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FUZZY LINGUISTIC MAJORITIES IN GROUP DECISION MAKING AND CONSENSUS REACHING

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Abstract: The concept of a fuzzy majority, expressed by a fuzzy linguistic quantifier (most, almost all, much more than a half, ...), is employed in group decision making (group DM) and consensus reaching. Yager's fuzzy-logic-based calculus of linguistically quantified propositions is used as a formal tool. In group DM, taking additionally as a point of departure individual and social fuzzy preference relations, new solution concepts are presented which are basically those "best" accepted by, say, *most* of the individuals. In consensus reaching, a new concept of a degree of consensus is shown which is basically a degree to which, say, *most* of the *important* individuals agree as to *almost all* of the *relevant* options.

Keywords: group decision making, consensus reaching, degree of consensus, fuzzy preference relation, fuzzy majority, fuzzy logic, fuzzy linguistic quantifier.

1. INTRODUCTION

The essence of *group decision making* (group DM) models, which can provide powerful tools for the analysis of many real problems in the area of decision making, is that there are m individuals (decision makers) whose testimonies are assumed here to be *individual fuzzy preference relations* over a set of n options, and the problem is to find an option (or a set of options) best reflecting the preferences of the group of individuals as a whole. *Consensus reaching* is evidently strongly related to group DM.

Unfortunately, since its very beginning group DM has been plagued by negative results (theorems on impossibility, manipulability, instability of solutions, etc.) (cf. Nurmi, 1982, 1983, 1987, 1988; or Nurmi, Fedrizzi and Kacprzyk, 1990). They all can be summarized as that no group DM procedure can satisfy all plausible conditions. An immediate idea is therefore that instead of devising more sophisticated procedures, maybe we should modify some basic underlying assumptions of the problem's very setting; this line of reasoning is also assumed here.

First, since the human preferences are inherently imprecise, *individual* and *social fuzzy preference relations* will be assumed. Second, we assume that a crucial concept of *majority* may also be inherently imprecise as, e.g., in many cases an option preferred by *most, almost all, much more than a half,* ... individuals may be adequate to a real human perception of the problem's very essence. A good illustration is here (Loewer and Laddaga, 1985): "...It can correctly be said that there is a *consensus* among biologists that Darwinian natural selection is an important cause of evolution though there is currently *no consensus* concerning Gould's hypothesis of speciation. This means that there is a *widespread agreement* among biologists concerning the first matter but *disagreement* concerning the second ...". Needless to say that a crisp majority cannot reflect the very sense of this statement.

Natural manifestations of a fuzzy majority are *linguistic quantifiers* exemplified by: most, almost all, much more than a half,.... They may be handled by fuzzy-logic-based calculi of *linguistically quantified propositions* as, e.g., those due to Zadeh (1983) and Yager (1983). The latter will be used here (for the use of the former, see, e.g., Kacprzyk, Fedrizzi and Nurmi, 1992).

2. A FUZZY-LOGIC-BASED CALCULUS OF LINGUISTICALLY QUANTIFIED PROPOSITIONS

A *fuzzy set* A in X is equated with its *membership function* $\mu_A : X \to [0,1]$; $\mu_A(x) \in [0,1]$ is the *grade of membership* of x in A. If X is finite, we write $A = \mu_A(x_1)/x_1 + \cdots + \mu_A(x_n)/x_n$ where "$\mu_A(x_i)/x_i$" is the pair "grade of membership – element" and "+" is meant in the set-theoretic sense. Moreover: $a \wedge b = \min(a,b)$, $a \vee b = \max(a,b)$, and "$\Rightarrow$" stands for the implication.

A *linguistically quantified proposition* as, e.g., "most (Q) experts (y's) are convinced (F)", is generally written as Qy's are F where Q is a (fuzzy) linguistic quantifier (most), $Y = \{y\}$ is a set of objects (experts), and F is a (fuzzy) property (convinced). Moreover, different *importances* of y's may be added, yielding QBy's are F, e.g., "most (Q) of the important (B) experts (y's) are convinced (F)". The problem is to find truth(Qy's are F) or truth(QBy's are F).

To briefly present Yager's (1983) approach, we introduce the statements: P_i: "y_i is F", whose truth$(P_i) = \mu_F(y_i)$, $i = 1,\ldots,p =$ card$Y$. We introduce the set $V = \{v\} = \{P_{k1},\ldots,P_{km}\}$ $= 2^{\{P_1,\ldots,P_p\}} \backslash \emptyset$. Then, $\mu_T(v) = \text{truth}(v) = \bigwedge_{i=1}^{m} \mu_F(y_i)$.

The fuzzy linguistic quantifier Q is defined as a fuzzy set in V. For instance, if $p = 3$, then $V = \{P_1, P_2, P_3, P_1 \text{ and } P_2, \ldots, P_2 \text{ and } P_3, P_1 \text{ and } P_2 \text{ and } P_3\}$, and

$$\mu_{\text{"most"}}(v) = \begin{cases} 1 & \text{for } v \in \{P_1 \text{ and } P_2 \text{ and } P_3\} \\ 0.7 & \text{for } v \in \{P_1 \text{ and } P_2, P_1 \text{ and } P_3, P_2 \text{ and } P_3\} \\ 0.3 & \text{for } v \in \{P_1, P_2, P_3\} \end{cases} \tag{1}$$

The so-called *monotonic quantifiers*, defined as $\mu_Q(v_1 \text{ and } v_2) \geq \mu_Q(v_1) \vee \mu_Q(v_2)$, for each $v_1, v_2 \in V$, are the most relevant; such quantifiers mean basically "the more the better", and "most" (1) is evidently proportional.

Now

$$\text{truth}(Qy\text{'s are } F) = \max_{v \in V}(\mu_Q(v) \wedge \mu_T(v)) \tag{2}$$

or, with importance,

$$\text{truth}(QBy\text{'s are } F) = \max_{v \in V}(\mu_Q(v) \wedge (\bigwedge_{i=1}^{m}(\mu_B(y_{ki}) \Rightarrow \mu_F(y_{ki})))) \tag{3}$$

where $\Rightarrow$ is an implication whose most widely used form is $a \Rightarrow b = (1-a) \vee b$; $\wedge$ (*bigwedge*) and max may be replaced by a $t-$norm or $s-$norm, respectively.

Since (2) and (3) are complicated, their simplifications are often used. For the case without importance, under sime mild restrictions: (1) Q is a monotonic quantifier, (2) there is a finite number of distinct values of $\mu_Q(v)$ as, say, $b_1 \leq b_2 \leq \ldots \leq b_s$, (3) d_i is the $i-$th largest element of the set $\{\mu_T(v_1), \ldots, \mu_T(v_{\text{card}\,V})\}$, then

$$\text{truth}(Qy\text{'s are } F) = \max_{i \in \{1,\ldots,s\}} (d_i \wedge b_i) \tag{4}$$

A noteworthy simplification is also provided by Yager's (1988) OWA (ordered weighted average) operators.

3. GROUP DM UNDER FUZZY PREFERENCES AND FUZZY MAJORITY

The second relevant elements is an *individual fuzzy preference relation* defined as $\mu_{R_k} : S \times S \rightarrow [0,1]$, where $S = \{s_1, \ldots, s_n\}$ is a set of options; $\mu_{R_k}(s_i, s_j) \in [0,1]$ is the *intensity of preference* of option s_i over option s_j as perceived by individual k: from 1 for definite preference of s_i over sj to 0 for a definite preference of s_j over s_i through all intermediate values (0.5 for indifference). For a finite S, R_k is represented by a matrix $[\mu_{R_k}(s_i, s_j)] = [r^k_{ij}]$. And similarly for a *social fuzzy preference relation* representing preferences of the whole group.

Now, a solution of group DM is sought, i.e. an option (or a set of options) which is "best" acceptable by the group of individuals as a whole. Two lines of reasoning may be used:

- a *direct* approach: $\{R_1, \ldots, R_m\} \rightarrow$ solution, i.e., a solution is found just from the individual preference relations, and
- an *indirect* approach: $\{R_1, \ldots, R_m\} \rightarrow R \rightarrow$ solution, i.e., first a social fuzzy preference relation R is determined that is then used to find a solution.

The concept of a solution is not obvious (cf. Nurmi, 1983, 1987), and we will present some of them under individual fuzzy preference relations and a fuzzy majority.

For the direct approach, a solution concept of much intuitive appeal is a *fuzzy $Q-$core* (Kacprzyk, 1985, 1986). We start with

$$h^k_{ij} = \begin{cases} 1 & \text{if } r^k_{ij} < 0.5 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

and then

$$h^k_j = \frac{1}{n-1} \sum_{i=1, i \neq j}^{n} h^k_{ij} \tag{6}$$

which is the extent to which individual k is not against option s_j; we introduce the statements P^k_j: "individual k is not against s_j", and truth1, $P^k_j = h^k_j$. Then, we construct the set $V_j = \{v_j\} = 2^{\{P^1_j, \ldots, P^m_j\}}$, and define a linguistic quantifier Q as a fuzzy set in V_j.

Next, we introduce the statements P^Q_j: "Q individuals are not against option s_j", and truth $P^Q_j = \max_{v_j \in V_j} (\mu_T(v_j) \wedge \mu_Q(v_j))$.

Finally, the *fuzzy $Q-$core* is defined as a fuzzy set

$$C_Q = \text{truth}\, P^Q_1 / s_1 + \cdots + \text{truth}\, P^Q_n / s_n \tag{7}$$

i.e. as a fuzzy set of options that are not defeated by Q (e.g., most) individuals.

Notice that in (5) a strength of "being not against s_j" is not accounted for; this may be done too. First, instead of "$\ldots > 0.5$" we may use "$\ldots \geq \alpha > 0.5$", and define analogously a *fuzzy $\alpha/Q-$core.* Moreover, we can redefine h_{ij}^k in (5) to explicitly express the strength of "being not against s_j" as, in general, $h_{ij}^k = s(r_{ij}^k)$ where h is, e.g., a nondecreasing function. Then, we can analogously define a *fuzzy $s/Q-$core* (cf. Kacprzyk, 1985, 1986).

For the indirect approach, we derive first a *social fuzzy preference relation*, $R = [r_{ij}]$, from $\{R_1, \ldots, R_n\}$. This will not be discussed here, and some details can be found in, e.g., Nurmi (1981). A solution concept with much intuitive appeal is here the *consensus winner.* We start with

$$g_{ij} = \begin{cases} 1 & \text{if } r_{ij} > 0.5 \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

and then we introduce P_j^i: "s_i is preferred over s_j", truth $P_j^i = g_{ij}$. We construct the set V_i (analogously as mentioned before), determine $\mu_T(v_i)$, introduce P_i^Q: "s_i is preferred over Q other options", and determine its truth, truth P_i^Q, due to (2). Finally, we define the *fuzzy $Q-$consensus winner* as

$$W_Q = \text{truth} P_1^Q / s_1 + \cdots + \text{truth} P_n^Q / s_n \tag{9}$$

i.e., as a fuzzy set of options that are preferred over Q other options. We can account – similarly as in the case of the fuzzy cores – for the strength of preference in (8), and define similarly a *fuzzy $\alpha/Q-$consensus winner* and *fuzzy $s/Q-$consensus winner.* For details and other definitions, see Kacprzyk (1985, 1986), and for newer, more sophisticated solution concepts, mainly of the uncovered and undominated type, see Nurmi and Kacprzyk (1991).

Consensus reaching is strongly related to group DM. The main issue will be here how to measure the *degree of consensus* since our position is that consensus, traditionally meant as a full and unanimous agreement, is utopian, too rigid, and often unnecessary in practice.

Starting again with a set of individual fuzzy preference relations, the degree of consensus is derived in four steps of aggregation. First, for each pair of individuals we derive a degree of agreement as to their preferences between *all* the pairs of options, next we obtain a degree of agreement of each pair of individuals as to their preferences between $Q1$ (most, almost all, much more than a half, ...) pairs of relevant options. Then , we obtain a degree of agreement of $Q2$ pairs of important individuals as to their preferences between $Q1$ pairs of relevant options. Finally, we obtain e degree of agreement of $Q2$ pairs of important pairs of individuals as to their preferences between $Q1$ pairs of relevant options. This is meant to be the *degree of consensus* sought.

If $\mu_B(s_i) \in [0,1]$ is the relevance of option s_i, then the relevance of a pair (s_i, s_j) is, e.g., $b_{ij}^B = \frac{1}{2}(\mu_B(s_i) + \mu_B(s_j))$. And analogously for the importance of individuals, $\mu_I(ki)$, and the importance of the pair $(k1, k2)$, $b_{k1,k2}^I$.

We start with the degree of strict agreement between $k1$ and $k2$ as to their preferences between s_i and s_j

$$v_{ij}(k1, k2) = \begin{cases} 1 & \text{if } r_{ij}^{k1} = r_{ij}^{k2} \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

and introduce the propositions $P_{ij}(k1, k2)$: "individuals $k1$ and $k2$ agree as to their preferences between s_i and s_j", and truth$(P_{ij}(k1, k2) = v_{ij}(k1, k2)$. We construct the set $W_{ij}(k1, k2) = 2^{\{P_{12}(k1,k2), \ldots, P_{(n-1)n}(k1,k2)\}}$, and determine $\mu_T(w_{ij}(k1, k2))$.

We introduce the propositions $P^B_{Q1}(k1,k2)$: "individuals $k1$ and $k2$ agree as to their preferences between $Q1$ relevant (B) pairs of options", and determine truth $P^B_{Q1}(k1,k2)$ due to (3) or (4). Using the $P^B_{Q1}(k1,k2)$'s, we construct the set $W^B_{Q1}(k1,k2)$ analogously as $W_{ij}(k1,k2)$, and determine truth $P^B_{Q1}(k1,k2)$.

We introduce the propositions $P^{I,B}_{Q1,Q2}$: "$Q2$ important (I) pairs of individuals agree as to their preferences between $Q1$ relevant (B) pairs of options", determine its truth $P^{I,B}_{Q1,Q2}$.

This is the *degree of* $Q1/Q2/I/B-$*consensus* sought, i.e. $\mathrm{con}(Q1,Q2,I,B) = \mathrm{truth}\, P^{I,B}_{Q1,Q2}$, meant as the degree to which $Q2$ pairs of important (I) individuals agree as to their preferences between $Q1$ pairs of relevant (B) options. Moreover, one can consider the strength of agreement in (10), and derive extensions of the above degree of consensus (cf. Kacprzyk and Fedrizzi, 1989). The new degrees of consensus have been used in an implemented DSS for consensus reaching (cf. Fedrizzi, Kacprzyk and Zadrożny, 1989).

4. CONCLUDING REMARKS

We showed briefly how to account for a fuzzy majority in group DM and consensus reaching by using a fuzzy-logic-based calculus of linguistically quantified propositions.

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