

Modern Approaches in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics Volume II: Applications

Editors

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Generalized net model of slow learning algorithm of unsupervised ART2 neural network

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Abstract

This paper describes the slow learning algorithm of unsupervised adaptive resonance theory ART2 neural network. At first, the algorithm is presented step by step with formulas, and it is shown how an individual vector affects the network. At the end of the process, we have a learned network with stable recognition clusters according to the vectors. The process of the learning algorithm is presented by a generalized net model.

Keywords: Generalized Nets, Neural Networks, Adaptive Resonance Theory.

1 Introduction

Adaptive resonance theory (ART) [3, 4] was introduced by Stephen Grossberg in 1976. In this work ART2 [2, 3, 4] neural network [8], slow learning algorithm [5, 6, 7] is taken into consideration. ART2 is designed to perform operation over continuous valued input vectors. It consists of two layers – the first one has complex units or neurons that support a combination of normalization and noise suppression. The second layer is a competitive one. Both of them are fully connected with bottom-up and top-down weights. In addition, the bottom-up and

top-down signals are needed for the reset mechanism that takes the design whether or not the input vector takes place in the winner cluster. The neural network is learned by modification of the bottom-up and top-down weights. The structure of the ART2 neural network is presented below:

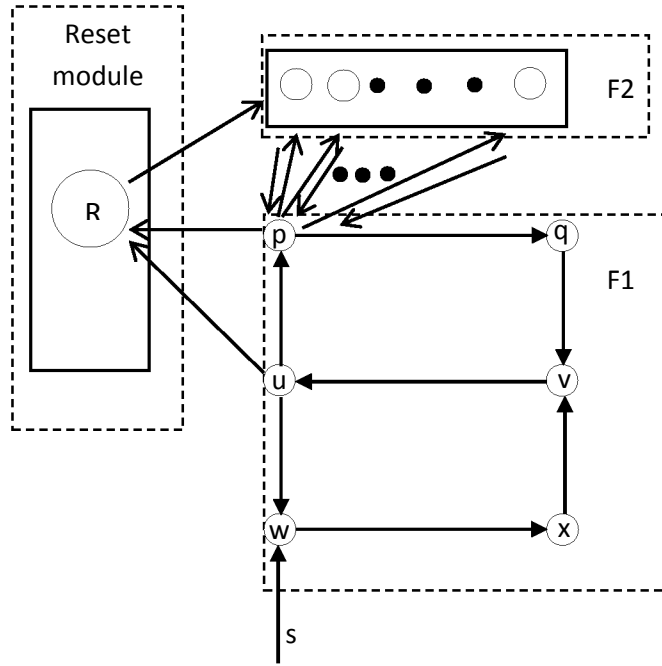


Fig. 1. Structure of the ART2 neural network

The slow learning algorithm according to [6, 7] can be expressed by the following steps:

Step 1. Initialize parameters: $a, b, \theta, c, d, e, \alpha, \delta, EP, l$ where:

- a, b – fixed weights in the F_1 layer;
- θ - noise suppression parameter;
- c – fixed weight used in testing for reset;
- d – activation of winning F_2 unit;
- e – small parameter using preventing division by zero;
- S -matrix with input vectors (s_1, s_2, \dots, s_n) ;
- n – number of the neurons on the input layer;
- m – number of the neurons in the second layer;
- α – learning rate;
- ρ – vigilance threshold;
- b_j – initial bottom-up weights;
- t_j – initial top-down weights;

- EP – number of epochs;
- l – number of loops in F_1 layer.

Step 2. For each input vector s do Steps 3-11.

Step 3. Update F_1 unit activation:

$$u_i = 0, \quad x_i = \frac{w_i}{e + \|w\|}, \quad q_i = 0$$

$$w_i = s_i, \quad p_i = 0, \quad v_i = f(x)_i$$

The activation function is $f(x) = \begin{cases} x & \text{if } x \geq \theta \\ 0 & \text{if } x < \theta \end{cases}$

Update F_1 activations again

$$u_i = \frac{v_i}{e + \|v\|}, \quad x_i = \frac{w_i}{e + \|w\|}, \quad q_i = \frac{p_i}{e + \|p\|}$$

$$w_i = s_i + a * u_i, \quad p_i = u_i, \quad v_i = f(x)_i + bf(q)_i$$

Step 4. Compute the signals to F_2 units:

$$y_j = \sum_j b_j * p_i$$

Step 5. While reset is true, do Steps 6-7

Step 6. Find F_2 unit with largest signal. (Define J such that $y_j \geq y_j$ for $j = 1, 2, \dots, m$.)

Step 7. Check for reset:

$$u_i = \frac{v_i}{e + \|v\|}, \quad r_i = \frac{u_i + cp_i}{e + \|u\| + c\|p\|}, \quad p_i = u_i + dt_{Ji}$$

If $\|r\| < p - e$ then $y_J = -1$ (inhibit J)

(Reset is true; repeat Step 5)

If $\|r\| \geq p - e$

Reset is false; proceed to Step 8

Step 8. Update weights for winning unit J

$$t_{Ji} = \alpha du_i + \{1 + \alpha d(d-1)\} t_{Ji}$$

$$b_{iJ} = \alpha du_i + \{1 + \alpha d(d-1)\} b_{iJ}$$

Step 9. Test stopping condition for number of epochs.

2 GN-Model

Initially the following tokens enter the generalized net [1]: In place $L_1 - \alpha$ -token with initial characteristic $x_0^\alpha = \langle s_1, s_2, s_3, \dots, s_k \rangle$, where k is the number of input vectors. In place $L_2 - \beta$ -token with initial characteristics

$$x_0^\beta = \langle a, b, c, e, d, n, m, \rho, \alpha, \theta, EP, l \rangle.$$

Generalized net is presented by a set of transitions:

$$A = \{Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7, Z_8\},$$

where the eight transitions describe the following processes:

Z_1 = "Extraction of a vector from the matrix";

Z_2 = "Calculation values of the vector";

Z_3 = "Normalization of the data";

Z_4 = "Assignment of weights";

Z_5 = "Calculation of resonance state";

Z_6 = "Suppression of the noise";

Z_7 = "Updating weights";

Z_8 = "Determination of winning neuron".

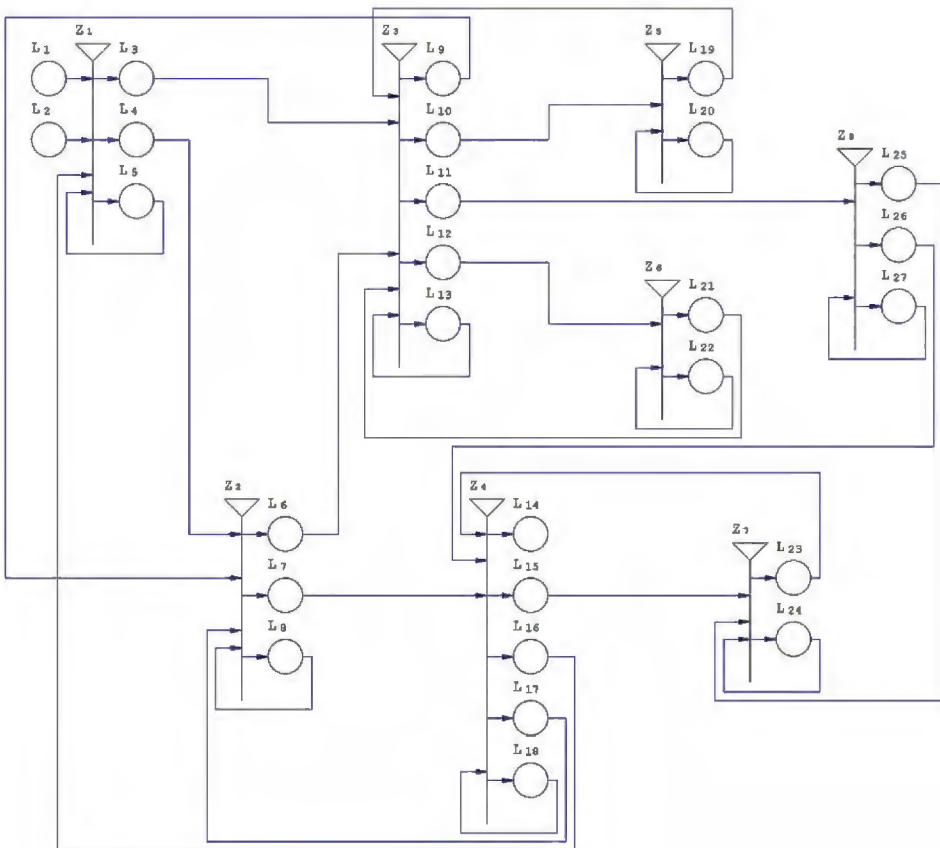


Fig. 2. GN model of slow learning algorithm of unsupervised ART2 neural network

The constructed GN-Model consists of eight transitions with the following descriptions:

$$Z_1 = \langle \{L_1, L_2, L_5, L_{16}\}, \{L_3, L_4, L_5\}, R_1, \vee(L_1, L_2, \wedge(L_5, L_{16})) \rangle$$

$$R_1 = \begin{array}{c|ccc} & L_3 & L_4 & L_5 \\ \hline L_1 & False & False & True \\ L_2 & False & False & True, \\ L_5 & W_{5,3} & W_{5,4} & True \\ L_{16} & False & False & True \end{array}$$

where $W_{5,3} = W_{5,4} =$ “There is an active signal from place L_{16} ”. Token β that enters place L_5 from place L_2 unites with token α' (from place L_1) in one δ -token with characteristic

$$x_{cu}^\delta = \langle "pr_1x_0^\beta; pr_2x_0^\beta; pr_3x_0^\beta; pr_4x_0^\beta; pr_5x_0^\beta; pr_6x_0^\beta; pr_7x_0^\beta; pr_8x_0^\beta; \\ pr_9x_0^\beta; pr_{10}x_0^\beta; pr_{11}x_0^\beta; pr_{12}x_0^\beta; pr_1x_0^\alpha; pr_2x_0^\alpha, \dots, pr_kx_0^\alpha" \rangle.$$

Token δ' enters place L_3 from place L_5 with characteristic

$$x_{cu}^{\delta'} = \langle pr_1x_0^\beta; pr_2x_0^\beta; pr_3x_0^\beta; pr_4x_0^\beta; pr_5x_0^\beta; pr_6x_0^\beta; pr_7x_0^\beta; \\ pr_8x_0^\beta; pr_9x_0^\beta; pr_{10}x_0^\beta; pr_{11}x_0^\beta; pr_{12}x_0^\beta; pr_ix_0^\alpha \rangle,$$

where $i \in [1; k]$ represents the current number of iteration. Token α' enters place L_4 from place L_3 with characteristic

$$x_{cu}^{\alpha'} = "pr_ix_0^\alpha".$$

$$Z_2 = \langle \{L_4, L_9, L_{17}, L_{18}\}, \{L_6, L_7, L_8\}, R_2, \vee(\wedge(L_4, L_9), \wedge(L_8, L_{17})) \rangle$$

$$R_2 = \begin{array}{c|ccc} & L_6 & L_7 & L_8 \\ \hline L_4 & False & False & True \\ L_9 & False & False & True, \\ L_8 & W_{8,6} & W_{8,7} & True \\ L_{17} & False & False & True \end{array}$$

where:

- $W_{8,6} =$ “The values of p and w units are calculated and $pr_{12}x_{cu}^\beta = 1$ ”;
- $W_{8,7} =$ “The values of p units are calculated and $pr_{12}x_{cu}^\beta > 1$ ”.

Token δ'' enters place L_8 from place L_9 with characteristic

$$x_{cu}^{\delta''} = \langle pr_1 x_0^\beta; pr_2 x_0^\beta; pr_3 x_0^\beta; pr_4 x_0^\beta; pr_5 x_0^\beta; pr_6 x_0^\beta; pr_7 x_0^\beta; pr_8 x_0^\beta; \\ pr_9 x_0^\beta; pr_{10} x_0^\beta; pr_{11} x_0^\beta; pr_{12} x_0^\beta; pr_i x_0^\alpha; x_{cu}^{\epsilon'}; \eta; \zeta \rangle,$$

On the first activation on the transition token γ enters place L_6 with characteristic

$$x_{cu}^\gamma = \langle x_{cu}^{\epsilon'}, \eta, \zeta \rangle, \text{ and } \eta = 0 \text{ and } x_{cu}^\zeta = \langle x_{cu}^{\alpha'} + pr_1 x_0^{\delta''} * x_{cu}^{\epsilon'} \rangle.$$

On the second activation $\eta = x_{cu}^{\epsilon'} + \kappa' * pr_5 x_0^{\delta''}$. The token that enters place L_7 from place L_8 obtain characteristic

$$\delta^{IV} = \langle pr_5 x_0^{\delta''}; pr_6 x_0^{\delta''}; pr_7 x_0^{\delta''}; pr_9 x_0^{\delta''}; x_{cu}^{\epsilon'}; x_{cu}^{\alpha'} \rangle \\ Z_3 = \langle \{L_3, L_6, L_{13}, L_{19}, L_{21}\}, \{L_9, L_{10}, L_{11}, L_{12}, L_{13}\}, R_3, \\ \vee(L_3, L_6, L_{13}, L_{19}, L_{21}, L_{26}) \rangle$$

$R_3 =$	L_9	L_{10}	L_{11}	L_{12}	L_{13}
L_3	False	False	False	False	True
L_6	False	False	False	False	True
L_{13}	$W_{13,9}$	$W_{13,10}$	$W_{13,11}$	$W_{13,12}$	True'
L_{19}	False	False	False	False	True
L_{21}	False	False	False	False	True

where:

- $W_{13,9}$ = "The values of "u" units are calculated";
- $W_{13,10}$ = "The values for resonance verification are calculated";
- $W_{13,11}$ = "The value of resonance is normalized";
- $W_{13,12}$ = "The values of "x" and "q" units are calculated".

Token δ'' enters place L_{13} from place L_3 united with one ϵ' – token and obtain characteristic

$$x_{cu}^{\delta''} = \langle pr_1 x_0^\beta; pr_2 x_0^\beta; pr_3 x_0^\beta; pr_4 x_0^\beta; pr_5 x_0^\beta; pr_6 x_0^\beta; pr_7 x_0^\beta; pr_8 x_0^\beta; \\ pr_9 x_0^\beta; pr_{10} x_0^\beta; pr_{11} x_0^\beta; pr_{12} x_0^\beta; pr_i x_0^\alpha; x_{cu}^{\epsilon'}; \eta; \zeta \rangle,$$

where

$$x_{cu}^{\varepsilon'} = \frac{x_{cu}^{\varepsilon}}{\text{pr}_4 x_0^{\delta''} + \|x_{cu}^{\varepsilon}\|}$$

Token γ' enters place L_{10} from place L_{13} with characteristic

$$x_{cu}^{\gamma'} = \text{"pr}_2 x_{cu}^{\gamma}, \varepsilon', \varepsilon'', \eta', \text{pr}_3 x_0^{\delta''}, \text{pr}_4 x_0^{\delta''}\text{"}$$

where

$$x_{cu}^{\varepsilon''} = \sqrt{\sum_{i=1}^n x_1^2 + x_2^2 + \dots + x_n^2}$$

and

$$x_{cu}^{\eta'} = \sqrt{\sum_{i=1}^n x_1^2 + x_2^2 + \dots + x_n^2},$$

where x are the different elements of the vector s_i .

Token μ'' enters place L_{11} from place L_{13} with characteristic

$$x_{cu}^{\mu''} = \text{"}\mu', \text{pr}_8 x_0^{\delta''}, \text{pr}_4 x_0^{\delta''}\text{"},$$

where

$$x_{cu}^{\mu'} = \sqrt{\sum_{i=1}^n x_1^2 + x_2^2 + \dots + x_n^2}.$$

Token γ'' enters place L_{12} from place L_{13} with characteristic

$$x_{cu}^{\gamma''} = \delta''', \zeta', \text{pr}_2 x_0^{\delta''}, \text{pr}_6 x_0^{\delta''}, \text{pr}_{10} x_0^{\delta''},$$

where

$$x_{cu}^{\zeta'} = \frac{\text{pr}_2 x_{cu}^{\gamma}}{\text{pr}_4 x_0^{\delta''} + \|\text{pr}_2 x_{cu}^{\gamma}\|}$$

and

$$x_{cu}^{\delta'''} = \frac{\text{pr}_1 x_{cu}^{\gamma}}{\text{pr}_4 x_0^{\delta''} + \|\text{pr}_1 x_{cu}^{\gamma}\|}.$$

$$Z_4 = \langle \{L_7, L_{18}, L_{23}, L_{27}\}, \{L_{14}, L_{15}, L_{16}, L_{17}, L_{18}\}, R_4, \vee \{L_7, L_{18}, L_{23}, L_{27}\} \rangle$$

	L_{14}	L_{15}	L_{16}	L_{17}	L_{18}
L_7	False	False	False	False	True
L_{18}	$W_{18,14}$	$W_{18,15}$	$W_{18,16}$	$W_{18,17}$	True,
L_{23}	False	False	False	False	True
L_{27}	False	False	False	False	True

where:

- $W_{18,14} = \text{"}i > m\text{"}$;
- $W_{18,15} = \text{"Bottom-up and top-down weights are determined"}$;
- $W_{18,16} = \text{"Request for next input vector"}$;

- $W_{18,17}$ = "Top-down weights are determined".

The token that enters place L_{18} obtain characteristics

$$\delta^V = "pr_{10}^{\delta^{IV}}, pr_{20}^{\delta^{IV}}, pr_{30}^{\delta^{IV}}, pr_{6_{cu}}^{\delta^{IV}}, x_{cu}^{\theta}, x_{cu}^k, x_{cu}^l, x_{cu}^{k'}, x_{cu}^{\theta'}"$$

where

$$x_{cu}^{\theta} = \frac{1}{(1-pr_{10}^{\delta^{IV}})*pr_{20}^{\delta^{IV}}},$$

$$x_{cu}^k = ("t_j"),$$

$$pr_{30}^{\delta^{IV}}$$

$$x_{cu}^l = \sum_1 pr_{cu}^{\varepsilon'} * x_{cu}^{\theta}.$$

$$x_{cu}^{k'} = \max(x_{cu}^l(t_j)).$$

The tokens $x_{cu}^{\theta}, x_{cu}^k$ update their values from the tokens coming from the transition Z_7 . The tokens x_{cu}^l obtain characteristic "next max value". The token that enters place L_{15} from place L_{18} obtains the characteristic

$$x_{cu}^{\rho} = "x_{cu}^{k'}, x_{cu}^{\theta'}, pr_{10}^{\delta^{IV}}, pr_{40}^{\delta^{IV}}; pr_{5_{cu}}^{\delta^{IV}}";$$

Tokens ζ enter place L_{16} from place L_{18} and obtain characteristic

$$x_{cu}^{\zeta'} = ("next vector")$$

The token in L_{14} obtains characteristic $pr_{4_{cu}}^{\delta^V}$ = "reject set".

$$Z_5 = \langle \{L_{10}, L_{20}\}, \{L_{19}, L_{20}\}, R_5, \vee(L_{10}, L_{20}) \rangle$$

$$R_5 = \begin{array}{c|cc} & L_{19} & L_{20} \\ L_{10} & False & True, \\ L_{20} & W_{20,19} & True \end{array}$$

where $W_{20,19}$ = "Resonance state is calculated".

The tokens that enter place L_{20} obtain characteristics

$$x_{cu}^{\gamma'''} = "pr_2 x_{cu}^{\eta}, \varepsilon', \varepsilon'', \eta', pr_3 x_0^{\delta''}, pr_4 x_0^{\delta''}, x_{cu}^{\mu}";$$

where

$$x_{cu}^{\mu} = \frac{pr_2 x_{cu}^{\gamma'} + pr_5 x_0^{\gamma'} * pr_1 x_{cu}^{\gamma'}}{pr_6 x_0^{\gamma'} + pr_3 x_{cu}^{\gamma'} + pr_5 x_0^{\gamma'} * pr_4 x_{cu}^{\gamma'}};$$

The token that enters place L_{19} from place L_{20} obtain characteristic

$$\mu' = pr_7^{\gamma'''}.$$

$$Z_6 = \langle \{L_{12}, L_{22}\}, \{L_{21}, L_{22}\}, R_6, \vee(L_{12}, L_{22}) \rangle$$

$$R_6 = \begin{array}{c|cc} & L_{21} & L_{22} \\ L_{12} & False & True, \\ L_{22} & W_{22,21} & True \end{array}$$

where $W_{22,21}$ = "Noise suppression is determined".

The token that enters place L_{22} obtain characteristic $x_{cu}^y = x_{cu}^{y''}, x_{cu}^{y^e}$; where $x_{cu}^{y^e} = f(pr_2^{y''}) + pr_3 x_0^{y''} * f(pr_1 x_{cu}^{y''})$;

The token that enters place L_{21} from place L_{22} obtain characteristic

$$x_{cu}^{e'} = "pr_2^{y^v}".$$

$$Z_7 = \langle \{L_{15}, L_{24}, L_{25}\}, \{L_{23}, L_{24}\}, R_7, \vee(L_{24}, \wedge(L_{15}, L_{25})) \rangle$$

	L_{23}	L_{24}
$R_7 =$	$False$	$True$
L_{15}	$W_{24,23}$	$True'$
L_{24}	$False$	$True$
L_{25}	$False$	$True$

where $W_{24,23}$ = "The weights are updated".

The token that enters place L_{24} obtain characteristic

$$x_{cu}^{\delta''} = \langle pr_1 x_0^\beta; pr_2 x_0^\beta; pr_3 x_0^\beta; pr_4 x_0^\beta; pr_5 x_0^\beta; pr_6 x_0^\beta; pr_7 x_0^\beta; pr_8 x_0^\beta; pr_9 x_0^\beta; pr_{10} x_0^\beta; pr_{11} x_0^\beta; pr_{12} x_0^\beta \rangle$$

$$x_{cu}^{\rho'} = x_{cu}^\rho, x_{cu}^\lambda, x_{cu}^v, x_{cu}^\xi;$$

where

$$x_{cu}^\lambda = pr_4 x_0^\rho * pr_3^\rho * pr_5^\rho + (1 + pr_4^\rho * pr_3^\rho (pr_3^\rho - 1) * pr_1 x_{cu}^\rho);$$

$$x_{cu}^v = pr_4 x_0^\rho * pr_3^\rho * pr_5^\rho + (1 + pr_4^\rho * pr_3^\rho (pr_3^\rho - 1) * pr_2 x_{cu}^\rho);$$

The token that enters place L_{23} from place L_{24} obtain characteristic

$$x_{cu}^s = "x_{cu}^\lambda, x_{cu}^v".$$

$$Z_8 = \langle \{L_{11}, L_{27}\}, \{L_{25}, L_{26}, L_{27}\} \vee (L_{11}, L_{27}) \rangle$$

	L_{25}	L_{26}	L_{27}
$R_8 =$	$False$	$False$	$True,$
L_{11}	$W_{27,25}$	$W_{27,26}$	$True$
L_{27}			

where:

- $W_{27,25} = "\mu' \geq pr_8 x_0^\beta - pr_4 x_0^\beta"$;
- $W_{27,26} = ",, \neg W_{27,25}"$;

The token that enters place L_{27} obtains the characteristic

$$x_{cu}^{\mu''''} = x_{cu}^{\mu''''}, x_{cu}^\xi, x_{cu}^o;$$

where

$$x_{cu}^\xi = "true";$$

$$x_{cu}^o = "false".$$

The token that enters place L_{25} from place L_{27} obtain characteristic

$$x_{cu}^{\xi'} = pr_2 x_{cu}^{\mu''''};$$

The token that enters place L_{26} from place L_{27} obtain characteristic

$$x_{cu}^{o'} = pr_3 x_{cu}^{\mu'''}$$

3 Conclusion

The process of slow learning algorithm of ART2 neural network was presented with GN model. It was showed how an individual vector passes across the network and changes its activities of the neurons, in the end we have stable recognition clusters that have both “stability and plasticity” and also if there is noise, the network is able to suppress it according to the user’s choice of parameter.

The algorithm might be of help for people who need to solve problems that cannot be solved with other networks.

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The papers presented in this Volume 2 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.

It may be viewed as a result of fruitful discussions held during the Twelfth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2013) organized in Warsaw on October 11, 2013 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, Prof. Asen Zlatarov University, Burgas, Bulgaria, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Twelfth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2013) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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