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SUPPORT SYSTEMS FOR DECISION AND NEGOTIATION PROCESSES

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PREFERENCES, AGREEMENT, CONSENSUS - MEASURING, AGGREGATION AND CONTROL -

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Abstract: The paper presents the model for analysis and the positive method of management of a session meant at attainment of consensus regarding preferences over a set of multiaspect options, these preferences being expressed in pairwise comparisons or orderings. The model and the method accept and process fuzzy preferences, while avoiding typical arbitraryness of numerous definitions related to majority of fuzzy approaches. The aggregation method presented previously in Owsiński and Zadrożny (1989,1990) is referred to and expanded for the case of clusterwise preference aggregation and consensus measurement.

Keywords: pairwise comparisons, fuzzy preferences, preference aggregation, clusterwise preference aggregation.

1. The Model: What is the Common Opinion?

1.1. Introductory remarks

Assume that m judges (experts, voters,...) give their preferences with regard to n items (options, policies, candidates, ...). These preferences are expressed as pairwise preference (precedence) coefficients d^k_{ij}, where $k \in \{1,...,m\}$ is the judge index, while i and j are item indices, $i,j \in \{1,...n\}$, with $d^k_{ij} \in [0,1]$, thus allowing for fuzzy pairwise preferences. Each judge provides, therefore, in a certain manner (see Owsiński, 1990b, for various ways of specifying preferences within such a setting), $0.5 \cdot n(n-1)$ preference coefficients ranging from 0 to 1. This set of preference coefficient values, called preference relation, is denoted $D^k = \{d^k_{ij}\}_{ij}$.

Assume further that judges meet at a session whose broadly conceived goal is elaboration of common opinion within the context outlined. The definition of common opinion is therefore crucial for the management of the session and for its outcome.

1.2. The soft extreme

At one extreme, admitting entirely fuzzy form of output, it would be possible to take the averages of d_{ij}^{k} over k and treat their set as the proper result of the session, e.g.

$$\bar{D} = \{\bar{d}_{ij}\}_{ij} \tag{1a}$$

where

$$\bar{d}_{ij} = \frac{1}{m} \sum_{k=1}^{m} d_{ij}^{k} \qquad \forall i,j \in I = \{1,\ldots,n\} \tag{1b}$$

Thus, the session would be (positively!) terminated after just one round of voting by the judges.

Two remarks on the "common opinion" form (1) are due:

* First, it allows for a wide disparity of preference relations, where even quite opposing ones are treated as composing the common opinion. This disparity could be measured with, for instance

$$\bar{M}^{A}(\{D^{k}\}) = \frac{2}{n(n-1)} \cdot \frac{1}{m} \sum_{k} \sum_{i<j} |d_{ij}^{k} - \hat{d}_{ij}| \tag{2a}$$

(see Owsiński, 1990b), or, in relative terms,

$$\bar{M}^{R}(\{D^{k}\}) = \frac{\max\limits_{\{D^{k}\}} \bar{M}^{A}(\{D^{k}\}) - \bar{M}^{A}(\{D^{k}\})}{\max\limits_{\{D^{k}\}} \bar{M}^{A}(\{D^{k}\})} \tag{2b}$$

so that the values of $\bar{M}^{R}(.)$ range from 0 to 1, reaching 0 for the maximum diversity of opinions and 1 when all the precedence coefficients given by all the judges are identical. For the sake of simplicity an approximate measure could be used, namely

$$\bar{M}_{a}^{R}(\{D^{k}\}) = 1 - 2 \cdot \bar{M}^{R}(\{D^{k}\}) \tag{2c}$$

resulting from the fact that

* for even m: $\max\limits_{\{D^{k}\}} \bar{M}^{R}(\{D^{k}\}) = \frac{1}{2}$

* for odd m: $\max\limits_{\{D^{k}\}} \bar{M}^{R}(\{D^{k}\}) = \frac{1}{2} \frac{(m-1)(m+1)}{m^{2}}$.

This question can also be treated via the fuzzy majority approach as e.g. introduced by Kacprzyk (1985) and then expanded by Kacprzyk and Fedrizzi (1986), whereby the vote is accepted under some mild conditions on the agreement ("a majority of judges sufficiently agree as the majority of options"). Such an approach, quite pragmatic and effective in session management, has two shortcomings: it assumes a number of arbitrary notions and definitions (majorities, sufficiencies etc.), and it gives no solution to the second problem, commented upon below.

* In case of wide disparity of relations (but not necessary only in this case) it is highly probable that the $\bar{d}_{ij}$ from (1b) will near 0.5, implying indifference with regard to the set of options under choice; not only is such a result insatisfactory from the point of view of the choice problem with which, as assumed, the judges are confronted, but it also conveys no information - in fact, conceals it - on the preferences of particular judges or groups of judges, which may be far from indifference.

Thus, the result of the kind of (1) may carry some cognitive or statistical, but certainly not much of practical informative or decision oriented value.

1.3. The hard extreme

At the other extreme, farthest, it seems, from (1), it might be required that the session end with a strictly "crisp" ordering (admitting ties where unresolvable) over which "complete consensus" is reached meaning, in fact, unanimosity. This, indeed, is a very tall order and it could easily happen that in spite of repetitive votings and discussions no such result (common opinion in this sense) is generated.

A step back from this extreme would consist in determination of a crisp ordering on the basis of relations D^k given by the judges, assuming that some natural agreement measure attains a predefined level. In this case the results, in terms of crisp preferences, forming an ordering, would be obtained after each voting as the solution $\hat{D} = \{\hat{d}_{ij}\}$ to the following problem:

$$\max_{D=\{d_{ij}\}} \{Q_1(D) = \sum_{i<j} (d_{ij}\bar{d}_{ij} + d_{ji}\bar{d}_{ji}) \tag{3a}$$

subject to constraints

$$d_{ij} \in \{0,1\} \quad \forall i,j \in I \tag{3b}$$

$$d_{ij} + d_{ji} = 1 \quad \forall i,j \in I \tag{3c}$$

$$d_{ij} + d_{jl} - d_{il} \leq 1 \quad \forall i,j,l \in I \tag{3d}$$

with $\bar{d}_{ij}$ defined as in (1). Thus, $\hat{D} = \arg\max_D Q_1(D)$. This LP problem, originally formulated by Marcotorchino and Michaud (1979) poses an additional difficulty, besides the potential substantial one, of the kind mentioned before. One is namely obliged to solve at each voting an LP problem which in view of (2d) may get very large.

Before defining the agreement measures for (3) we give the following properties:

* $\arg\min_{\bar{D}} \max_D Q_1(D,\bar{D}) = \{\frac{1}{2}\}_{i,j}$,
* $\min_{\bar{D}} \max_D Q_1(D,\bar{D}) = \frac{1}{4}\cdot n\cdot(n-1)$, and
* $\max_{\bar{D}} \max_D Q_1(D,\bar{D}) = \frac{1}{2}\cdot n\cdot(n-1)$,

the latter corresponding to any argument $\bar{D}$ (here made explicit) representing an ordering. We can now define the agreement measures:

$$M^1(\bar{D}) = \frac{Q_1^{opt}(\bar{D}) - \frac{1}{4}\cdot n\cdot(n-1)}{\frac{1}{4}\cdot n(n-1)} = \frac{Q_1^{opt}(\bar{D})}{\min_{\bar{D}} Q_1^{opt}(\bar{D})} - 1 \tag{4a}$$

with $M^1(\bar{D})\in[0,1]$, reaching 0 for all $\bar{d}_{ij}=\frac{1}{2}$, and 1 for $\bar{D}$ representing an ordering. Note, again, that $M^1(\bar{D})$ does not reflect that much the agreement <u>among</u> judges as the agreement with respect to <u>ordering</u> of options. Another agreement measure related to (3) is

$$M^2(\bar{D}) = \frac{\max_D Q_1(\bar{D},D) - \min_D Q_1(\bar{D},D)}{\max_D Q_1(\bar{D},D)} \tag{4b}$$

with, again, $M^2(\bar{D})\in[0,1]$, reaching 0 when $\max_D Q_1(\bar{D},D) = \min_D Q_1(\bar{D},D)$ i.e. when $\bar{D}=\{\frac{1}{2}\}_{ij}$, and 1 when $\bar{D}$ represents a crisp ordering. The latter results from the fact that (E_O denoting the space of orderings):

* $\min_D \{Q_1(\bar{D},D) \mid \bar{D}\in E_O\} = 0$.

Both measures (4) are easy to calculate since the minimum appearing in (4a) can be obtained from the D maximizing $Q_1(\bar{D},D)$ by reversing the ordering obtained thereby.

1.4. The way out or what do we really need?

The questions to be asked at this point are:

* what do we really want from the session in which the judges are involved for their effort to be effective?, and

* are we dealing with an irreconciliable alternative of the soft and hard extremes, with two options over which a compromise might be reached (say, a Pareto-like solution), or with two different views of the same situation?

Thus, if what we are after were just the opinions of the judges then there would be no sense in speaking of consensus. Hence we are after something more than opinions. Consensus, however, must be reached with respect to a definite outcome of the session. We can of course agree that this outcome could be any relation D as defined at the outset. Imagine, though, the task of presenting to any kind of a body, interested in the outcome of this session, the result in the form of a matrix $\{d_{ij}\}$. It is obvious that the simplest and most effective presentation would be given in the form of one or few alternative orderings with a comment concerning their validity as suggested by the structure of particular relations specified by the judges. This is insofar true as the ultimate goal of such a session ought to be a decision as to selection of options. Still, of course, one has to take into account the very fact that the judges have specified definite preference coefficient values. Thus, the information we need from a session (or from each of its voting rounds, in fact) is:

** the average relation (1) and the structure of the set of relations in terms of (2), and the groups of similar relations,

and

** the resulting ordering (see (3)), the nearest one to (2), and the structure of the set of relations with that respect, i.e. groups of relations which are similar in terms of the closest ordering.

Thus, we have also the answer to the two questions asked before: we are dealing with two different perspectives on the same set of data. Both of these perspectives must be taken into account simultaneously and although the bicriterial approaches can be devised for this setting, they have to be very carefully formulated, so as not to lose the sense of the two perspectives.

2. Clusterwise aggregation of preference relations

2.1. The two perspectives

Thus, we will be looking, after each voting round, for two aggregate solutions for the whole set of judges, namely: the average relation and the resulting ordering, i.e. the one which is the closest to the average relation. Besides this, we will be looking for the structures of the set of judges in the form of partitions into groups of relations (judges) similar in both these senses.

In the first case we will define distances between pairs of relations, $\delta(D^k,D^l) = \delta_{kl}$ and on the basis of these distances we will perform clustering of relations using the method described in Owsiński (1990a). Resulting will be a suboptimal partition of relations (judges). This partition is accompanied by the parameter values indicating the validity interval ("stability") of such a partition and the objective function values compared to those related to other partitions. Within each group an average can be calculated so as to show the "ideological cores" of these groups. Note that this problem (of simultaneous determination of clusters and their cores) is in general a very difficult one and finds only approximate solutions in which clustering is performed first and finding of the cores after, ultimately in an iterative manner: clustering → finding of cores → reallocation to cores → redefinition of cores → etc., although this does not ensure finding of an optimum solution, either. Optimum could be found through simple clustering only under definite assumptions concerning distance definitions $\delta(D^k,D^l)$ and $\delta_K(D^k,D_q)$, where D_q is a core relation of cluster q. Since the clustering method to be used was described in detail elsewhere, we will only give here just a few comments.

In the second perspective a simile of the programming problem (3) has to be formulated taking into account simultaneous optimum partition into clusters of relations (judges) with regard to the closest ordering. The formulation of the problem so as to avoid trivial solutions is by no means an easy task as we will see. Not only, though, will it be presented here, but also a very simple method for attaining a suboptimum solution will be given in Section

2.3. of the paper.

2.2. Clustering of relations around averages - the soft extreme

In further course of this section we will be assuming that the average relation is given by (1) both for the whole set of judges of for their subsets (clusters). This section is based upon Owsiński (1984, 1990b, 1991).

Denote the set of judge indices by M. We are looking for the partition of M which would reflect in the optimum manner the differences and similarities between the D^k. For this purpose we take the objective function

$$Q_S^D(P) = \bar{Q}_D(P) + Q_S(P) \rightarrow \max_P \qquad (5)$$

or, in the algorithmic form,

$$Q_S^D(P,r) = r\bar{Q}_D(P) + (1-r)Q_S(P) \qquad (6)$$

in which $r \in [0,1]$, P is a partition of M, $\bar{Q}_D(P)$ reflects the distances between the clusters forming partition and $Q_S(P)$ reflects the proximities (similarities) of relations forming clusters in the partition. Owsiński (1991a) gives conditions for (5) to be suboptimizable through a simple progressive merger procedure. The procedure starts with the algorithmic coefficient $r=1$, to which $P^{opt}(r=1) \equiv M$ corresponds, and then proceeds through mergers of selected pairs of previously determined clusters for successively decreasing values of r. These values of r result from the condition

$$\max_{A_q, A_{q'} \in P^t} \{r: Q_S^D(P^t,r) = Q_S^D(P_H^t(q,q'),r)\} \qquad (7)$$

where A_q, $A_{q'}$ are clusters forming a partition P^t, obtained in the preceding step, and $P_H^t(q,q')$ is a partition formed out of P^t by aggregating clusters indexed q and q'. Under certain simple conditions the sequence of $\{r^t\}$, where t is the merger number, is nonincreasing.

The result is the partition of M into p^* non-overlapping suboptimal clusters A_q^* of relations D^k, obtained for the lowest value of $r^t \geq 0.5$ together with the corresponding values of objective function elements, as in (5). Additionally, the values of $\{r^t\}$ at around the suboptimal solution define the validity intervals of consecutive partitions close to the subotimal one, giving an additional important information. For each A_q^* appropriate averages can be calculated, to be treated as representative relations ("ideological cores") for clusters of relations (judges).

Thus, while the method given in Owsiński (1990a,1991a) guarantees easy and effective solution of the clustering problem, it heavily relies upon the definition of distances (and therefore also proximities) to be used as the basis for clustering. The question of distances is taken up in Owsiński (1991b).

Note, however, that when the definitions of distances $\delta(D^k,D^l)$ and $\delta_K(D^k,D_q)$ are taken for purposes of concrete formulation of (5) and (6) in such a way as to make them correspond to formulation (1) (e.g. Euclidean distances), then the suboptimization procedure described in this section applies in a similar manner to the task of simultaneous determination of relation clusters A_q and their cores D_q. Thus, we can be sure we are not making a too big an error in clustering of judges according to their preference relations and in determination of "ideological cores" of the clusters obtained.

2.3. Clustering of relations around orderings - the hard extreme

The problem can be verbally stated as follows:

* to find the (crisp) ordering (allowing for ties where unresolvable) which is the closest to relations given by the judges, and, simultaneously

* to determine the groups of relations (judges) which are possibly close to each other in terms of indication of the same, or similar crisp order, with the differences among groups being, simultaneously, possibly big, the groups being determined together with the orderings corresponding to them.

We know already that the first part can be given the proper answer by solving the directly the mathematical programming problem (3) or by application of the simple suboptimization procedure given, for instance, in Owsiński and Zadrożny (1986). Formulated along the same lines, though, the objective function for the second part of the problem, ensuring avoidance of such trivial solutions as every judge forming a separate cluster represented by the closest order, would have a rather complex form of

$$Q_S^D(P,\hat{D}) = r\bar{Q}_D(P,\hat{D}) + (1-r)Q_S(P,\hat{D}) \rightarrow \max_{\{A_q,\hat{D}_q\}} \quad (8)$$

where

$$\bar{Q}_D(P,\hat{D}) = \frac{2}{p(p-1)} \sum_{q=1}^{p-1} \sum_{q'=q+1}^{p} \sum_{k\in A_q} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (\hat{d}_{ji}^{q'} d_{ij}^{k} + \hat{d}_{ij}^{q'} d_{ji}^{k}) \quad (9)$$

and

$$Q_S(P,\hat{D}) = \frac{1}{p} \sum_{q=1}^{p} \sum_{k\in A_q} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (\hat{d}_{ij}^{q} d_{ij}^{k} + \hat{d}_{ji}^{q} d_{ji}^{k}) \quad (10)$$

in the analogy to (5) and (6), where $\hat{D} = \{\hat{D}_1,\ldots,\hat{D}_q,\ldots,\hat{D}_p\}$, p being the number of clusters of partition P.

Note, though, that (8), together with (9) and (10), need not comply with the conditions to which (5) and (6) was subject in order to ensure existence of a simple suboptimization algorithm. The analogy here relates to global optimality of partitions implied by both objective functions only and not to the algorithm resulting. In fact, the algorithm proposed is of entirely different character.

There are in general as yet no effective algorithms for maximization of the objective function (8), neither in this form nor in the form similar to (3). It appears, though, that a relatively simple and effective algorithm for suboptimization of (8) can be based upon the following general procedure:

* solve (3) for each of D^k separately, thus obtaining the set of m closest orderings $\hat{D}^k$;

* assign relations D^k to clusters defined by identical orderings $\hat{D}^k$, i.e. $A_q^0 = \{k\colon \hat{D}^k = \hat{D}_q^0\}$ and there are as many A_q^0 as there are different orderings $\hat{D}^k$;

* the partition $P^0 = \{A_q^0\}_q$ together with the set of orderings $\{\hat{D}_q^0\}_q$ maximize (8) for r=0;

* for increasing r the condition analogous to (7) is checked for a class of operations on the existing clusters and corresponding orderings and whenever an improvement in the value of the objective function is detected, the operation is performed;

* for $\min_t \{r^t \geq 0.5\}$ the suboptimal solution is found and the improving operations can be stopped.

The nature of the operations performed will depend upon the dimensions of the problem, defined by values of m, n and p^t. Dependence upon the last parameter indicates that the nature of the operations would be changed dynamically in the course of the procedure. Since it is in general envisaged that the number of clusters will grow in the course of the procedure, it is possible that certain, more time consuming operations, will be excluded from the proceuure in its course.

Note that definition of $\hat{D}^k$ through the suboptimizing procedure described, for instance, in Owsiński and Zadrożny (1986), allows for definition of a specific kind of distance between the preference relations D^k. For every D^k we obtain a sequence of orderings, denote it $\{\hat{D}^k_t\}_t$, corresponding to consecutive values of r^t, from $r^0=1$ down to the last value of $r^t \geq 0$. These values of r^t have nothing to do with the ones from the procedure analyzed in this section, and they result from the procedure of Owsiński and Zadrożny (1986), through which $\hat{D}^k$ are obtained. Thus, we can define the distance between D^k as the distance between $\{\hat{D}^k_t\}_t$, by e.g. taking a definite number of orderings from that sequence which are the closest to the suboptimal one, and by applying appropriate weight derived from the corresponding values of r^t. Thereby potential operations on $\{A^0_q\}$ and the clusters from the following partitions, envisaged in the procedure, can be made simpler.

3. What the session manager gets

On the basis of previous, more general considerations, we will now list the kind of information that are provided during the session to the session manager. The list is limited to the information which is in a way obligatory and which is from the point of view of the two perspectives necessary for the conduct of the session and for steering in the direction of consensus, as defined before, and for directing discussions.

At the beginning of the session judges are asked to define distances between relations to be used by the clustering procedures. The potential definitions are presented in Owsiński (1991b).

The information that the session manager obtains after each voting by the judges (each specification of preference relations or of changes in these relations) is as follows:

* Round number
* Totals:
  ** Average relation and:
    *** difference (distance) with respect to the previous round average;
    *** agreement measure;
    *** difference with respect to the previous round agreement measure;
  ** Central ordering and:
    *** difference with respect to the previous round central ordering;
    *** agreement measure;
    *** difference with respect to the previous round agreement measure;
  ** Distance between average relation and central ordering, and
    *** difference with respect to the previous round distance between average relation and central ordering;
* Clusters:
  ** Clusters around averages:
    *** number of clusters;
    *** list of cluster averages, cluster cardinalities and cluster agreement measures;
    *** differences with respect to previous round concerning: number of clusters, cluster agreement measures (average), cluster averages and cluster cardinalities;
  ** Clusters around orderings:
    *** number of clusters;
    *** list of cluster-proper orderings, cluster cardinalities and cluster agreement measures;
    *** differences with respect to previous round concerning: number of clusters, cluster agreement measures (average), cluster-proper orderings and cluster cardinalities.

Although this seems to be quite a lot of information for just one voting round of a session in which a limited number of participants are present, it must be borne in mind that a session manager can hardly grasp the meaning of even a limited number of matrices of preference coefficients with numbers ranging from 0 to 1. The purpose of the information provided is to summarize and not multiply the numbers available as the output of a voting round. Thus, information outlined would be drastically limited if, for instance, the number of clusters approached the number of relations (judges). On the basis of information listed a session manager should be capable of steering the discussion through his knowledge of judges and options crucial for the attainment of consensus. Ultimately, he/she would state that consensus would have been reached or that a stalemate ensued.

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