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SUPPORT SYSTEMS FOR DECISION AND NEGOTIATION PROCESSES

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EXTENT ANALYSIS AND SYNTHETIC DECISION

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ABSTRACT

In this paper, the concept of extent analysis and the value of extent analysis are given by using the triangular fuzzy number. To start with, we give the definition of triangular fuzzy number and its method of operation, then the concept of extent analysis and the value of extent analysis are introduced. Finally, we will use the extent analysis method to make synthetic decision.

Keywords: Fuzzy decision, Fuzzy number, Extent analysis, Synthetic decision.

1. Introduction

There are much problems of estimation in the real world which needs to consider the extent of the object to be satisfied for the goal. For example, many people always say "satisried extent", "confidence extant", "stable extent", "reliable extant", "resemblance extent", etc. We need to quantify for them. Moreover, in some complex decision–making problems, we need to be in progress the comprehensive evaluation for the multiple goals, in order to find a optimal decision.

In this paper, we will use fuzzy number for the metric "extent". In section 2 the concept of fuzzy number is introduced. The concept of extent analysis and the value of extent analysis calculation method are presented in section 3. In section 4, we will use the extent analysis method to make synthetic decision. The conclusion is discussed in section 5.

2. Fuzzy Number

We consider a special class of fuzzy number, which is suitable for the application discussed here. First we define the triangular fuzzy number and next, we define its oprcration law.

Definition1. We define a fuzzy number M on R to be a triangular fuzzy number, if its membership function $\mu_M(x)$: $R \to [0, 1]$ is equal to

$$\mu_M(x) = \begin{cases} \dfrac{x}{m-l} - \dfrac{l}{m-l} & x \in [l, m] \\ \dfrac{x}{m-u} - \dfrac{u}{m-u} & x \in [m, u] \\ 0 & otherwise. \end{cases}$$

where $l \leqslant m \leqslant u$, l and u stand for the lower and upper value of the support of M, respectively. And m for the modal value. The triangular fuzzy number will be denoted by (l, m, u). The support of M is the set of elements $\{x \in R | l < x < u\}$ (Figure 1). When l = m = u, it is a nonfuzzy number by comention.

For any two fuzzy numbers M and N defined by their membership function μ_M and μ_N, the membership function of the fuzzy number T = f (M, N) is assumed to be continuous function on R.

$$\mu_T(z) = \sup_{z=f(x,y)} \min[\mu_M(x), \mu_N(y)] \tag{1}$$

or

$$\mu_{M \circledast N}(z) = \sup_{z=x*y} \min[\mu_M(x), \mu_N(y)] \tag{2}$$

where $*$ is a binary operation and $\circledast$ represents to combine two fuzzy numbers M and N.

2.1 Addition and multiplication

Consider two triangular fuzzy numbers $M_1 = (l_1, m_1, u_1)$ $M_2 = (l_2, m_2, u_2)$.
Eq. (2) implies, for addition

$$\mu_{M_1 \oplus M_2}(z) = \sup_{z=x+y} \min[\mu_{M_1}(x), \mu_{M_2}(y)] = \sup_{x \in R} \min[\mu_{M_1}(x), \mu_{M_2}(z-x)] \tag{3}$$

for multiplication

$$\mu_{M_1 \odot M_2}(z) = \sup_{z=xy} \min[\mu_{M_1}(x), \mu_{M_2}(y)] = \sup \min[\mu_{M_1}(x), \mu_{M_2}(\frac{z}{x})] \tag{4}$$

Hence we have

$$(l_1, m_1, u_1) \oplus (l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2) \tag{5}$$

and the following approximation formula

$$(l_1, m_1, u_1) \odot (l_2, m_2, u_2) \approx (l_1 l_2, m_1 m_2, u_1 u_2) \tag{6}$$

We also have got the result of scalar multiplication as follows

$$(\lambda, \lambda, \lambda) \odot (l, m, u) = (\lambda l, \lambda m, \lambda u) \quad \lambda > 0, \ \lambda \in R \tag{7}$$

2.2 Inverse

Let M = (l, m, u,), we obtain following approximation formula

$$(l,\ m,\ u,\ )^{-1} \approx \left(\frac{1}{u},\ \frac{1}{m},\ \frac{1}{l}\right)$$

2.3 Comparison of fuzzy numbers

When comparison of fuzzy numbers, two kinds of question may arise.

(A) What is the fuzzy value of the least or greatest number from a family of fuzzy numbers?

(B) Which is the greatest or the least among several fuzzy numbers?

The answer to the first question is given by the use of the operation max and min [2]. But, the answer to the second question is more diffcult. We must evaluate the degree of possibility for $x \in R$ fuzzily restricted to belong to M, to be greater than $y \in R$ fuzzily restricted to belong to M.

Definition 2. The degree of possibility of $M_1 \geqslant M_2$ is defined as below

$$V(M_1 \geqslant M_2) = \sup_{x \geqslant y} \min\left(\mu_{M_1}(x),\ \mu_{M_2}(y)\right) \tag{8}$$

When a pair (x, y) exists such that $x \geqslant y$ and $\mu_{M_1}(x) = \mu_{M_2}(y) = 1$, then we have $V(M_1 \geqslant M_2) = 1$. Since M_1 and M_2 are convex fuzzy numbers, it can be seen Fig.2 that

$$\begin{aligned} &V(M_1 \geqslant M_2) = 1 && \text{iff } m_1 \geqslant m_2 \\ &V(M_1 \geqslant M_2) = hgt(M_1 \cap M_2) = \mu_{M_1}(d) && \end{aligned} \tag{9}$$

where d is the ordinate of the highest intersection point D between μ_{M_1} and μ_{M_2}.

When $M_1 = (l_1, m_1, u_1)$ and $M_2 = (l_2, m_2, u_2)$, the ordinate of D is given by the equation (Fig.3)

$$V(M_2 \geqslant M_1) = hgt(M_1 \cap M_2) = \frac{l_2 - u_1}{(m_1 - u_1) - (m_2 - l_2)} \tag{10}$$

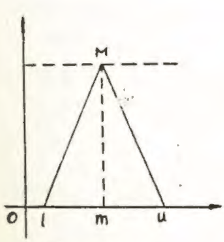

Figure 1.

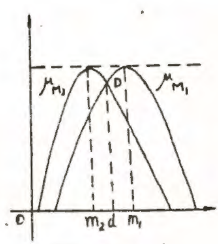

Figure 2.

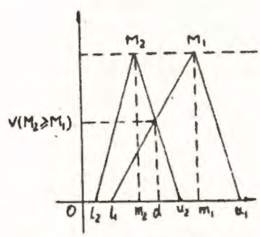

Figure 3.

To compare M_1 and M_2, we need both V $(M_1 \geqslant M_2)$ and V $(M_2 \geqslant M_1)$. If, for instance, V $(M_1 > M_2)$ = 1, we know that either $M_1 \geqslant M_2$, or M_1 and M_2 are too close to be separated.

Definition 3. The degree of possibility for convex fuzzy number M to be greater than K convex fuzzy numbers M_i (i=1, 2, ⋯, k) can be defined by

$$V(M \geqslant M_1, M_2, \cdots, M_K) = V[(M \geqslant M_1) \text{ and } (M \geqslant M_2) \text{ and} \cdots \text{and } (M \geqslant M_K)] = \min_{i=1,k} V(M \geqslant M_i) \quad (11)$$

3. Extent Analysis

First at all, we will give some basic concepts.

Assume that $X = \{x_1, x_2, \cdots, x_n\}$ is an object set, and $U = \{u_1, u_2, \cdots, u_m\}$ is a goal set .Let us consider how to appraise the extent whose element of object set is to be satisfied for the each goal. We consider a case which contains only one goal in the goal set , we write it as $U_0 = \{u\}$.

Definition 4. Let $X = \{x_1, x_2, \cdots, x_n\}$ is an object set and $U_0 = \{u\}$ is a goal set then

$$E_i = \{(x_i, u), \mu_E(x_i) \mid (x_i, u) \in X \times U_0\}, \quad i = 1, 2, \cdots, n$$

is called the extent of the ith object satisfied requirement of the goal.

Where $\mu_E(x_i)$ is membership function of extent of ith object, (x_i, u) is a binary relation on $X \times U_0$.

Our problem is how to calculate value of membership function for each object. We can use an approach which give point from judges. Assume that we invite t judges, let everybody of them gives a fuzzy number for ith object (in fact, the judges need to give the lower and upper value only) .Thus we obtain definition as follows

Definition 5. Let M_k^i k = 1, 2, ⋯, t i = 1, 2, ⋯, n are some triangular fuzzy numbers, then

$$M_E^i = \frac{1}{t} \odot [M_1^i \oplus M_2^i \oplus \cdots \oplus M_t^i] \quad (12)$$

is called the value of extent of ith object, and the lower and the upper value all take value on [0, 1] by convention.

If we note

$$M_k^i = (l_k^i, m_k^i, u_k^i) \quad k = 1, 2, \cdots, t \quad i = 1, 2, \cdots, n,$$

then

$$M_E^i = \frac{1}{t} \odot [(l_1^i, m_1^i, u_1^i) \oplus (l_2^i, m_2^i, u_2^i) \oplus \cdots \oplus (l_t^i, m_t^i, u_t^i)] \quad (13)$$

By applying (6) and (7), we have

$$M^i_E = (\frac{1}{t}\sum_{k=1}^{t} l^i_k,\ \frac{1}{t}\sum_{k=1}^{t} m^i_k,\ \frac{1}{t}\sum_{k=1}^{t} u^i_k) \qquad i=1,\ 2,\ \cdots,\ n. \tag{14}$$

We stipulate l^i_k, $u^i_k \in [0,\ 1]$, of course, $m^i_k \in [0,\ 1]$is natural for each k.

Definition 6. If we note $\delta^i_k = u^i_k - l^i_k$ $k=1,\ 2,\ \cdots,\ t,$ $i=1,\ 2,\ \cdots,\ n,$ then δ^i_k is called degree of spread of kth point.

Where δ^i_k represent fizzy degree of kth point.i.e.The greater δ^i_k is, the fuzzier the point is.On the contrary, the less δ^i_k is, the clearer the point is.

Definition 7. Let $\delta^i = \frac{1}{t}\sum_{k=1}^{t} (u^i_k - l^i_k)$, and we note $\lambda^i = (1-\delta^i)$ 100% then

λ^i is called degree of self–assurance concerning ith object.

Evidently, if the δ^i increases, the degree of self– assurance dicreases, conversely, if δ dicreases, the degree of self–assurance increases, when $\delta^i = 0$, it will be able to represent that the appraisal have got absclute self–assurance.

4. Synthetic Decision

The task of extent analysis is to be in progress decision between some goals of contradict each other.

For example, assume that we have some plans, we will consider for them from two aspects of requirement and feasible, in oder to chose a best plan.The process is called the synthetic decision.

Now, we will use the method of extent analysis to solve the problem.

Let $X = \{x_1,\ x_2,\ \cdots,\ x_n\}$ is an object set, and $U = \{u_1,\ u_2,\ \cdots,\ u_m\}$ is a goal set, for example, $U = \{u_1,\ u_2\}$, where u_1 represents requisite goal and u_2 represents feasible goal.

According to method of extent analysis, we will take each plan, which is to be in progress extent analysis for each goal, respectively.Therefore, we can get m values of extent analysis for each plan, with the following signs:

$$M^1_{E_i},\ M^2_{E_i},\ \cdots,\ M^m_{E_i} \qquad i=1,\ 2,\ \cdots,\ n$$

To find the best plan, we must be give some definitions as dbelow

Definition 8. Let $M^1_{K_i}$, $M^2_{K_i}$, $\cdots$, $M^m_{K_i}$ are values of extent analysis of ith plan for m goals, then the weighted–sum–type fuzzy synthetic extent is defined as below

$$S_i = \sum_{j=1}^{m} M^j_{E_i} \cdot r^j_i \odot [\sum_{i=1}^{n}\sum_{j=1}^{m} m^j_{E_i} \cdot r^j_i]^{-1} \tag{15}$$

Where r are weights and $\sum_{j=1}^{m} r^j_i = 1$ for all i.For short S_i is synthetic extent for ith plan.

In this paper, we do not have to consider weight.i.e. we have

$$S_i = \sum_{j=1}^{m} M_{E_i}^j \odot [\sum_{i=1}^{n} \sum_{j=1}^{m} M_{E_i}^j]^{-1} \quad (16)$$

Next, we may prefer to have a scalar measure of the dominance d (x_i) of a plan x over other ones.By using the formula (11),

$$d(x_i) = \min_{\substack{k=1,2,\cdots,n \\ k \neq i}} V(S_i \geq S_k) \qquad because\ V(S_i \geq S_i) = 1 \quad (17)$$

At last, we given the concept of optimal decision.

Definition 9.Let

$$d(x^*) = \max_{i=1,2,\cdots,n} d(x_i) \quad (18)$$

then the optimal decision will be able to corresponds the plan x^*, where $x^* \in x$.

Now, let us use an example to illustrate the synthetic decision process.

Example. Let X= (x_1, x_2, x_3) is a plan set, and U= (u_1, u_2) is a goal set, where u_1 represents requirement goal and u_2 represents feasihility goal.

First at all, we can use the approach of extent analysis to find the value of requirement extent and the value of feasibility extent for each plan, respectively.The data is shown as below

extents \ goals / plans	Requirement (u_1)	feasibility (u_2)
x_1	$M_{E_1}^1$ = (0.5, 0.6, 0.7)	$M_{E_1}^2$ = (0.5, 0.65, 0.8)
x_2	$M_{E_2}^1$ = (0.6, 0.8, 1)	$M_{E_2}^2$ = (0.7, 0.8, 0.9)
x_3	$M_{E_3}^1$ = (0.8, 0.9, 1)	$M_{E_3}^2$ = (0.5, 0.6, 0.8)

By using the formula (16) we obtain

$$S_1 = \sum_{j=1}^{2} M_{E_1}^j \odot [\sum_{i=1}^{3} \sum_{j=1}^{2} M_{E_i}^j]^{-1}$$

$$= (1.00, 1.25, 1.50) \odot (\frac{1}{5.2}, \frac{1}{4.35}, \frac{1}{3.6})$$

$$= (0.19, 0.29, 0.42)$$

$$S_2 = (1.30, 1.60, 1.90) \odot (\frac{1}{5.2}, \frac{1}{4.35}, \frac{1}{3.6})$$

$$= (0.25, 0.37, 0.52)$$

$$S_3 = (1.30,\ 1.50,\ 1.80) \odot (\frac{1}{5.2},\ \frac{1}{4.35},\ \frac{1}{3.6})$$
$$= (0.25,\ 0.34,\ 0.50)$$

And then, by using the formula (10), we have

$$V\ (S_1 \geqslant S_2) = \frac{0.25-0.42}{(0.29-0.42)\ -\ (0.37-0.25)} = \frac{0.17}{0.25} = 0.68$$
$$V\ (S_1 \geqslant S_3) = \frac{0.25-0.42}{(0.29-0.42)\ -\ (0.34-0.25)} = \frac{0.17}{0.22} = 0.77$$
$$V\ (S_2 \geqslant S_1) = 1$$
$$V\ (S_2 \geqslant S_3) = 1$$
$$V\ (S_3 \geqslant S_1) = 1$$
$$V\ (S_3 \geqslant S_2) = \frac{0.25-0.50}{(0.34-0.50)\ -\ (0.37-0.25)} = \frac{0.25}{0.28} = 0.89$$

And then, we use the formula (11)

$$d\ (x_1) = V\ (S_1 \geqslant S_2,\ S_3) = min\ (0.68,\ 0.73) = 0.68$$
$$d\ (x_2) = V\ (S_2 \geqslant S_1,\ S_3) = min\ (1,\ 1) = 1$$
$$d\ (x_3) = V\ (S_3 \geqslant S_1,\ S_2) = min\ (1,\ 0.89) = 0.89$$

At last, by using the formula (18)

$$d\ (x^*) = max\ (d\ (x_1)\ ,\ d\ (x_2)\ ,\ d\ (x_3)\ )$$
$$= max\ (0.68,\ 1,\ 0.89)$$
$$= 1$$

Therefore, we have got the optimal decision whose is the plan x_2 i.e. the optimal plan is that

$$x^* = x_2.$$

When the value of weights is given, we can use formula (15) to be calculate the synthetic extent.

5. Conclusion

In this paper, "extent" was defined a fuzzy number and by using the operation of fuzzy number to calculated the value of extent and degree of self–assurance.

A new method of decision was given which used extent analysis to make syunthetic decision, in oder to choose an optimal plan in a certain number of plans.

In this paper, formulars (15) and (16) are a situation of the weights was fixed.In fact, we sould be allso consider a situation of the change of weight, we have not been to touch on it in here.

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