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SUPPORT SYSTEMS FOR DECISION AND NEGOTIATION PROCESSES

Preprints of the IFAC/IFORS/IIASA/TIMS Workshop

Warsaw, Poland

June 24-26, 1992

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VOLUME 2:

Names of first authors: **L-Z**

SYSTEMS RESEARCH INSTITUTE, POLISH ACADEMY OF SCIENCES

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COR-1: A SUPPORT TO CONSENSUS REACHING IN GROUP DECISION

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Abstract: *In this paper we propose an approach when several cooperative decision makers seek to reach a final decisionor recommendation. We propose an interactive aid that facilitates consensus reaching. This approach draws from a previous results which posits that multitudinous ranking can be obtain from a single relational system.*

Keywords: *hierarchical decomposition, relational preference system, consensus, aggregative isolation, aggregative decomposition, modular system architecture*

INTRODUCTION

Decision-making in any organization is only exceptionally entrusted to a single individual. Besides, though the autonomous decision maker may be the sole responsible, he cannot shut out the host of outside influences and pressures that enter his decision-making process. To investigate the case where several decision makers are involved, and the nature of their purpose, is to enter the field of collective action, the subject of this paper. Numerous such group situations have been identified and analyzed over the last years [Jelassi, 1990].

We shall focus on group decisions where several cooperative decision makers seek to reach a final decision (or recommendation) for which they are collectively responsible and committed. Many reasons vie for the development of effective tools and means of assisting group decisions. Huber (1982) mentions these: discussions dominated by overpowering participants, lack of communication, peer pressure that leads participants to conform to prevailing ideas rather than speak their own mind. Huber contends that lack of information, distortion of information and shallow treatment of issues contribute to lower the group participants' productivity. He argues that Group Decision Support Systems (GDSS) hold considerable promiss in clearing these hurdles.

So far, experimentation with GDSS [Lewis, 1987; Watson and al., 1988; Benbasat and Nault, 1990; etc.] seems to corroborate Huber's prognosis. These experiments suggest that GDSS increase participation, enhance decision quality and improve the group's ability to concentrate on the task at hand. They would also seem to dampen individual influence and domination, bolster the group's confidence in its decisions, augment individual participants' satisfaction with both the process and the result of the group's work, facilitate consensus and shorten the time needed to reach a decision. Moreover, Huber (1984) pleads a convincing case for the need to develop group decision aids by conjuring up this common dilemma: decision makers are asked to partake in evermore frequent and lengthy meetings, thus preventing them from attending to other equally pressing duties. The solution to this dilemma is to make these meetings shorter and more productive. Such are the motives driving the need for GDSS.

Needless to say, experimentation with GDSS has embraced many different dimensions. Gray and al.(1990) have proposed a method for distinguishing (and classifying) these experiments, thus facilitating their interpretation. Research in this field has drawn from group psychology and the study of individuals' behavior within groups. Assessing the link between electronically assisted group meetings and the ensuing results rests on three sets of factors: context, process and effects on group interaction. For instance, Bui and Jarke (1986) insist on group architecture and interaction between members; Jelassi and Beauclair (1987) point out that technical specifications of GDSS must consider group behavioral aspects and allow for interaction between group members. Though GDSS may take on a variety of configurations, the basic elements invariably include: hardware, software, users and procedures. We shall focus on software, an element comprising several interlocking components which, together, embody the essence of a GDSS.

To be more specific, we aim to develop an interactive aid that facilitates consensus reaching. To do so, we refer to a previous analysis [Kiss and Martel, 1991] wherein we demonstrate that for a given relational preference system (RPS) there exist many possible hierarchical decompositions (or rankings), and vice versa. Thus, given a RSP for each group member and the associated rankings, we seek to determine a ranking that satisfies all the participants (a consensus) or at least a strong majority.

FORMULATING THE SITUATION

Let us consider a case where N, $r = 1,2,\ldots,N$, interacting decision makers acknowledge a common decision problem, share a range of decision-making concerns or at least agree to subscribe to such whilst retaining their respective preferences and judgements, and are confronted with m, $X_{[m]} = \left\{X_1,\ldots,X_i,\ldots,X_m\right\}$, options.

The first step is for each of the N decision makers to specify his/her own relational indifference system (RIS):

$$\mathcal{R}(I)_{r,[b_r]} = \left\{ \bigcup_{v=1}^{b_r} \left(X_i I X_j\right)_v \;\middle|\; i,j = 1,\ldots,m\ ;\ i\neq j;\ \ m \geq 2\right\};\quad 1 \leq b_r \leq \frac{m(m-1)}{2};$$

$\forall r;\ r = 1,\ldots,N.$

Applying an aggregative isolation procedure [Kiss & Martel, 1991] to each RIS so obtained results in N partitions of the $X_{[m]}$ elements, thus yielding the sets:

$$A^{(r)}_{[n_r]} = \left\{A^{(r)}_{1,[h_1(r)]},\ldots,A^{(r)}_{j,[h_j(r)]},\ldots,A^{(r)}_{n_r,[h_{n_r}(r)]}\right\};\ r = 1,\ldots,N,$$

whose elements we shall denominate macroentities, "conglomerates" of the elements contained in $X_{[m]}$, with

$$\sum_{j=1}^{n_r} h_j(r) = m;\ n_r \le m \text{ and } \bigcup_{j=1}^{n_r} A_{j,[h_j(r)]} = \bigcup_{j=1}^{n_r}\left[\bigcup_{s=1}^{h_j(r)} A^{(r)}_{j,s}\right] = \bigcup_{i=1}^{m} X_i;\ \forall r;\ r = 1,\ldots,N.$$

The next step is for each decision maker to specify his/her relational preference system (RPS) by referring to the set of macroentities established through the aggregative isolation procedure:

$$\mathcal{R}(P)_{r,[c_r]} = \left\{ \bigcup_{z=1}^{c_r}\left(A_{i[h_i(r)]} P A_{j[h_j(r)]}\right)_z \ \middle|\ i,j = 1,\ldots,n_r;\quad i\ne j;\quad n_r \ge 2\right\};$$

$$1 \le c_r \le \frac{n_r(n_r-1)}{2};\ \forall r;\ r = 1,\ldots,N.$$

A new series of partitions is found by applying an aggregative decomposition procedure [Kiss & Martel, 1991] to each RPS and yields the sets:

$$D^{(r)}_{0[\nu_r]} = \left\{D^{(r)}_{1,[\eta_1(r)]},\ldots,D^{(r)}_{k,[\eta_k(r)]},\ldots,D^{(r)}_{\nu_r,[\eta_{\nu_r}(r)]}\right\};\ r = 1,\ldots,N,$$

which constitute the initial raw decompositions (or groupings) of sets $A^{(r)}_{[n_r]}$, where each of the r partitions has ν_r hierarchical levels $D^{(r)}_{k,[\eta_k(r)]}$ and each partition level contains $\eta_k(r) \ge 1$ elements (macroentities) originating from the r^{th} set $A^{(r)}_{[n_r]}$; $k = 1,\ldots,\nu_r$; $r = 1,\ldots,N$. Thus, we have

$$\sum_{j=1}^{n_r} h_j(r) = \sum_{k=1}^{\nu_r} \eta_k(r) = m\ ;\ \nu_r \le n_r \le m \quad\text{and}$$

$$\bigcup_{k=1}^{\nu_r} D^{(r)}_{k,[\eta_k(r)]} = \bigcup_{k=1}^{\nu_r}\left[\bigcup_{t=1}^{\eta_k(r)} D^{(r)}_{k,t}\right] = \bigcup_{j=1}^{n_r}\left[\bigcup_{s=1}^{h_j(r)} A^{(r)}_{j,s}\right] = \bigcup_{i=1}^{m} X_i;\ \forall r;\ r = 1,\ldots,N.$$

We shall assume that the indifference relation I is at once reflexive, symmetrical and transitive whereas the preference relation P is reflexive, asymmetrical and non-intransitive. The system cannot admit a preference relationship that violates the transitivity condition. In order to establish these relations, the N decision makers may or may not refer to the same set of criteria. These relations depend upon *their overall assessment* of the m options considered. Going successively from the set of options $X_{[m]}$ to the sets of macroentities $A^{(r)}_{[n_r]}$, then to the hierarchical decompositions (groupings) $D^{(r)}_{0[\nu_r]}$, in other terms, going from $X_{[m]}$ to $D^{(r)}_{0[\nu_r]}$, constitute "non-biunivoque" (non-mutually congruous) relations. We know that a given RSP may lead to many different hierarchical decompositions and, conversely, that several RSP may stem from a single hierarchical decomposition [Kiss & Martel, 1991]. It is this "non-biuni-vocité" (non-mutual congruity) and the multitudinous decomposition possibilities that we wish to exploit in our search for group consensus.

PROPOSITION 1

A consensus as to hierarchical decomposition D_c may be obtained by seeking the intersections of all possible hierarchical ranks for each element of the $X_{[m]}$ set by referring to the initial raw decompositions $D^{(r)}_{0[\nu_r]}$; $r = 1,\ldots,N$, and abiding by the constraints posed by the RIS and RPS established by each of N decision makers.

REACHING CONSENSUS AS TO HIERARCHICAL DECOMPOSITION

UNANIMOUS CONSENSUS WITHOUT CONCESSION

Let $\delta^{(r)}_{j,p}$ stand for the p^{th} possible hierarchical rank of the j^{th} element of $X_{[m]}$ and let $k_{r,j} \geq 1$ designate the number of such possible hierarchical ranks for the j^{th} element for the r^{th} initial decomposition $D^{(r)}_0$. $\mathcal{D}^{(r)}_{j[k_{r,j}]}$ shall denote the $k_{r,j}$ cardinal set containing all possible hierarchical ranks for the j^{th} element given the r^{th} RIS $\mathcal{R}(I)_{r,[b_r]}$ and the r^{th} RPS $R(P)_{r,[c_r]}$, where

$$\mathcal{D}^{(r)}_{j[k_{r,j}]} = \left\{ \bigcup_{p=1}^{k_{r,j}} \delta^{(r)}_{j,p} \right\} \; ; \; k_{r,j} \geq 1 \; ; \; j = 1,\ldots,m; \; r = 1,\ldots,N. \qquad (1)$$

The hierarchical consensus rank(s) for the j^{th} element, while taking simultaneous account of all initial decompositions $D^{(r)}_0$; $r = 1,\ldots,N$, is no other than a $\kappa_j > 0$ cardinal set comprising all intersections of the sets defined in (1). Thus,

$$\mathcal{D}_{j[\kappa_j]} = \bigcap_{r=1}^{N} \mathcal{D}^{(r)}_{j[k_{r,j}]} \; ; \; k_{r,j} \geq 1 \; ; \; j = 1,\ldots,m. \qquad (2)$$

An unanimous consensus as to hierarchical decomposition D_c is reached without concession if none of the sets defined in (2) are empty.

MAJORITY CONSENSUS WITHOUT CONCESSION

However, if $\exists j;\ 1 \le j \le m$ so that $\mathcal{D}_{j[\kappa_j]} = \{\emptyset\}$, then that (those) element(s) will be reexamined for $\tilde{N}$ (<N) participants; in other words, the search for intersections specified in (2) proceeds until $\mathcal{D}_{j[\kappa_j]} \ne \{\emptyset\};\ \forall j\ ;\ j = 1,\dots,m$. (see figure 1)

Before going any further, we must define a majority rule that states what number of decision makers ($\tilde{N}$) among the total (N) must share concurring hierarchical decompositions before consensus is declared; for instance $N/2 < \tilde{N} \le N$. For this kind of consensus we need only find the $\tilde{N}$ by $\tilde{N}$ intersections defined in (2) rather than N by N. Also, we should have

$$\mathcal{D}^{(\tau)}_{j[\kappa_j]} = \bigcap_{r=1}^{\tilde{N}} \mathcal{D}^{(r,\tau)}_{j[k_{r,j}]}\ ;\ k_{r,j} \ge 1\ ;\ j = 1,\dots,m\ ;\ \tilde{N} \le N\ ;\ \tau = 1,\dots,\binom{N}{\tilde{N}}. \tag{3}$$

A majority consensus as to the hierarchical decomposition exists without concession if there is at least one τ series where none of the sets defined in (3) is empty. We can now formulate the conditions required for a hiarerchical consensus decomposition:

$$\circ\quad \exists D_c \mid \mathcal{D}_{j[\kappa_j]} \ne \{\emptyset\} \Rightarrow \forall \kappa_j \in \mathbb{N}^+;\ j = 1,\dots,m. \quad \text{(unanimous)} \tag{4.a}$$

$$\circ\quad \exists D_c \left|\ \begin{array}{l} \exists\tau;\ \tau = 1,\dots,\binom{N}{\tilde{N}} \text{ so that} \\ \mathcal{D}^{(\tau)}_{j[\kappa_j]} \ne \{\emptyset\} \Rightarrow \forall \kappa_j \in \mathbb{N}^+;\ j = 1,\dots,m. \quad \text{(majority)} \end{array}\right. \tag{4.b}$$

PROPOSITION 2

Whether in reference to condition **(4.a)** *or* **(4.b)**, *the uniqueness or multiplicity of a decomposition* D_c *can be directly verified by calculating the* $\lambda \in \mathbb{N}^+$ *multiplicity coefficient, as follows:*

$$\lambda = \prod_{j=1}^{m} \kappa_j\ . \tag{5}$$

(D is unique if $\lambda = 1$ and multiple if $\lambda > 1$.)

CONSENSUS THROUGH CONCESSION

When conditions (4.a) or (4.b) are not met at the outset, we launch into an interactive man/machine procedure that allows decision makers to reach a final consensus.

∘ Let $X_{c[m_c]}$ denote an m_c cardinal set containing those elements of $X_{[m]}$ which do satisfy conditions (4.a) or (4.b) and let $X_{d[m_d]}$ denote the complementary set, in other words, the set of discordant elements, i.e., those elements whose appraisal by N decision makers differ. Of course $X_{c[m_c]} \subseteq X_{[m]}$, $X_{d[m_d]} \subseteq X_{[m]}$, $m_c \le m$, $m_d \le m$, $X_{c[m_c]} \cup X_{d[m_d]} = X_{[m]}$ and $m_c + m_d = m$.

∘ $\mu_c(j_c)$ and $\mu_d(j_d)$ refer to the two tables of index-pointers containing the identification indices that distinguish between those elements of $X_{[m]}$ contained in $X_{c[m_c]}$ and those contained in $X_{d[m_d]}$, $0 \le \mu_c(j_c) \le m$, $0 \le \mu_d(j_d) \le m$, $j_c = 0,\ldots,m_c$; $j_d = 0,\ldots,m_d$.

∘ Let $\mathcal{D}_{\mu_c(j_c)}[\kappa_{\mu_c(j_c)}] \neq \{\emptyset\}$ denote the set containing the hierarchical consensual ranks for the $\mu_c(j_c)^{th}$ element of $X_{[m]}$, i.e., for the j_c^{th} element of $X_{c[m_c]}$, onward $j_c > 0$ (refer to (2)).

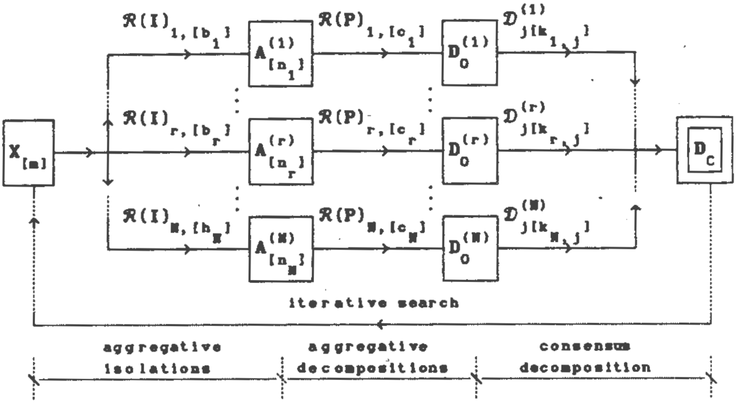

Figure 1: *Synopsis of consensus search*

For instance, by considering the ***quasi-median rank*** in each $\mathcal{D}_{\mu_c(j_c)}[\kappa_{\mu_c(j_c)}]$; $j_c = 1,\ldots,m_c$, we can generate a partial RIS $\tilde{\mathcal{R}}(I)$ and a partial RPS $\tilde{\mathcal{R}}(P)$, which serve to summarize the useful information gathered as of the current step of the procedure. These relational ranking systems point the ***way towards consensus***. In principle, we could generate not only a single RIS and RPS pair but a whole $\prod_{j_c=1}^{m_c} \kappa_{\mu_c(j_c)}$ cardinal set of partial RIS and RPS with each RIS-RPS pair containing m_c ranking relations. However, we shall focus on the median rank to ensure the interactive man/machine procedure does not bog down in complexity.

The decision makers are asked to restate their individual RIS and RPS, this time in reference to the elements contained in the $X_{d[m_d]}$ and $X_{c[m_c]}$ sets whilst abiding by the conditions posed by the RIS $\tilde{\mathcal{R}}(I)$ and RPS $\tilde{\mathcal{R}}(P)$ derived from the quasi-median elements of $X_{c[m]}$. In other words, throughout this iterativeand interactive search, aggregative isolations and aggregative decompositions are performed by virtue of the N (Ñ) RIS and RPS specified by the decision makers and in compliance with this structure:

$$\left.\begin{aligned} \mathcal{R}(I)_{r,t} &= \mathcal{R}\left\{I \,\middle|\, \left(\tilde{\mathcal{R}}(I) \cup \tilde{\mathcal{R}}(P)\right)_{t-1}\right\}_{r,t} \cup \tilde{\mathcal{R}}(I)_{t-1} \\ \mathcal{R}(P)_{r,t} &= \underbrace{\mathcal{R}\left\{P \,\middle|\, \left(\tilde{\mathcal{R}}(I) \cup \tilde{\mathcal{R}}(P)\right)_{t-1}\right\}_{r,t}}_{\text{first terms}} \cup \underbrace{\tilde{\mathcal{R}}(P)_{t-1}}_{\text{second terms}} \end{aligned}\right\} ;\ r = 1,\dots,N;\ 1 < t \le T, \quad (6)$$

where T stands for the number of iterations required to complete the search. Notice that in (6), as t increases, the importance of the first terms weakens while that of the second terms becomes more dominant, i.e., the cardinality m_c of X_c converges, though stochastically, towards that of $X_{[m]}$, m, while the cardinality m_d of X_d converges towards zero.

Should a rupture occur, i.e., a lack of monotony in the progression towards consensus, in other terms, if $X_{c[m_c(t)]} \subseteq X_{c[m_c(t-1)]}$, $m_c(t) \le m_c(t-1)$, we note the rankings (and their originators) causing the rupture, i.e., we identify those decision makers whose views differ. These dissenters are encouraged to make concessions along the lines of the preferences expressed by their colleagues in order to reduce the observed discrepancies (see figure 2).

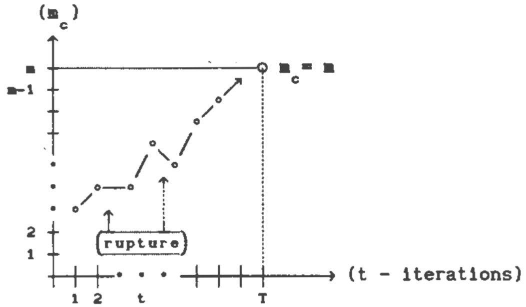

Figure 2: *Progressing towards consensus*

- Let $r_1 < N$ denote the number of dissenters.
- $\rho(j_\rho)$ refers to a table of index-pointers containing the identification indices for the r_1 dissenters; $\rho(j_\rho) \in [1,\dots,N]$; $j_\rho = 1,\dots,r_1$.
- $\phi(j_\phi)$ is the complement of $\rho(j_\rho)$ and refers to a table of index-

pointers containing the identification indices for the $N - r_1$ consensus-sharing participants; $\phi(j_\phi) \in [1,\dots,N]$; $j_\phi = 1,\dots,N - r_1$.

We can now formalize the structure of the r revised RIS and RPS as such:

$$\left.\begin{aligned}\mathcal{R}(I)_{\rho(j_\rho),t} &= \mathcal{R}\left[I\left|\left[\tilde{\mathcal{R}}(I) \cup \tilde{\mathcal{R}}(P)\right]_{t-1};\mathcal{R}(I)_{\phi(j_\phi),t}\right.\right)_{\rho(j_\rho),t} \cup \tilde{\mathcal{R}}(I)_{t-1}\\ \mathcal{R}(P)_{\rho(j_\rho),t} &= \mathcal{R}\left[P\left|\left[\tilde{\mathcal{R}}(I) \cup \tilde{\mathcal{R}}(P)\right]_{t-1};\mathcal{R}(P)_{\phi(j_\phi),t}\right.\right)_{\rho(j_\rho),t} \cup \tilde{\mathcal{R}}(P)_{t-1}\end{aligned}\right\}; \quad (7)$$

$$j_\rho = 1,\dots,r_1;\; j_\phi = 1,\dots,N - r_1;\; t = 2,\dots,T.$$

Of course, the structure of the N - r RIS and RPS remains the same as in (6).

BRIEF INTRODUCTION TO THE SYSTEM

To prevent a credibility gap from alienating the researcher (whose outlook is academic) from the manager (whose concerns are more practical), we have developed a group decision support system (GDSS) called COR-1 (Consensus Research, version 1), conceived along the mathematical conception presented herein.

The software is structured to facilitate consensus reaching among decision group participants placed in a cooperative decision setting and whose task is to rank a finite number of options (i.e., the elements contained in the $X_{[m]}$ set). COR-1's system architecture (intelligent junction of aggregation programs, access to expanded memory, and so on) conforms, as far as possible, to the SAA (System Application Architecture) standard. Its points of entry were defined in keeping with the CUA (Common User Access) standard, for instance, in assigning functions keys, posting menu entries and managing screen division. The result is a user-friendly system that is remarkably simple to use.

COR-1's system architecture is modular, a feature which favors smooth adjustement to various hardware configurations and allows future enhancements to be implemented without altering the software's basic structure (see figure 3). Thanks to the integration of various algorithmic-mathematical devices and the Monitoring Module's efficient tracking of the Math, Dialogue and Data Base Management Modules, the system is both speedy and efficient. To accelerate performance, we opted for a mixed programming technique, each time choosing the best-suited programming language: interface in MS QuickBASIC 4.5, heavy-duty internal computations in MS C, toroïdal scrolling windows were handled with MS Assembler. Given the appropriate hardware, COR-1's performance can be optimized by defining a 1 Mbyte RAM-DRIVE (virtual disk).

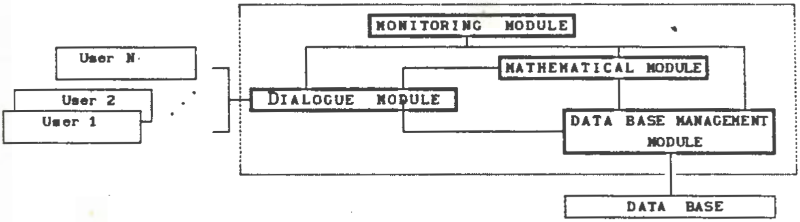

Figure 3: *COR-1's Modular System Architecture*

CONCLUSION

Experience seems to show that GDSS are liable to improve the quality of group decisions. Consequently, we have developped a support whose purpose is to facilitate consensus reaching among group participants.

In this paper we propose a novel approach to tackle the problems plaguing group decisions where several cooperative decision makers are asked to use pairwise comparisons in ranking a finite number of options. This approach draws from a previous result which posits that the options under consideration may be ranked in several different ways given a single relational preference system. Thus, the intersection of these rankings (one per decision maker) may lead to a consensus (see proposition 1). If the intersection is void, we may either content ourselves with a majority of decision makers, or request some decision makers (the dissenters) to make concessions, i.e. to revise their relational preference system.

The search process considers three possible situations, namely: unanimous consensus without concession, majority consensus without concession and consensus through concession. All three situations can be dealt with in hierarchical sequence or according to user preferences, thus attesting to the great flexibility of our software.

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