IFAC/IFORS/IIASA/TIMS

The International Federation of Automatic Control
The International Federation of Operational Research Societies
The International Institute for Applied Systems Analysis
The Institute of Management Sciences

SUPPORT SYSTEMS FOR DECISION AND NEGOTIATION PROCESSES

Preprints of the IFAC/IFORS/IIASA/TIMS Workshop
Warsaw, Poland
June 24-26, 1992

Editors:
Roman Kulikowski
Zbigniew Nahorski
Jan W. Owsiński
Andrzej Straszak

Systems Research Institute
Polish Academy of Sciences
Warsaw, Poland

VOLUME 2:

Names of first authors: **L-Z**

SYSTEMS RESEARCH INSTITUTE, POLISH ACADEMY OF SCIENCES

SUPPORT SYSTEMS FOR DECISION AND NEGOTIATION PROCESSES
Preprints, IFAC/IFORS/IIASA/TIMS Workshop, June 24-26, 1992, Warsaw, Poland

PROMISE : A DSS FOR MULTIPLE OBJECTIVES STOCHASTIC LINEAR PROGRAMMING PROBLEMS

Raymond NADEAU[(1)], Bruno URLI[(2)], Lazlo KISS[(1)]
[(1)] Sciences de l'administration, Université Laval, Québec, Canada.
[(2)] Université du Québec à Rimouski, Rimouski, Québec, Canada.

Abstract: In the last ten years, some multiple objectives linear programming (MOLP) methods have been proposed to deal with MOLP problems in stochastic contexts. In this paper we briefly present a DSS called PROMISE: it enables the decision maker (DM) to identify the characteristics of many current stochastic contexts and it helps him to choose the stochastic MOLP method which is the most appropriate to each context, if such a method exist.

Keywords: Multiple objectives, linear programming, stochastic, decision support system

1. Introduction

Many real problems can be modeled as MOLP problems but in order to obtain an adequate modeling, one often has to take into account the uncertainty which surrounds the parameters of these problems. So, in the last ten years, some methods have been developped to deal with MOLP problems in stochastic contexts. The way of modeling uncertainty depends on the specific nature of that uncertainty. In particular, there are risky situations and situations with partial uncertainty: moreover, in some contexts, random parameters can be summarily expressed by scenarios.

In order to help the DM to well identify the characteristics of the uncertain context, the best way of modeling it and to choose the stochastic MOLP (MOSLP) method which is the most appropriate in that context, we have built a decision support system called PROMISE (in french: **PRO**grammation **M**ulti-objectifs **I**nteractive **S**tochastiqu**E**). In section 2, we present a typology of uncertain situations. After, taking into account the nature of uncertainty, many ways of modeling the MOSLP problems are summed up in section 3. Finally the section 4 presents the general structure of our DSS which enables the DM to choose the appropriate MOSLP method with regard to the stochastic context.

2. The uncertain situations

In this paper, we deal with the following MOSLP problem

$$\max \quad Z_k(x) = c_k'.x \ , \ k=1,..,h$$

(1) $\quad$ s.t. $\quad A.x \le b$,

where $x \in R^n$ and c_k, A, b are matrices with dimensions m*n, m*1 and n*1 respectively and of which the coefficients a_{ij}, b_j and c_{kj} are random variables defined on some probability space. According that the DM possesses more or less information with respect to those random parameters, he is supposed to be placed in a situation of risk or in a situation of partial uncertainty .

In a *situation of risk*, the DM is able to associate probability distributions to the stochastic parameters of (1); that is certainly not the most frequent situation in practice but that is this context which is supposed in most of stochastic linear programming methods with a single objective and also in some MOSLP models (for example, in the PROTRADE method from Goicoechea and al.[1982]). In practice, it is often more realistic to suppose that the DM is placed in a *situation of partial uncertainty*, i.e. a situation where he possesses only *incomplete information* about the stochastic parameters. In such a partial uncertainty, one can try either to directly model the stochastic parameters from the available information either to approximate those parameters by scenarios. In the framework of the *direct approach of modeling*, the uncertain situations which have been considered are those where the DM is able to specify the intervals of variation of the stochastic parameters; moreover, when the DM is able to determine goals with respect to the objective functions, these goals can be used in the modeling.

An indirect approach to modeling a situation of partial uncertainty (which is interesting particularly in planning problems) consists of using *scenario approach*. Scenarios represent plausible values of the random variables of the system in the future; so each continuous random parameter of (1) is in a way transformed into a discrete one, that is approximated by some representative and well contrasted values. Those scenarios can be global or partial. In the framework of *global scenarios*, there are some states of nature and each of those states simultaneously affects all the random parameters of (1). Moreover, in front of such global scenarios, the DM can be able or unable to specify (or estimate) the probabilities of those scenarios; the first situation has been recently tackled by Klein and al.[1990]. In the framework of *partial scenarios*, there are also some states of nature but, for example, some of them affect only the stochastic objective functions and some others of them affect only the stochastic constraints; again the DM can be able or unable to specify the probabilities of those scenarios. For example, in the STRANGE method, Teghem and al.[1986] have used partial scenarios with specified probabilities. The different uncertain situations that we have previously described are represented in the diagram of figure 1.

3. Modeling the MOSLP problems

Up to now the MOSLP methodologies has been few in number but there are nevertheless some which are referred by Stancu-Minasian [1984] and Slowinski and Teghem [1990]. Generally, from appropriate transformations of the objective functions and the stochastic constraints, the initial MOSLP problem is changed into an equivalent multiobjective deterministic problem. In order to obtain a good compromise solution, it is generally necessary to obtain supplementary information from the DM about his preference structure. This is done using an a priori articulation of preferences or a progressive articulation of preferences, what is called the interactive approach. The different approaches which have been proposed with regard to modeling the uncertainty of the objectives functions and the constraints are summed up in the following section.

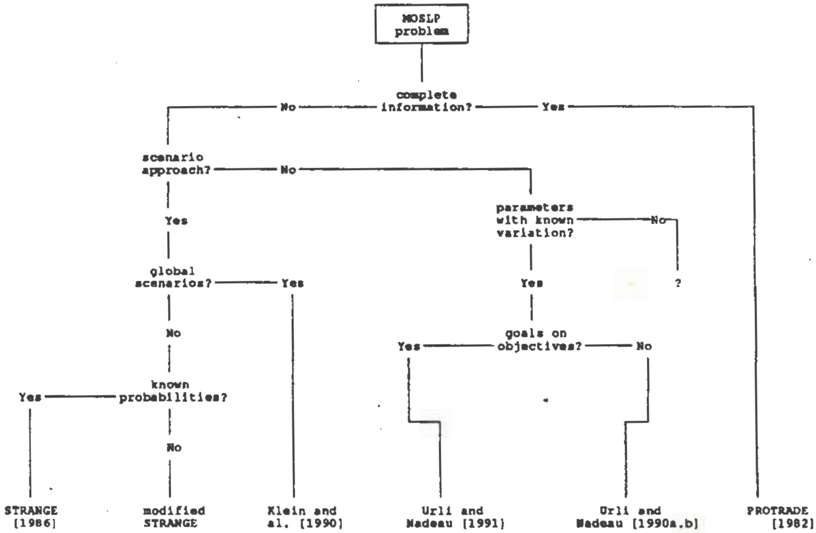

Figure 1. Diagram of the PROMISE algorithm

3.1. Modeling of objective functions

-*Mean value approach*. If the DM is able to specify probabilities for the c_k of (1), this approach simply consists of replacing those c_k by their mean values in the objective functions.

-*Minimum risk approach*. If the DM is able to specify the probability distributions for the c_k of (1) and the goals $G=\{G_1,..,G_k\}$ with respect to the objectives, instead of maximizing $Z_k(x)$, one tries to maximize the probabilities $P(Z_k(x)\geq G_k)$, $k=1,..,h$, what provides a deterministic equivalent.

-*Scenario approach*. The stochastic parameters c_{kj} of (1) are approximated by some representative values c^s_{kj}, $s=1,..,r$; so the stochastic objective function $Z_k(x)=c_{k1}x_1+..+c_{kn}x_n$ is replaced by r deterministic functions $Z^s_k(x)=c^s_{k1}x_1+..+c^s_{kn}x_n$; $s=1,..,r$; $k=1,..,h$.

-Mean-dispersion approach. If the DM is placed in a situation of partial uncertainty which enables him to specify for each c_{kj} the interval of variation $[c_{kj}^-, c_{kj}^+]$ and a central value c_k^o, then one can replace $Z_k(x)$ by a "central tendency" criterion $Z_k^o(x)$ and a "dispersion" criterion $Z_k^\Delta(x)$. One can choose $Z_k^o(x)=c_k^o.x$. Moreover, Z_k^Δ can be defined by the following generalized distance $Z_k^\Delta=D_p(Z_k^s(x), Z_k^o(x))=(\Sigma_s \tau_s^p.|Z_k^s-Z_k^o|^p)^{1/p}$ where s ($s=1,..$ S ($S \leq 2^n$)) is the index of the combinations of the extreme values of c_{kj}, τ_s is the weight of each of these combinations and p is a positive integer which expresses the importance that the D.M. places on the maximal deviation between Z_k^s and Z_k^o.
-Degree of satisfaction approach. In a partial uncertainty situation, one associates to each stochastic Z_k a stochastic inequality in the form $\{Z_k(x) \geq g_k\}$, where g_i represents the goal to be attained relatively to the objective $Z_k(x)$. It is supposed that $c_{kj} \in [c_{kj}^-, c_{kj}^+]$ and $g_k \in [g_k^-, g_k^+]$; g_k^+ represents the goal that the DM wishes to attain for $Z_k(x)$, whereas g_k^- represents the inferior bound which the DM is ready to accept for the goal g_k. So we look for solution x* which minimize the difference $(g_k-Z_k(x))$. More precisely, the degree of satisfaction of the DM relatively to the attainment of the goals g_k is expressed by a decreasing function P_k of $(g_k-Z_k(x))$; consequently, the initial problem which consisted of "maximizing" the objective functions $Z_k(x)$ is replaced by the problem of maximizing the DM's degree of satisfaction. If one chooses for P_k a linear form, then the equivalent deterministic objective functions are also linear.

3.2. Modeling of stochastic constraints

In a risky situation, two well-known approaches of transforming the stochastic constraints consist of using constraints with recourse or constraints with probability thresholds.
-Constraints with recourse. The problem (1) is approached as a two-stage problem: one chooses here and now and, because this solution eventually implies a violation of the constraints $A.x \leq b$, one is able to take a recourse decision which comes to compensate that violation by penalties. In the literature, several forms of recourse have been considered (see, for example, Kall [1976]).
-Constraints with probability thresholds. Here, the constraint $A_i.x \leq b_i$ (where A_i is the i-th row of A) is replaced by $P(A_i.x \leq b_i) \geq \alpha_i$, where α_i, $i=1,..,n$, is a probability threshold provided by the DM. The new deterministic constraints are generally non-linear. It is also possible to use a joint probability threshold on all the constraints taken together.

In a situation of partial uncertainty, according to the nature of the available information, the transformation of the stochastic constraints can be realized by using constraints with recourse, constraints with satisfaction thresholds and a scenario approach.

-Constraints with satisfaction thresholds. If the DM is able to specify the intervals of variation $[A_i^-, A_i^+]$ and $[b_i^-, b_i^+]$ for A_i and b_i, the constraint $A_i.x \le b_i$ is interpreted as it follows: "the DM hopes that $A_i^-.x$ will not be larger than b_i^+ and his satisfaction level will be even higher that $A_i^+.x$ will be closer to b_i^+". Consequently, we introduce the DM's degree of satisfaction $\mu(A_i.x \le b_i)$ relatively to each stochastic constraint: this degree of satisfaction can be defined as a linear decreasing function of $(b_i^+ - A_i^+.x)$. After, if the DM is able to specify a satisfaction threshold α_i on each constraint, the stochastic constraint $A_i.x \le b_i$ is replaced by $\mu(A_i.x \le b_i) \ge \alpha_i$ which is deterministic and eventually linear.

-Scenario approach. As for the objective functions, each stochastic constraint $A_i.x \le b_i$ is replaced by r deterministic constraints $A^s{}_i.x \le b^s{}_i$ where $A^s{}_i$ and $b^s{}_i$, s=1,..,r, are plausible scenarios for A_i and b_i.

4. Structure of the DSS

In order to help the DM to choose the MOSLP method which is the most appropriate to his MOSLP problem, we have built an interactive algorithm called PROMISE; it is graphically summed up in the diagram of figure 1. That algorithm enables the DM to identify the characteristics of his stochastic problem and consequently to choose an appropriate MOSLP method. For the moment, our system includes only some methods but it can be generalized in order to include more methods. We will say only some words about those MOSLP methods which are completely described in the reference papers. In most of those methods, after that the MOSLP problem has been transformed into a deterministic MOLP problem, an interactive algorithm (as, for example, STEM) is used to solve this last problem.

If the DM has a complete information about the stochastic parameters of (1), his problem can be solved by the PROTRADE method from Goicoechea and al.[1982]; essentially the stochastic parameters are replaced by their mean values and an interactive algorithm which uses the DM's utility function (calculated for mean values) leads to a compromise solution.

In a context of partial uncertainty, one can use the scenario approach to model the problem (1). On the one hand, the idea of separated scenarios for the objective functions and for the constraints are used in the STRANGE method from Teghem and al.[1986]; it is also supposed that the DM is able to estimate the probabilities of those scenarios. Obviously, it should be easy to use the same idea of scenario and recourse but without resorting the probabilities of scenarios: that should be the modified STRANGE method. On the other hand, Klein and al.[1990] have recourse to the idea of global scenarios which affect both objective functions and constraints and also suppose that the probabilities of those scenarios are known.

In a context of partial uncertainty, one can also try to model that uncertainty directly from the available incomplete information. When the DM can estimate the intervals of variation for the stochastic parameters, Urli and Nadeau [1990a,b] propose to replace each Z_k by a criterion of central tendency and a criterion of dispersion and to use the idea of constraints with recourse or constraints with satisfaction thresholds; moreover, if the DM is able to specify goals for the objective functions and constraints, Urli and Nadeau [1991] propose to use the degree of satisfaction approach both for the objective functions and constraints.

5. Concluding remarks

This paper gives only an outline of our system. Obviously, the list of contexts of uncertainty and the MOSLP methods that it includes is not exhaustive: it will be easy to extend the system in order to include others contexts and others methods (existing or to develop). We are now in the process of implementing our algorithm on a microcomputer in such a manner that it becomes a real interactive decision support tool for the DM.

6. References

Goicoechea, A., Hansen, D.R., and Duckstein, L.(1982), Multiobjective Decision Analysis with Engineering and Business Applications, John Wiley, New York.

Kall, P. (1976), Stochastic linear programming, Springer, Berlin.

Klein, G.,Moskowitz, H., Ravindran, A.(1990),"Interactive multiobjective optimization under uncertainty", Management Science, 36(1), 58-75.

Slowinski, R., Teghem, J.(1990), Stochastic versus fuzzy approach to multiobjective mathematical programming under uncertainty, Kluwer Academic Publishers, Dordrecht.

Stancu-Minasian, I.M.(1984), Stochastic programming with multiple objective functions, D. Reidel Publ., Amsterdam.

Teghem, J., Dufrane, D., Thauvoye, M., and Kunsh, P.(1986), STRANGE: an Interactive Method for Multiobjective Linear Programming Under Uncertainty, E.J.O.R., 26(1), 65-82.

Urli, B., Nadeau, R.(1990.a), "Multiobjective Stochastic Linear Programming With Incomplete Information: a General Methodology", in Stochastic versus Fuzzy Approaches to Multiobjective Mathematical Programming Under Uncertainty, Slowinski R. and Teghem J., Op.cit.

Urli, B., Nadeau, R.(1990.b), Stochastic MOLP with Incomplete Information: An Interactive Approach with Recourse, J.O.R.S., 41(12), 1143-1152.

Urli, B., Nadeau, R.(1991), "An interactive method to multiobjective linear programming problems with interval coefficients", to appear in INFOR.