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POWER INDICES FOR POLITICAL AND FINANCIAL DECISION MAKING

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Abstract: The most important power indices are presented. Effectiveness of these indicators is discussed in order to describe political and financial events. Some recent studies and applications are shown.

Keywords: cooperative games, weighted majority games, power indices, voting, shareholding.

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1. Why Power Indices

The problem of decision making regarding political and economic matters is a topical question both inside the Eastern and Western countries, and in the prospects of cooperation between the East and the West.

Within the Eastern countries, the rapid evolution of the second perestroika leads to the requirement of studying the democratic systems in order to take advantage of its positive aspects while trying to avoid the negative ones. Thus, the strong interest in voting systems both in a political area (electoral and government systems) and in an economic area (introduction of shareholding in the new market economy).

Within Western Countries, the European Economic Community regulations related to the opening of markets in '93, offer important questions about the tactical and strategic decisions to be made regarding mergers, acquisitions and in general firms' control. There is, moreover, a stronger interest in electoral legislation, in an attempt to improve the present systems both at a local and national level.

As regards the whole East-West area, new, important questions have risen concerning both, the economic level (swapping, joint-ventures, etc.) and the political level (for example, the regulations regarding the entrance of new countries into the EEC).

From the information mentioned above, we can deduce the importance of the study of majority coalitions and their main indicators (power indices) for decision-making, both at a normative and operative level.

2. What Power Indices

The formation of majority coalitions is often hardly explicable to the participants themselves. In fact, these coalitions are the result of human relationships, feelings, aversions, external influences and psychological attitudes more than of simple strength ratios. The analysis of such processes can be quite interesting, especially when it leads to forecasting the resulting configuration. This is then taken to be the essential basis for achieving optimality in subsequent decisions as well as equity in problems of social rules and regulations. A first step towards such a forecast must necessarily be that of neglecting the least quantifiable components. Then the gap will have to be bridged, by means of subsequent improvements to the proposed model. However, everybody has to be aware of the fact that the truth of the conclusions depends on the reliability of the hypotheses and on the credibility of the inference rules upon which the model is based. The power indices fulfill the above mentioned preliminary steps. They assume unquestionably defined and numerically given values for the "weights" of the members of various coalitions. In other words, the indices are based exclusively on weights, such as shares of a firm, thousands of joint-owners, votes, seats and the like.

Let $N = \{1,\ldots,n\}$ be the set of participants in a certain committee ("players"), let w_i $(i=1,\ldots n)$ be the real not negative weight of the i-th player. Let $t = \sum_{i=1}^{n} w_i$ be the total weight, and q be a real number ("majority quota") so that $t/2<q\leq t$. For every subset S of N ("coalition") the win v(S) is:

$$v(S) = (1 \quad \text{if} \quad \sum_{h\epsilon S} w_h \geq q; \quad 0 \quad \text{elsewhere})$$

i.e. a coalition is winning, and gains 1, if it is a majority; otherwise it is losing and gains 0. We say that, v(S) is the characteristic function of the weighted majority game $[q; w_1,\ldots w_n]$. Clearly, the vector $w = [w_1,\ldots,w_n]$ belongs to the

simplex $\quad Z = \{x: \sum_{h=1}^{n} x_h = t \ , \ x_h \geq 0 \ (h=1,\ldots,n)\}$

We denote by Z^1 the simplex corresponding to Z for t=1. We say that i-th player is crucial in the game V for the coalition S if $v(S)=1$ and $v(S-\{i\})=0$, that is the coalition is winning with him and losing without him.

A power index is a function Φ: $Z \rightarrow Z^1$ fit to represent a reasonable expectation of the percent share of the decisional power among the various players, in relation to their strength in the game. We denote by $\Phi_i(v)$ the quota of power that the index Φ grants to the i-th player in the game v; such a quota is called "power index of the i-th player".

3. The Most Important Power Indices

Several authors have proposed various power indices on the basis of different axiomatic grounds and bargaining models. The most widely used of them are the Shapley-Shubik index (Shapley and Shubik, 1954) and the Banzhaf-Coleman index (Banzhaf, 1965 and Coleman, 1971). Some others can be found in: (Ameljanczyk, Holubiec and Piasecki, 1984), (Deegan and Packel, 1978 and 1980), (Holler, 1982), (Holler and Packel, 1983), (Lemaire, 1974), (Pressacco, 1978) and (Tijs, 1981). For further information see (Lucas, 1976) and (Nurmi, 1987). The Shapley-Shubik index is a particularization, for weighted majority games, of the Shapley value of a game (Shapley, 1953). For its axiomatic derivation see (Dubey, 1975b). Such an index derives from a model of bargaining which provides the forming of the whole coalition through equiprobable additions of single players to all possible subcoalitions. It assigns to the i-th player (i=1,...,n) the quota of power $\Phi_i = \Sigma\ (s-1)!(n-s)!/n!$ where the sum is extended to all the coalitions (of s members) for which the i-th player is crucial. The normalized index of Banzhaf-Coleman assigns to every player the ratio between the number of coalitions for which he is crucial and the sum of such numbers extended to all the players (coefficient of normalization). For its axiomatic derivation see (Owen, 1978). While the second index seems to be the better solution for problems of arbitration, the first is more suitable to represent the result of bargaining. For instance, the validity of this model has been proved in the representation of the course of the control-stock quotes on the Swedish market (see Rydqvist, 1985).

A property introduced in (Gambarelli, 1980), the strong monotony, has been found common to various power indices (Shapley-Shubik, normalized Banzhaf-Coleman, Nash, Tijs) in (Sagonti, 1991).

It can be interesting to study how the index varies with the variation of the distribution of the "weights" among the players. For example, in the trading of shares, if the initial distribution of shares gives a certain power index, how many shares must be bought and by whom, so that the index varies? How many shares can be sold and to whom, so that the index remains unchanged? In (Gambarelli, 1983) it was proved that, for strongly monotonic power indices, when the weight of a player increases to the disadvantage of other players, its power index is a monotonic step function of the weight variation. The discontinuity points of such a function are generated by the solutions of a system. Research on the most dangerous partner in the exchange was carried out in (Gambarelli and Szegö, 1982). Another case, which is particularly interesting for applications, concerns the variation of the weight of the i-th player, without varying the weights of all the other players, the only majority quota changing in proportion to the variation of the total weight (see Gambarelli, 1983). In the financing field, a typical case is that of the takeover of a firm by an investor, who buys shares from small shareholders, who are not interested in control. In the political field these variations occur when, for example, some electors migrate from or to a certain electoral college, or after the introduction of laws that allow new classes of electors with common political inclinations to vote (handicapped and young people, prisoners, emigrants and so on). Even in such a case the power index (if strongly monotonic) of the i-th player is a monotonic step function, whose discontinuity points can be generated by a simple formula.

For computation of the Shapley value in general games we quote the algorithms of Gambarelli (1980 for superadditive games; 1989a for all general games). Such algorithms are linear in the number of significant coalitions and employ a theorem of early stop of calculation. But for weighted majority games having small total weight, the algorithm of (Mann and Shapley, 1962) is preferable. It is possible to link this algorithm with the results mentioned above, so that we are able to calculate automatically the power of the i-th player in relation to the variation in the distribution of weights. Obviously, this function has to be computed only in the discontinuity points, since it is constant in the other segments. But using the algorithm (Mann and Shapley, 1962) every time, we lose the results of the previous iterations, and consequently we waste calculation. In order to avoid this inconvenience, a new tecnique is proposed in (Arcaini and Gambarelli, 1986). This result answers the question raised in (Milnor and Shapley 1961 p. 18), on the recursive generation of the value ("Strangely, no such recursive method for computing values has ever been found for finite games").

4. Some Applications to Decision Making in Politics

A problem arises in the application of power indices to political situations, in which the propensity of members to certain alliances more than others is particularly strong. In fact, it is obvious that some coalitions, possible in theory, are not possible or hardly probable in practice, so that the indices must be modified. Briefly, we observe that all parties having (with regards to the possible alliances) a higher agreement index, increase in practice their power index. An interesting approach (Myerson, 1977) uses the Graph theory. More important results are given in (Owen, 1977), where the modifications of the Shapley-Shubik index by using different probabilities of coalitions are studied. This has been done for the Banzhaf-Coleman index in (Owen, 1981). For a recent application see (Carreras and Owen, 1988). Another study about different probabilities of coalitions is due to Zhang Sheng-Kai (1989). The recent work (Gambarelli, 1989b) allows the forecast of the set of the most probable government coalitions and the relative sharing of power among the party members. Such a model takes into consideration, for each party with respect to all possible coalitions, the internal cohesion indices ("snipers"), the direct damage (i.e. discontent of voters, decrease of support) and the indirect damage (i.e. probability of "putsches"). It has been successfully experimented on the results of the latest Italian elections.

All these modifications to the power indices do not mean that such indices in their pure state are not suitable to describe political situations. It happens that when majority balances, which are easily destabilized, come into play, the negotiation does not exclude the threat of separation from the closest parties, in order to take advantage of the division of power. There are more decisional situations (e.g. referendum or presidential elections) where numerical strength is more important than political closeness. Besides, in situations of a normative nature (e.g. for the Nations at the European Parliament or for the democratic roundings, which we will see later), the possible coalitions must be considered equally probable a priori.

For the Nations at the European Parliament, the entry of new members induces many discussions, because there is no deterministic normative on the matter. Then, each Nation can study by simulation, which number of seats to attribute to the new members is optimal (in terms of power indices) for his consequent power of control. For a detailed study see (Gambarelli, 1991).

A further observation is made concerning the method used at present to determine the number of seats in relation to the number of voters, or the thousandths to the joint-owners in relation to the values of properties, or the representation of certain people in relation to the number of voters, or the members of a board of directors in relation to the shares owned by the shareholders. The problem is to find a transformation which leads the integer simplex of votes to the integer simplex of seats. Such a transformation has to minimize the distance (properly defined) between the vector of votes and the projection of the vector of votes on the simplex of seats, and the distance of the corresponding power indices.

Another proposal of institutional reform concerns the elimination from the Chambers of the parties gaining votes which are inferior to a certain percentage.

A simulation can be made, in order to know which power indices correspond to different theresholds. From this simulation it is then possible to find the optimal thereshold for each party (see Gambarelli, 1991).

5. Some Applications to Decision Making in Finance

It can happen that a shareholder owns in a firm more shares than another, but their power index is the same. Then, the first investor can consider the possibility of selling shares and buying other shares in other companies, in order to improve his power position in them. All this is linked to the clear economic benefits derived from controlling firms, either directly or indirectly through holding. Actually, the more attempts to gain control are made, the more the share price increases; in such cases, shares were bought at up to four times their official price. A first attempt at the modelization of this phenomenen, dating back to (Amihud and Barnea, 1974), to (Batteau, 1980) and to (Gambarelli, 1981), met an obstacle in the determination of the function control. That obstacle was overcome by the results in (Gambarelli, 1982a) (exchange of shares between two players), in (Gambarelli and Szegö, 1982) (estimation of purchaser danger) and in (Gambarelli, 1983) (share takeover by an investor buying shares from minor shareholders not interested in control). An essential support to such results is given by the algorythms shown in section 3. In these models the Shapley-Shubik index is employed, bacause it is closer to the context for the solution construction mechanism (subsequent additions of the coalition members).

Another particular problem concerns the steadiness of the control position reached. In (Gambarelli, 1982b), in relation to each discontinuity point of the step function, a further security amount to purchase was determined, so that the investor would not be damaged in the transaction, if counter-actions from other major shareholders take place. In a forthcoming work (Gambarelli, Owen, 1992) the control on a firm by using shares of other firms is studied. An interesting problem, then, arises in the natural application of these works to the Portfolio Theory. This theory studies the optimal share of an investor's capital among a certain set of risky investments, in order to minimize risk and maximize expected return (see Szegö, 1980). Now, the investment diversification involved in classical theories in order to minimize risk, is opposed to the share concentration necessary to control. The problem becomes then the choice, to be made by a risk averse investor, of the investments that optimize both usual returns and risk or returns and risk from control. A solution proposed in (Gambarelli, 1982b) for the Portfolio Selection is now under examination for an application to the case of Portfolio Management. To give an idea of the timeliness of financial studies in this field (not available hitherto), only a few cases will be quoted. Texaco had to repurchase, at 1.28 billion US dollars, 10% of its capital scraped up by a group of Texan raiders, paying it much more than the official quotation; Warner paid 181 million dollars in order to recover 8,6% of its capital, paying 42% more than the official quotation; a similar operation cost Walt Disney nothing less than 32 million dollars, and it is known that elsewhere blocks of controlling shares have been exchanged at up to four times the price quoted on the Stock Exchange: De Benedetti's takeover bid on the Societé Generale de Belgique has quadrupled the price of this share in just a few weeks. For other information see (Ragazzi, 1981). Legislative measures are being considered in some countries, in order to limit such situations. For instance, present American legislation does not allow investment funds to have more than 5% of shares in a company. Nevertheless, even 5% can be very important for control in some cases. The opening of the markets in Western Europe in 1992 with new possibilities of takeover, makes the study of these models very topical. The determination of the critical stocks (Gambarelli, 1983) and the relative applications to the portfolio selection (Gambarelli, 1982b) are now being improved, with the determination of destabilization indices, in relation to takeover.

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