therefore an important piece of information as it is usually more effective and efficient to "work" within the individuals belonging to the clusters determined.

## 5. AN INTERACTIVE DSS FOR CONSENSUS REACHING

Now we will briefly present the DSS developed, and sketch an example of a session concerning the choice of an investment option.

From the *software point of view*, the system may be described in terms of its basic modules. The **Data Elicitation Module** elicits in a simple, user-friendly way from the moderator and the particular individuals data needed by the system and for the particular session as, e.g., individual $FPR$'s, relevance of options, importance of individuals, fuzzy linguistic quantifiers $Q1$ and $Q2$, etc. The **Managing Module** is an "operating system" for decoding the moderator's commands, the individuals' responses, etc. Basically, it: (1) controls data elicitation, (2) sets parameters, and (3) activates an appropriate reporting facility as, e.g., display of the value of a consensus measure, some "troublesome" (causing a low value of a degree of consensus) options or individuals, clusters of individuals, etc. The **Parameter Setup Module** determines necessary parameters and their values, needed for the particular session. The **Consensus Degree Computation Module** calculates the value of a consensus degree by an algorithm (with its parameters) determined by the Parameter Setup Module. The **Clustering Module** provides information on the structure of the set of $FPR$'s at a given step of the session. The **Reporting Module** provides reporting facilities for the moderator and the individuals, mainly related to the values of the degree of consensus and its temporal evolution.

The system is implemented for the IBM PC AT/386/486 and compatibles (for details on the system's "shell", without the clustering part, see Fedrizzi, Kacprzyk and Zadrożny, 1989).

The system is *menu-driven*. From the top level its operation is governed by the **Main Menu** meant for the moderator and including basic options for initializing, running and quitting the session. First, to initialize the system, the $FPR$'s of the particular individuals are introduced. Then, the parameters are set (mainly in the beginning of the session), as, e.g., the names (labels) of the problem, options and individuals, the number of individuals and options, the values of importances and relevances, etc. Next, the membership functions of $Q1$ and $Q2$, which are piecewise linear for simplicity (i.e. four values are needed for their specification), are input in a user-friendly way. The type of a degree of consensus and of a $t$-norm is chosen.

We are now ready for running the session. We can review the values of parameters, update them, compute a degree of consensus, etc. If we have reached a sufficient value of a degree of consensus, we can quit. Otherwise, we can interact with the individuals to persuade them to change their $FPR$'s; when in trouble, the moderator may request a help as, e.g., the display of "troublesome" options and individuals or of clusters of individuals with whom it may be fruitful to "work". At the end of the session, we obtain sufficiently close $FPR$'s. This is the end of the scope of the paper; in the system, one can go further and try to determine some group DM solutions (cf. Kacprzyk, 1985, 1986; Nurmi, 1987; Nurmi and Kacprzyk, 1991).

**Example.** This short example concerns the choice of an investment option in a small community. The four options are: school, movie theater, shopping center, and swimming pool. The ten individuals represent the local and upper level authorities, social and political organizations, some informal groups, a "man-in-the-street", etc. For lack of space we cannot present the (initial) individual $FPR$'s.

We obtain first con(*most*, *most*, $B$, $I$) = 0.8069 which is viewed unsatisfactory. Then, some "help" is requested. The cluster of individuals 1, 2 and 8 is suggested, and the pair of options "2 (movie theater) - 3 (shopping center)" is indicated as a "troublesome" one, i.e. for which a considerable disagreement occurs. We obtain con(*most*, *most*, $B$, $I$) = 0.9232. Then, by utilizing another "help", we obtain con(*most*, *most*, $B$, $I$) = 0.9934. Finally, we obtain by another "help" con(*most*, *most*, $B$, $I$) = 1 which is evidently fully satisfactory.

## 6. CONCLUDING REMARKS

A DSS for consensus reaching has been presented. It uses individual $FPR$'s, a fuzzy majority expressed by a fuzzy linguistic quantifier, a concept of a "soft" degree of consensus, and a clustering approach to find groups of individuals whose $FPR$'s are similar. This all enhances the system's capabilities, and contributes to its effectiveness and efficiency in supporting the consensus reaching process.

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