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UNCERTAINTY ANALYSIS AND ITS MATHEMATICAL BASIC SYSTEM

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ABSTRACT

In the process of analyzing the system optimization, this paper after drew the conclusion "Uncertainty Information Processing Is The Key Problem In System Optimization", discussed the uncertainty mathematical basis, put forward the basic uncertainty analysis concept and its theory system.

Keywords: system optimization, uncertainty information, uncertainty mathematics, uncertainty analysis.

The continuous development of science and technology and social production enable people to become more and more to attach importance to system optimization issue (esp. big and complex systems). And with the development of computer technology and modern mathematics, information processing is becoming an important problem daily among the problems of system optimization. Nevertheless the information received by people always has uncertainty due to outside various interference and the subjective knowledge limitation of the people, this created a situation that it is difficult for people to apply accurate mathematical analysis method to carry out effective real system optimization. In order to solve this problem, we, after analyzed the mathematical theory of uncertainty information, put forward uncertainty analysis theory system.

I. UNCERTAINTY INFORMATION PROCESSING IS THE KEY ISSUE IN SYSTEM OPTIMIZATION

1. Information processing is of incomparable importance in system optimization

A successful optimization system should be a optimum ordered combination of all elements (i. e. the subsystems) and the entire function best embodiment, and finally achieve preset aim. To achieve this requirement, it is necessary on the basis of determining the basic purpose and basic structure, to carry out elements correlativity analysis, high precision forecasting and right decision, and further more conduct an effective implementation and management, be sure to enable it to become an optimimum operation process. Every key stage and process there must be information processing.

Let the information be I, correlativity analysis be B, forecasting be F, decision be D, implementation and management be C, and Q be the optimized result, then every key element in the whole optimizing process can be expressed by the following function:

$$Q=\varnothing(B(I),F(I),D(I),C(I))$$

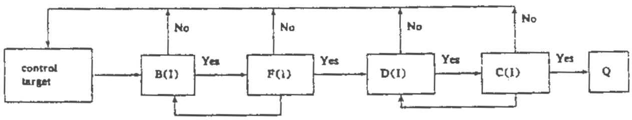
Where $\varnothing$ is the optimized combination of B(I), F(I), D(I), C(I).

2. Uncertainty information is found everywhere

To recognise a system is to recognise its information characteristics. The information characteristics mean mainly the triple unity of the system of elements, structure and function. A system cannot be described and expressed without these information characteristics. But every aspect from the generation of information to the description of information characteristics, makes the information have uncertainty because of different causes. In the process of information generation, transmission and reception, becauses of outside noise interference, the source information (i. e. the inherent information of the objective system itself) through information channel to host information (i. e. the information presented in the receiving system), always appears serious distortion phenomenon; when one describes the system information characteristics, it is impossible for one to portray objectively the existing things truthfully due to complexity and subjective knowledgement of the existing thing limitation of the quality presentation, quantity definition, relation expression and law discovering. All the consequences resulted from distortion and difficulty to reflect correctly renders the information to have uncertainty (such as randomness, fuzziness, greyness, extensionality difficult to forecast and develop, etc). We call the information with uncertainty the uncertainty information. And so long as there is system information characteristic description, there is uncertainty information accompanied. The system with uncertainty information may be called the uncertainty system.

3. The uncertainty information processing is the key problem for studying the system optimization

The system optimization process is the information processing and treatment process, its technical line is normally expressed as follows:



Note: All the symbols in the figure are the same as above.

Because every process in the system optimization includes uncertainty information, we can therefore claim that the normally called information processing should be the "uncertainty information processing". The system optimization without uncertainty information processing, one cannot perform effective correlativity analysis, forecasting and decision, not to say the implementation and management according to the controlling aim. Thus we can say for certain that the uncertainty information process is the key problem for system optimization.

II. UNCERTAINTY MATHEMATICAL BASIC SYSTEM

In order to process scientifically the uncertainty information, we must fully understand the uncertainty connotation, find out mathematical description method for the uncertainty information, establish a new theory system for studying and treating the uncertainty information——uncertainty analysis.

1. The uncertainty information connotation

No matter which way you consider from either quality, quantity, relation, or the law, information is of uncertainty, and information processing is the optimization key, we therefore must have a deep understanding toward the uncertainty. We shall compare the three kinds of uncertainties recognised by people as follows:

1. 1 **Randomness** Because of insufficient conditions or occasional element interference, it causes several results come up with occasionality, in a certain test, you cannot predict which one will happen, this is called randomness.

1. 2 **Fuzziness** Because of the complexity of things, the boundary of things are unclear, it makes the abstract concept not give out certained description, not give out certained appraisal criterion, makes the information present uncertainty, i. e. fuzziness. Just as the incompatible principle says: "When a system complexity increases, the ability to make it more precise descreases, once it reachea a certain limit, the comnlexity and precision will exclude each other, and what accompanys the complexity is fuzziness". This is an uncertainty presented by things themselves.

1. 3 **Greyness** Because of information channel (objective) noise interference and the receiving system (subjective) probability limit, we can only predict largely the scope presented by the information, not know the exact information, we call this partial known and partial unknown information the grey information, and also called uncertain information.

We know from abovementioned discussion that the three kinds of uncertainties are different. The randomness demonstrates its uncertained result before the test, but all the possible results are certained. Fuzziness manifests uncertainty by the things themselves, but it can be described by membership. The common point of the two is created by objective causes Whereas greyness is different from either randomness or fuzziness, it has either known or unknown cauies; has either objective causes or subjective causes. It can by way of membership determine the value taking scope of information.

The three kinds of uncertain information can be generally called potential information because they are caused by potentiality of things, either subjective or objective. "The logic from potentiality to exsitence is not myteriously to grow out from nothing or to make something out of nothing, but from potential existence to reality existence. Tbus potentiality essence is existence, whereas potentiality meaning is value. Though it looks like zigzag, but prosperious, and looms large and steps out with ease". In line of this thought, we say: Uncertainty information not only has to be studied but also can be studied. Nowadays, the rich results achieved toward the research the uncertainty of randomness, fuzziness and greyness clearly proved this point. But this is only just a beginning. As science and technology develop forward, there certainly will be other forms to show uncertainy, a lot of work are waiting ahead for us to improve, to develop. on the basis of this purpose, we established a unceutainty mathematical basic system.

2. Uncertainty mathematical basic system

2.1 General sense information aggregate concept

Definition 1 $\forall$ $y,z \in R$ (real number set) we call the set formed by the elememts y/z the suprareal number set, written as $\tilde{R}$

Definition 2 We call G one of the general information aggregate in the theory field U, it means we give the following mapping:

$$\mu_G(\cdot): U \to \tilde{R}, u \to [\underline{\mu}_G(u), \overline{\mu}_G(u)], u \in U; \underline{\mu}_G(u) \overline{\mu}_G(u) \in \tilde{R}$$

and we call $\mu_G(\cdot)$ the general information aggregate mambership function or characteristic function; interval $[\underline{\mu}_G(u), \overline{\mu}_G(u)]$ the general information field of u; $\underline{\mu}_G(u)$ and $\overline{\mu}_G(u)$ the lower and upper information limits of u, it can be simplified into $\underline{\mu}(u), \overline{\mu}(u)$ under the conditions that there should be no confusion.

Notes: Just as the sign $[\underline{\mu}(u), \underline{\mu}(u)]$ is called the general interval; in the concept, it contains the whole real numbera between $\underline{\mu}(u)$ and $\overline{\mu}(u)$ and the suprareal numbers $\frac{0}{0}$ and $\frac{a}{0}$ ($a \in R$); in operation, it denotes the aggregate formed by $\underline{\mu}(u)$ and $\overline{\mu}(u)$, and will not change its sequence in the operation.

In definition 1, when $\underline{\mu}(u) \neq \overline{\mu}(u)$, it rightly described the grey information "partial known and partial unknown"; when $\underline{\mu}(u) = \overline{\mu}(u) \in [0,1]$, it rightly described fuzzy information of "unclear boundary"; when $\underline{\mu}(u) = \overline{\mu}(u) = 1$, it rightiy described the certain information, or white information; when $\mu \in [a,b] \subseteq R$, $\underline{\mu}(u) = \overline{\mu}(u) = \mu(u) \in [0,1]$ and belongs to a certain probability distribution, it rightly described random information. We can see from above mention, general information aggregate is the mathematical basis for studying three kinds of uncertainty information.

2.2 Type A universal information mathematical basic system

For the sake of convenience, we call the uncertainty information the universal information. In definition 1, if U=R(real number set), $[\underline{\mu}(u), \overline{\mu}(u)] = [\underline{\mu}, \overline{\mu}] \subseteq [0,1]$ (real number interval), then we call G the type A universal information number set, written as F(R), and call the elements in F(R) type A universal information number, written as
$\alpha \triangleq (x, [\underline{\mu}(x), \overline{\mu}(x)])$

When $\underline{\mu}(x) = \overline{\mu}(x) \triangleq \mu(x)$, we call α the type A information number.

Then $\alpha = (x, \mu(x))$, written I(R) the type A information number set.

For the sake of convenience, this paper only deals with algebraic operation properties of type A information number.

Definition 3 $\forall \alpha_1, \alpha_2, \alpha \in I(R)$, define

$\alpha_1 = \alpha_2, \Leftrightarrow x_1 = x_2, \mu(x_1) = \mu(x_2)$;

$-\alpha = (-x, \mu(x))$ the negative element of $\alpha = (x, \mu(x))$

$\alpha_{(0)} = (0, \mu(0))$ the zero element of I(R);

$\alpha_{(1)} = (1, \mu(1))$ the unit element of I(R);

$\alpha^{-1} = (1/x, \mu(1/x))$ $(x \neq 0)$ the reverse element of α.

Definition 4 Let $\alpha_i (i=1,2,\cdots,n) \in I(R)$, define

$\Sigma\alpha_i = (\Sigma x_i, \mu(\Sigma x_i))$ the n type A algebraic addition of information number;

$\Pi\alpha_i = (\Pi\alpha_i, \mu(\Pi x_i))$ the n type A algebraic multiplication of information number.

we can prove

I(R) for the addition is the Abel Group; I(R) for the addition and multiplication is the replacible ring.

Definition 5 $\forall \alpha_i \in I(R), n \in J$(integer set), define

$\alpha_1^n = (x^n, \mu(x^n)$ the type A multiplication power of the information number, and there are $\alpha^0 = (x^0, \mu(x^0)) = (1, \mu(1))$; $\alpha^{-n} = (\alpha^{-1})^n = ((1/x)^n), u(1/x)^n)$

we can prove

$\forall \alpha_1, \alpha_2 \in I(R), (\alpha_1 \cdot \alpha_2)^n = \alpha_1^n \cdot \alpha_2^n; \alpha^m \cdot \alpha^n = \alpha^{m+n}; (\alpha^m)^n = \alpha^{mn}$

Tt's obvious, type A information number statisfies the operation law of classical mathematics; if one can find information function $\mu(x)$, we can conduct the information processing.

2.3 Type B universal information mathematical basic system

Because information function $\mu(x)$ sometimes is difficult to find out, but a certain information function can easily be found following reality. We give out algebraic operation definition from other angle; for the sake of operation (also have physical meaning) we have to expand the value taking scope of information value, i. e. let $\underline{\mu}(x), \overline{\mu}(x) \in \tilde{R}$ in the definition 2. At this time, we re-call the universal information number corresponding to type A universal information number the type B universal information number, and re-call the information number corresponding to type A infoumation number the Type B information number, similarly, for the sake of convenience we only discuss the algebraic operation properties of type B information number, we write G(R) the type B universal information number set, and $g(R)$ the type B infoumation number set.

Definition 6 $\forall g_i=(x_i,\mu(x))\in g(R)$ $(i=1,2,\cdots,n)$ we call

$\Sigma g_i=(\Sigma x_i,\Sigma x_i\mu_{(}x_i)/\Sigma x_i)$ the n type B algebraic operation addition of information number.

calling $\Pi g_i=(\Pi x_i,\Pi\mu(x_i))$ the n algebraic operation multiplication of type B information number.

We can prove, type B information number set $g(R)$ can form Abel Group for the addition and form replacible ring for the addition and multiplication.

On the basis of definition 6, we discussed type B universal information matrix theory, the operation property of the universal information determinant, the solution of universal information linear equation, the solution of one-element-two-order universal information equation, the solution of universal information linear planning as well as application in the interval analysis of type B universal information mathematics, etc. (owing to the limit of pages, we shall not discuss them one by one here).

2.4 Random information mathematical basis

In definition 2, if $U=R,\underline{\mu}(x)=\bar{\mu}(x)=\mu(x)\in[0,1],x\in[a,b]\leqslant R$, we call $G\underline{\triangle}([a,b],\mu(x))$ the interval information number.

When $\mu(x)$ is a random distribution function in $[a,b]$, we call $([a,b],\mu(x))$ the random information number. Here the $\mu(x)$ is the monotonous non-reducible function, and when $x<a$, defining $\mu(x)=0$, when $x\geqslant a$, defining $\mu(x)=\mu(b)\leqslant 1$.

The algebraic operation of random information number can be realized by Th. J. Stieltjes integration, this paper will not discuss further.

III. UNCERTAINTY ANALYSIS THEORETICAL SYSTEM

Nowadays, the theoretical basis for researching random information is probability statistics, the theoretical basis for researching fuzziness information is the frzzy aggregation, and the theoretical basis for researching the grey information is the grey system theory. They all have their own weak points; probability statistics require large data; fuzzy aggregation study on numbers is based on convex set; the mathematical basis of grey system theory is basically the classical mathematicas. In order to make up for these shortcomings we established above mentioned uncertainty mathematical theoretical basis, hut our starting point lies in solving the objective reality problems and focuses on large system optimization. Thus we need to study, on the basis of uncertainty mathematical theoretical basis, large system modeling problem with uncertainty and how to use modern computer technology to realize uncertainty large system optimization decision, etc. and we call the problems dealing with these uncertainty information processing the uncertainty analysis, it consists mainly of the following three parts:

1. Uncertainty mathematics theory

The uncertainty information mathematics consists principaly of type A universal information mathematics, type B universal information mathematics, random information mathematics, and the comprehensive processing methods for the three uncertainty mathematics theory.

2. The modeling theory for the uncertainy system

Since the involved systems in reality are multiple aspects, their proerty, and their stratification vary, their expressions of uncertainty are different. In order to study system performances, to realize the optimization control toward natural system or establish a certain optimization in the light of a certain need, it is necessary to establish different mathematical models according to different uncertainty information characteristics presented by different systems. We call such theory and method the modeling theory of uncertainty system.

3. Uncertainty system intelligence-aided decision theory

Now in the high-tech age, it is difficult to move a step toward realization of system optimization control without computer technology. To introduce the uncertainty mathematical model and its practice into computer is another complex process, we call such theory and method the intelligence-aided decision theory of uncertainty (such as computer intelligence analogy, uncertainty reasoning expert system, etc)

We believe that if the uncertainty analysis is improved and developed, uncertainty information processing will appear in the science field with new looks, and will promote system optimization study.

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