



**POLISH ACADEMY OF SCIENCES**  
**Systems Research Institute**

**ECO – INFO**  
**AND SYSTEMS RESEARCH**

**Editors:**

**Jan Studzinski**  
**Olgierd Hryniewicz**





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# **ECO – INFO AND SYSTEMS RESEARCH**

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The purpose of this publication is to present the information technology (IT) tools and techniques that have been developed at the Systems Research Institute of Polish Academy of Sciences in Warsaw (IBS PAN) and at the German Institute for Landscape System Analysis in Müncheberg (ZALF) in the area of applications of informatics in environmental engineering and environment protection. The papers published in this book were presented in the form of extended summaries during a special workshop organized by IBS PAN in Szczecin in September 2006 together with the conference BOS'2006 organized jointly by IBS PAN, University of Szczecin, and the Polish Society of Operational and Systems Research. In the papers the problems of mathematical modeling, approximation and visualization of environmental variables are described. Moreover, some questions concerning the environmental economy are also presented.

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CHAPTER 2

# Mathematical Modeling







## INTEGRATED AIR QUALITY MODELS - ASSESSMENT OF ENVIRONMENTAL IMPACT OF EMISSION SOURCES

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**Abstract:** *The evaluation and comparison of environmental impact of emission sources is an important task in integrated environmental models and decision support systems. The problem is especially difficult in case of a complex, multi-source emission field. The approach discussed in the paper is based on the forecasts of a dynamic model of air pollution dispersion. The aim is to get a quantitative evaluation of the contribution of the selected sources, according to the predefined, environmental cost function. The approach utilizes the optimal control technique for distributed parameter systems. The adjoint equation, related to the main transport equation of the forecasting model, is applied to calculate the sensitivity of the cost function to the emission intensity of the specified sources. An example implementation of a regional scale, multi-layer dynamic model of SO<sub>2</sub> transport is discussed as the main forecasting tool. The test computations have been performed for a set of the major power plants in a selected industrial region of Poland.*

**Keywords:** Air quality, mathematical modeling, optimization.

### 1. Introduction

The natural application of environmental models is forecasting of dispersion of pollutants, analysis of ecological results of some specific meteorological conditions or evaluation of environmental influence of emission sources. The integrated systems, being recently developed, try to combine a classical environmental model of pollutants transport with some economic, technological, social or medical constraints and standards (Juda-Rezler, 2004). Such a system, besides the natural scenario analysis, gives the possibility to formulate and solve optimization problems, which take into account certain specific environmental standards. Some optimization methods and recently developed environmental models (Haurie et al., 2004; Holnicki,

2006) give the possibility of implementation of complex air pollution control strategies.

In most of deterministic models of air quality, the process of pollution transport is considered as distributed parameter system and is described by the set of advection-diffusion equations, along with the respective boundary and initial conditions. The dimension of such a system of equations reflects the number of polluting species considered in the model as well as the vertical resolution of the domain. It is assumed in the sequel that the pollution transport process can be considered as distributed parameter system, governed by the respective set of transport equations. Implementation discussed below is sulfur-oriented, but the approach presented can be applied in a more general class of the forecasting models. The governing dynamic model generates forecasts of air pollution related to a specified, complex emission field. Examples of applications can be found in the cited literature, and relate both to long-term scenario analysis tasks as well as to on-line emission control problems.

For example, the long-term air pollution forecasting model was applied to evaluate the possible environmental consequences of the variant, emission abatement strategies applied to the energy sector in Poland (Ciechanowicz et al., 1996). The problem of the regional-scale strategy for emission abatement in a set of the major power plants was also discussed in (Holnicki and Kafuszko, 2004). The solution of the last task is searched by the optimal selection of the desulphurization technologies for emission sources considered. From the viewpoint of the mathematical formulation, the above tasks are stated as static optimization problems.

Another class of air quality management problems relates to the regional scale, real-time emission control. This is one of the possible applications of environmental models for supporting decisions related to environment protection and sustainable development (Holnicki, 2006). The approach is based on utilizing a dynamic forecasting model of air pollution dispersion and some general optimization methods, applied in formulation the basic emission control problem as well as in the design of the optimization algorithm. Dynamic properties of air pollution forecasting model constitute the key factor in constructing the real-time emission control systems. In such a case, the optimal control problem is formulated as on-line minimization of an environmental cost function, by the respective modification of emission level in a set of the controlled sources, according to the changing meteorological conditions.

The algorithms that solve such problems usually need certain procedure to evaluate the contribution of the controlled emission sources in the

final environmental damage. The paper addresses some related problems. We consider here a class of air pollution dispersion models, which can be applied as tools to support decisions in environment quality control, by evaluating contribution of each source in the resulting pollution field. Ultimately, those data can be utilized in decision making, concerning air quality control.

For simplicity of the presentation, the implementation discussed below concentrates on sulfur-type pollution, but the approach can be applied in a more general class of the forecasting models. The governing model generates short-term forecasts of air pollution related to a specified, complex emission field. Computation of the transport of sulfur pollution is carried out by Lagrangian type, multi-layer trajectory model (Holnicki, 1996; Holnicki, et al., 2000). The mass balance for the pollutants is calculated for air parcels following the wind trajectories. The model takes into account two basic polluting components: primary  $SO_2$  and secondary –  $SO_4^-$ . Transport equations include chemical transformations  $SO_2 \rightarrow SO_4^-$ , dry deposition, scavenging by precipitation. The governing equation, related to one polluting component, averaged over one vertical layer, has the following, general form

$$\frac{\partial c}{\partial t} + \bar{v} \nabla c - K_h \Delta c + \gamma c = Q \tag{1}$$

along with the boundary conditions

$$c = c_b \quad \text{on} \quad S^- = \{\partial \Omega \times (0, T) \mid \bar{v} \cdot \bar{n} \leq 0\}; \tag{1a}$$

$$c = K_h \frac{\partial c}{\partial \bar{n}} \quad \text{on} \quad S^+ = \{\partial \Omega \times (0, T) \mid \bar{v} \cdot \bar{n} > 0\}; \tag{1b}$$

and the initial condition

$$c(0) = c_0 \quad \text{in} \quad \Omega. \tag{1c}$$

Here we denote:

$\Omega$  – domain considered, with the boundary  $\partial \Omega = S^+ \cup S^-$ ,

$(0, T)$  – time interval of the forecast,

$c$  – pollution concentration,

$\bar{u}$  – wind velocity vector,

- $\vec{n}$  – normal outward vector of the domain boundary  $\partial \Omega$ ,  
 $K_h$  – horizontal diffusion coefficient,  
 $\gamma$  – pollution reduction coefficient (due to deposition and chemical transformation),  
 $Q$  – total emission field.

The emission field on the right-hand side of (1) is a sum of the uncontrolled, background emission, and the contribution of the controlled (or modernized) emission sources. It can be expressed as follows:

$$Q(x, y, t) = Q_0 + \sum_{i=1}^N \chi_i(x, y) \cdot q_i(t), \quad (2)$$

where

- $\vec{q} = [q_1, q_2, \dots, q_N]$  – emission vector of the controlled sources,  
 $q_i(t)$  – emission intensity of the controlled,  $i$ -th source,  
 $\chi_i(x, y)$  – characteristic function of the  $i$ -th source location,  
 $Q_0(x, y, t)$  – background (uncontrolled) emission field.

Transport equation of the form (1) can be base of air quality forecasting model (Holnicki et al., 2000; 2001), which generates maps of air pollution for a given time interval  $(0, T)$ . To implement a strategy of air quality control, the air quality damage (air quality cost) function must be defined. The environmental cost function, which depends on the concentration of pollutant and on the emission intensity of the controlled sources, can be considered in following, general form:

$$J(c((\vec{q}))) = \alpha_1 \int_0^T \int_{\Omega} \varphi_1(c(\vec{q})) d\Omega dt + \alpha_2 \int_0^T \varphi_2(\vec{q}) dt. \quad (3)$$

This measure is a weighted sum of two components, depending on the concentration of pollutant and on the emission intensity of the source. We assume in the sequel the respective differentiability of sub-integral functions,  $\varphi_1$  and  $\varphi_2$ . Time interval  $(0, T)$  depends on the temporal scale of analysis. It can be, for example, one year for optimal strategy of emission reduction or 6 hours for the real-time emission control.

In any optimization algorithm based on such an index it is necessary to calculate sensitivity of this index to emission of individual sources.

A direct method of evaluation of this impact can utilize the consecutive reduction of emission level of the sources under question - the impact is then represented by the related change of environmental cost index (3). In this approach, however, the main transport equation must be consecutively solved many times, for all the sources considered. This means that in case of emission control, the most time-consuming step of the analysis, has to be repeatedly performed many times.

Below another approach is presented. The first component of environmental cost function (3) depends indirectly on the emission intensity of the controlled sources, and is related to emission via the transport equation (1). This fact can be formally expressed by the gradient of this index. Basing on expression (3), the components of the gradient of this functional with respect to emission intensities is as follows:

$$\frac{\partial J}{\partial q_i}(\bar{q}) = \alpha_1 \int_0^T \int_{\Omega} \frac{\partial \varphi_1}{\partial c} \cdot \frac{\partial c}{\partial q_i}(\bar{q}) d\Omega dt + \alpha_2 \int_0^T \frac{\partial \varphi_2}{\partial q_i}(\bar{q}) dt \quad (4)$$

for  $(i = 1, \dots, N)$ .

To compute the components of the gradient function (4), the derivatives  $\partial c / \partial q_i$  are required, which can not be directly calculated. The applied procedure, based on the optimal control theory technique, allows us to calculate components of the gradient function in one simulation run of the transport equation. It is known (Lions, 1971; Martchuk, 1995) that the minimum of the index (3) is uniquely characterized as the solution of the state equation (1) and the solution ( $p^*$ ) of the following adjoint equation

$$-\frac{\partial p^*}{\partial t} - \bar{v} \nabla p^* - K_h \Delta p^* + \gamma p^* = \frac{\partial \varphi_1}{\partial c}(c) \quad \text{in } (0, T), \quad (5)$$

along with the boundary conditions

$$p^* = 0 \quad \text{on } S^-, \quad (5a)$$

$$K_h \frac{\partial p^*}{\partial \bar{n}} + (\bar{v} \cdot \bar{n}) p^* = 0 \quad \text{on } S^+, \quad (5b)$$

and the final condition (for the end of the time interval)

$$p^*(T) = 0 \quad \text{in } \Omega. \quad (5c)$$

It must be noted that equation (5) – similarly as for the state equation (1) – is of parabolic type, but it is solved for the negative time and the reversed direction of wind. Moreover, the right-hand side is the derivative of the subintegral function in environmental index (3). It can be shown (see Lions, 1971) that – due to the specific form of the boundary conditions in (1) and (5) – the solution of the adjoint equation allows us to calculate effectively the components of the gradient function (4). They have the following form

$$\frac{\partial J}{\partial q_i}(\bar{q}) = \alpha_1 \int_0^T \int_{\Omega} \chi_i(x, y) p^*(x, y, t) \cdot \frac{\partial \varphi_1}{\partial c} d\Omega dt + \alpha_2 \int_0^T \frac{\partial \varphi_2}{\partial q_i}(\bar{q}) dt \quad (6)$$

$$\text{for } (i = 1, \dots, N).$$

Components of the gradient function (6) allow us to evaluate quantitatively the contribution of emission sources in environment deterioration, which is measured in the sense of the objective function (3). Thus, to calculate the impact of the specified emission sources, the following computational steps have to be successively performed:

- solving the state equation – problem (1),
- solving the adjoint equation – problem (5), for the reversed time and the wind direction,
- substituting the adjoint variable ( $p^*$ ) to (6), to calculate the components of the gradient (4).

To get the final solution, the transport and the adjoint equations must be solved only once. The method presented above has been applied for the real-data case study concerning the industrial region of Upper Silesia. The objective function utilized in this test is a special case of the general form (3) discussed above.

## 2. Test computations. Evaluation of accuracy of the method

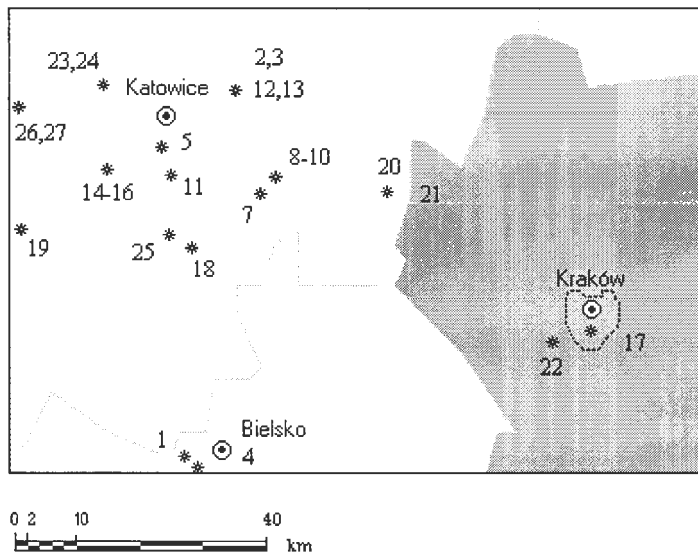
The test calculations have been performed for the set of 27 major power plants in the region of Upper Silesia and Kraków in Poland. The aim of the computational experiment was: i) to evaluate and compare the environmental impact of each source by the adjoint variable method, discussed in the previous section, ii) to examine accuracy of this technique, by comparing computational results with some reference data. Locations of the sources are indicated in Figure 1. For computational purposes, the rectangle domain

110 km × 74 km is discretized by the uniform grid, with the space step  $h = 2$  km. Grid coordinates and the main parameters of the controlled emission sources are presented in Table 1.

The cost functional used in the test is considered as a special case of (3), for  $\alpha_1 = 1/2$  and  $\alpha_2 = 0$ , in the following form

$$J(c(\bar{q})) = \frac{1}{2} \int_0^T \int_{\Omega} w [\max(0, c(\bar{q}) - c_{ad})]^2 d\Omega dt. \quad (7)$$

The area sensitivity function satisfies the inequality  $0 \leq w(x, y) \leq 1$ , and  $c_{ad}$  is a constant, admissible level of  $SO_2$  concentration.



**Figure 1.** Computational domain and location of the emission sources.

The area sensitivity function  $w(x, y)$  introduced in relation (7), was set to 1 for the Kraków urban area and 0 – outside this region (compare the dashed-line indicated area in Figure 1). Thus, the environmental impact of the sources under consideration was computed in the sense of deterioration of this domain and the area weight function is as follows:

$$w(x, y) = \begin{cases} 1 & (x, y) \in \text{Kraów area} \\ 0 & \text{outside Kraków} \end{cases} \quad (8)$$

**Table 1.** Emission parameters of the controlled sources.

No	Source	Coordinates	Stack [m]	Emission [kg/h]	
				Winter	Summer
1	Bielsko Biala	(14,20)	160	426.91	256.15
2	Będzin A	(18,31)	95	94.89	63.25
3	Będzin B	(18,31)	135	132.82	31.63
4	Bielsko-Kom.	(15,10)	250	426.9	189.74
5	Chorzów	(12,27)	100	363.66	180.25
6	Halemba	(8,25)	110	569.24	379.48
7	Jaworzno I	(20,23)	152	284.61	158.12
8	JaworznoII A	(21,24)	100	573.60	379.48
9	JaworznoII B	(21,24)	120	664.08	426.91
10	Jaworzno III	(15,10)	300	6324.60	4743.4
11	Katowice	(18,31)	95	1106.81	790.58
12	Łagisza A	(18,31)	160	948.69	695.71
13	Łagisza B	(18,31)	200	1359.79	1011.9
14	Łaziska I	(8,20)	200	1660.21	1185.8
15	Łaziska II	(8,20)	160	758.95	505.97
16	Łaziska III	(8,20)	100	727.95	505.97
17	Łęg	(46,12)	260	1106.81	790.58
18	Miechowice	(14,17)	68	161.28	117.01
19	Rybnik	(1,20)	300	4711.83	3510.1
20	Siersza A	(30,23)	150	1929.00	1423.0
21	Siersza B	(30,23)	260	2055.49	1739.2
22	Skawina	(43,11)	120	1992.25	1296.5
23	Szombierki A	(9,31)	110	164.44	113.84
24	Szombierki B	(9,31)	120	170.76	110.68
25	Tychy	(13,19)	120	110.68	177.09
26	Zabrze A	(2,29)	60	205.55	158.12
27	Zabrze B	(2,29)	120	221.36	145.47

In the case of the objective function (7), the respective components of the gradient function have the following form

$$\frac{\partial J}{\partial q_i}(\vec{q}) = \int_0^T \int_{\Omega} \chi_i(x, y) p^*(x, y, t) d\Omega dt \quad \text{for } (i=1, \dots, N), \quad (9)$$



and the adjoint variable is the solution of equation

$$-\frac{\partial p^*}{\partial t} - \bar{u} \nabla p^* - K_h \Delta p^* + \gamma p^* = w \cdot \max[0, c(\bar{q}) - c_{ad}] \quad \text{in } (0, T), \quad (10)$$

along with the boundary and initial conditions (5a-c).

The direct evaluation of the contribution of a source can be obtained by *reverse method*, based on calculation of the index (7) for switched off or reduced the emission of the source under question. In any optimization procedure, such an approach requires solving many times (repeatedly) the state equation (the most time-consuming part of computation) in any iteration. In some applications, e.g. real-time emission control, the computing time is a critical factor.

The reverse method was here used to get the reference influence of the emission sources, and then to estimate accuracy of the results obtained by the adjoint variable algorithm. The relative contribution of the specific source has been directly obtained as the solution to problem (1), for the emission of this source reduced by 50 %, with respect to the nominal intensity. This procedure was repeated in turn for all the sources, giving the reference contribution of each emission source.

Test computations were performed for a selected, representative year (1996). The meteorological conditions are characterized by the respective sequence of 12-h sets of the input data. One-year interval was split down into four 3-month periods, and calculations were performed for 4 quarters, respectively. Forecasting part of computations was based on the regional scale model (Holnicki et al., 2000). Approximation of equations (1) and (4) is based on the semi-Lagrangian scheme, which is combination of the linear finite element method and the method of characteristics (Holnicki, 1996).

The direct approach was utilized for evaluation of accuracy of the discussed method, based on application of the adjoint equation (5). For the consecutive 3-months periods of the selected year the following calculations were performed:

- averaged distribution of  $SO_2$  concentration for nominal emissions,
- nominal value of the index  $J^o$  according to (7),
- averaged distribution of  $SO_2$  for emission intensity reduced by 50% for the controlled sources,
- reduced value of the index,  $J^i$  related to a given source,
- relative contribution of the controlled sources (reference value), according to the formula

$$R_i = \frac{J^o - J^i}{J^o} . \quad (11)$$

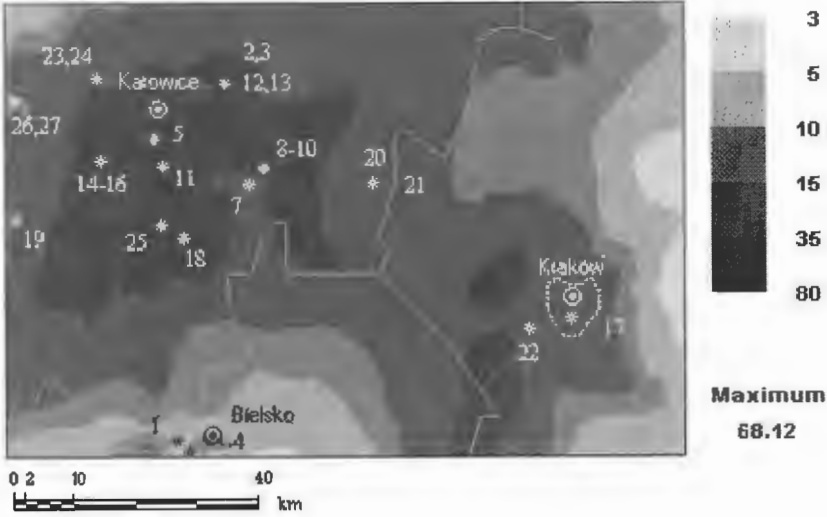
Selected numerical results of this evaluation are presented in Table 2. They show the relative contribution of the considered emission sources in the sense of the air quality index (7). For each quarter of the year considered, the relative contribution of the source is calculated as the respective component of the gradient function of the form (10). Then it is compared with the reference value, calculated according to relation (11).

**Table 2.** Assessment of relative contribution of emission sources.

No.	term 1		term 2		term 3		term 4	
	calc.	refer.	calc.	refer.	calc.	refer.	calc.	refer.
1	1.22	0.13	0.20	0.35	0.43	0.50	0.53	0.11
2	0.08	0.16	0.05	0.26	0.07	0.12	0.19	0.23
3	0.11	0.20	0.03	0.14	0.04	0.08	0.28	0.21
4	2.07	0.01	0.43	0.10	0.40	0.05	1.62	0.02
5	0.30	0.22	0.24	0.63	0.22	0.44	0.21	0.28
6	0.54	0.26	0.48	1.06	0.37	0.33	0.26	0.32
7	0.54	0.48	0.43	0.49	0.22	0.77	0.75	0.24
8	0.81	1.00	1.04	0.31	0.61	0.80	0.83	1.10
9	1.16	0.44	1.36	0.48	1.13	0.80	1.07	0.88
10	31.65	9.59	10.71	6.44	33.82	13.20	23.26	4.44
11	2.39	0.78	1.14	1.50	4.01	1.26	1.57	0.50
12	0.63	0.89	0.55	1.94	1.32	1.13	1.79	1.17
13	3.17	1.33	0.77	2.42	2.64	0.81	2.33	1.59
14	1.79	0.17	1.21	2.06	0.89	0.25	1.15	0.31
15	0.81	0.11	1.09	0.87	0.47	0.14	0.47	0.16
16	0.77	0.24	0.71	1.27	0.43	0.40	0.46	0.31
17	7.67	0.08	9.46	1.08	12.08	0.62	0.05	0.02
18	0.20	0.05	0.23	0.28	0.11	0.05	0.14	0.06
19	3.87	0.50	6.04	3.50	1.39	1.05	2.21	1.03
20	34.41	6.41	12.70	16.23	20.28	13.72	22.08	5.96
21	40.44	4.14	18.80	15.08	21.00	11.43	25.09	4.43
22	204.49	59.95	77.31	67.59	156.53	72.17	230.75	62.49
23	0.08	0.05	0.12	0.24	0.12	0.12	0.10	0.06
24	0.09	0.04	0.10	0.23	0.09	0.09	0.11	0.06
25	0.29	0.05	0.33	0.44	0.18	0.18	0.18	0.08
26	0.15	0.04	0.12	0.25	0.11	0.11	0.06	0.06
27	0.16	0.04	0.12	0.24	0.08	0.08	0.06	0.07

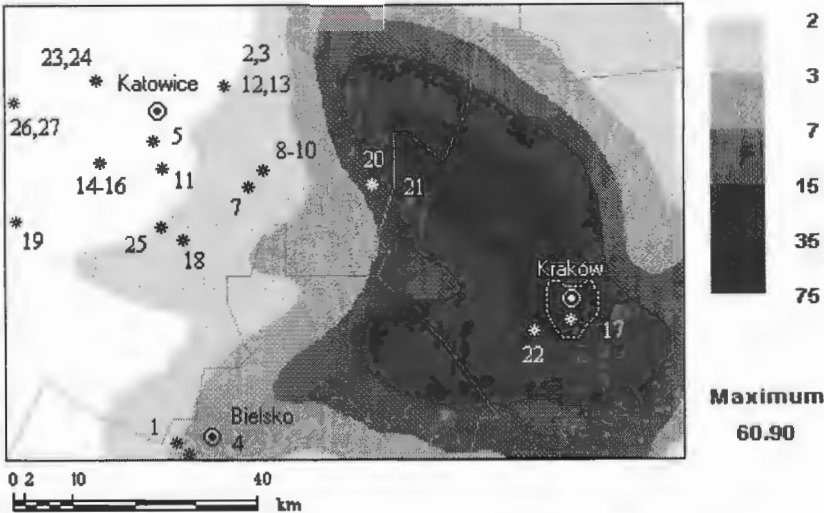
**SO<sub>2</sub> concentration forecast**

**Layer 1**

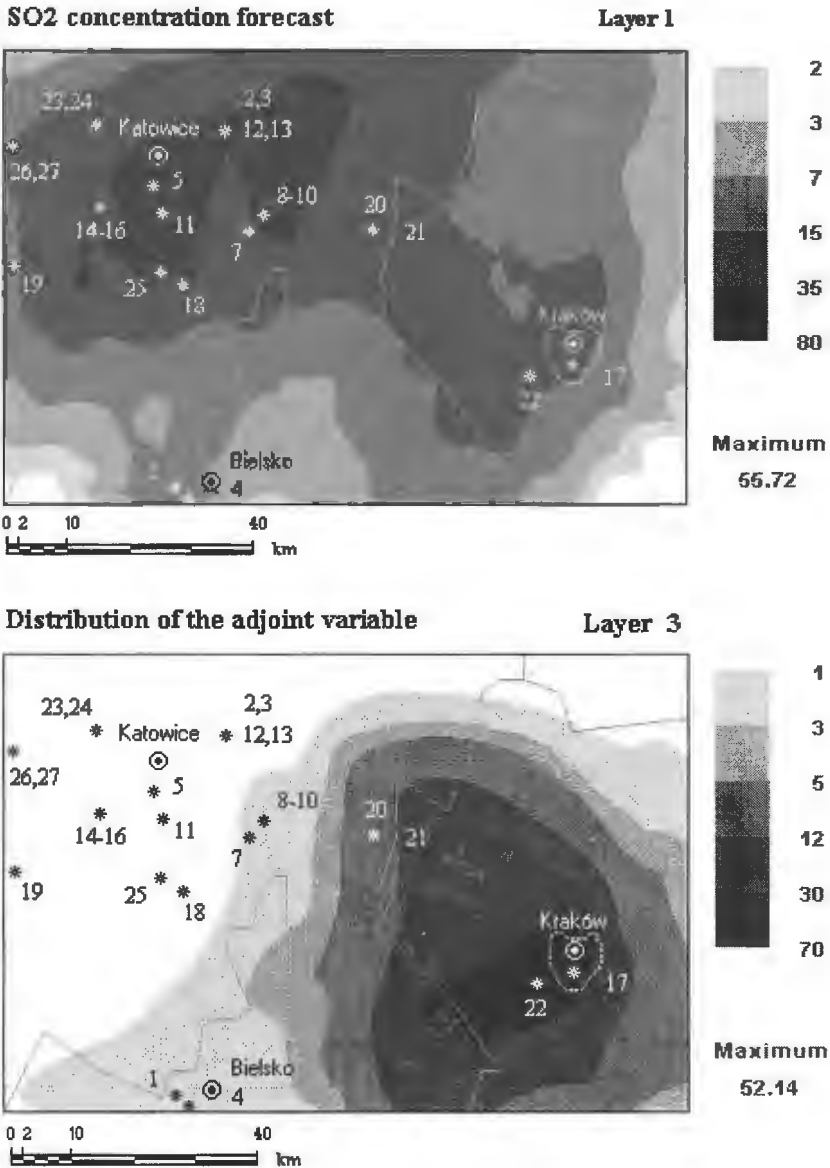


**Distribution of the adjoint variable**

**Layer 3**



**Figure 2.** Concentration of  $SO_2$  (top) and the adjoint variable (bottom) for Winter season (1996); (concentration in  $[\mu g/m^3]$ ).



**Figure 3.** Concentration of  $SO_2$  (top) and the adjoint variable (bottom) for Summer season (1996); (concentration in  $[\mu g/m^3]$ ).

For the selected quarter, the neighboring columns in Table 2 compare the relative impact of emission sources with the reference value. Both sets of results show the dominating impact of the source No. 22 (Skawina power plant) and the intermediate contribution of sources No. 10, 20, 21. On the other hand, there is a group of power plants with minor or negligible influence, in the sense of the assumed criterion function of the form (7).

It must be noted that the adjoint variable  $p^*$  in (5) has no unique physical dimension, because it always depends on the form of the objective function (3). Thus, the calculated and the reference data in the neighboring columns of Table 2, have different physical meaning and can not be directly compared. On the other hand, the results are sufficient to assess quantitatively the influence of the discussed sources. To facilitate a direct comparison, calculated values of the gradient components can be normalized, e.g. with respect to the maximum value for a given term. In any case, the correlation of two series of results can be calculated.

The results confirm good agreement (consistence) of two sets of results. Relatively low contribution of source No. 17 (Łęg power plant) to the protected area results from very high stack of this plant, so it affects rather distant receptors. The correlation coefficient of calculated and reference data is high, and for four quarters considered is over 0.97. This confirms correctness of the computational method discussed in the previous section. It can be useful in future applications concerning integrated environmental systems, decision support problems as well as the real-time emission control.

Figures 2 and 3 present the distribution maps for  $SO_2$  concentration and the respective adjoint variable. These are the aggregated maps, for Winter and Summer seasons, respectively. The maps illustrate the sense and the meaning of the adjoint variable in optimization task. It can be observed that the area of high values of this variable coincides with location of the most contributing sources.

### 3. Conclusions

The paper concentrates on two basic tasks: i) formulation of the optimization problem related to regional-scale air quality, and ii) testing the efficiency and accuracy of an implementation of the computational algorithm designed for solving the problem. As stated in Section 2, the emission field of the controlled sources and air pollution dispersion processes are considered as a distributed parameter system, which is governed by the respective set of transport equations. Consequently, the optimization technique for dis-

tributed parameter systems (Lions, 1971; Martchuk, 1995) is utilized in characterization of the optimal solution.

The key module of the system is the numerical model of air pollution transport. The quality of the respective finite-dimensional approximation scheme applied for solving the state and adjoint equations constitutes the basic problem. Numerical solution of evolutionary equations of this type is especially sensitive to the properties of approximation scheme applied. It is known that the crucial role in the final accuracy of the method is played by monotonicity and positivity of approximation method, as discussed by Holnicki (1996). A shape preserving scheme, based on a combination of the method of characteristics and the piecewise-quintic spatial interpolation (Holnicki 1996), is used for simulation of air pollution transport.

The above numerical algorithm was used in analysis of the state and adjoint equations. Since in the application considered, the emission field is composed of the point sources (power plants) – the case is especially sensitive to shape-preserving properties of the numerical approximation scheme. The test computations, performed for the selected periods, confirm good accuracy of the solution to transport equation as well as satisfactory integration of the method with the optimization algorithm. The obtained results also show that the method is computationally effective and the resulting accuracy of the optimal solution is sufficient, having in perspective the future applications.

The utilization of the techniques discussed in the paper concentrates on the problem of the long-term analysis of regional scale environmental tasks, e.g. in sustainable development problems, as discussed by Chang (2000) or Haurie et al. (2004). The technique can also be useful in supporting decisions concerning the planned energy sector investments and their location within the region. Presented results show that some elements of this approach can also be applied in the real-time emission control. The remark refers to the adjoint variable, which indicates the most influencing area, from the environmental perspective. Distribution of this variable is also the crucial factor in analysis of short time environmental effects and implementing on-line emission control.

The below last comments relate to the short term environmental management and the real-time emission control. In this case, the problem consists in minimizing environmental damage by the respective redistribution of emission intensity in controlled power plants, according to changing meteorological conditions. The cost functional (the objective function), which is to be minimized, in this case can be considered in the following, integral form

$$J(\bar{q}) = \alpha_1 \int_0^T \int_{\Omega} [\max(0, c(\bar{q}) - c_{ad})]^2 d\Omega dt + \alpha_2 \int_0^T \sum_{i=1}^N (q_i - q^*)^2 dt, \quad (12)$$

where the first component represents environmental damage depending on the concentration, and the second – cost of the controlling action. The area weight function  $0 \leq w(x, y) \leq 1$  represents sensitivity of the domain to this specific type of air pollution. Emission of the controlled sources  $\bar{q} = [q_1, \dots, q_N]$  is considered as the control function. Moreover, some additional economic and technological constraints can also be taken into account. They can be expressed in the following form

$$\underline{q}_i \leq q_i \leq \bar{q}_i \quad (i = 1, \dots, N) \quad \text{and} \quad \sum_{i=1}^N q_i(t) \geq d, \quad (13)$$

where the first inequality denotes lower and upper technological limits imposed on control function (emission intensity or the production level of the power plant), and the second – the total energy demand for the region under consideration.

Formulation of the real-time emission control problem, presentation of numerical optimization algorithm and selected results of the real-data test computations can be found in (Holnicki, 2006).

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**Jan Studzinski, Olgierd Hryniewicz (Editors)**

**ECO – INFO AND SYSTEMS RESEARCH**

This book presents the papers that describe the most interesting results of the research that have been obtained during the last few years in the area of applications of informatics in environmental engineering and environment protection at the Systems Research Institute of Polish Academy of Sciences in Warsaw (IBS PAN) and at the German Institute for Landscape System Analysis in Müncheberg (ZALF). The papers were presented in the form of extended summaries during a special workshop organized by IBS PAN in Szczecin in September 2006 together with the conference BOS'2006 dedicated to the applications of systems research in science, technology and economy and organized jointly by IBS PAN, University of Szczecin, and the Polish Society of Operational and Systems Research. They deal with mathematical modeling, approximation and visualization of environmental variables and with development of computer aided decision making systems in the area of environmental informatics.

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