POLSKA AKADEMIA NAUK
INSTYTUT BADAŃ SYSTEMOWYCH

PROCEEDINGS OF THE 3rd ITALIAN-POLISH CONFERENCE ON APPLICATIONS OF SYSTEMS THEORY TO ECONOMY, MANAGEMENT AND TECHNOLOGY

WARSZAWA 1977

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PREFACE

The present volume comprises the papers from the 3rd Italian-Polish Conference which was held in Warsaw in May 1980, [illegible]. The conference organised by the Centre of Control Systems and Computation of the Italian National Research Council in Rome and the Institute for Organisation, Management and Control Sciences of Polish Academy of Sciences and the Ministry of Science, Higher Education and Technology in Warsaw was the third conference of this type.

The first one took place in Kraków (Poland) in 1972, the second in Pugnochiuso (Italy) in 1974.

This conference, like both the previous ones, gathered the leading representatives of many scientific centres from Italy as well as from Poland, representing also the different aspects of the systems analysis applied to the economic, management, data processing and technological systems.

From the past, such meetings enrich the exchange of views and experiences between the specialists from both countries what is the [illegible] to both sides: above the general goal, the systematisation of the systems and control theory, especially in the econometrician's conception, the essential development of the quantitative analysis and the perspective application of large computer systems.

The meaning of such meetings, however, goes far beyond that, what is happening during the conference because the established contacts are also fruitful in form of common scientific research works and — "last but not least" — bring the friendly links between the scientists of both countries which last for years.

The proceedings of the conference were divided into three parts:

1. Optimization and Control Theory;
2. Systems Theory in Economics;
3. Technological, Managerial and Information Systems.

While the first two parts are of rather homogeneous character, the third part contains the papers concerning the different types of models — for the economic, technological, management and data processing systems.

W. Findeisen
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COORDINATION BY PRICE METHODS

1. INTRODUCTION

The aim of this paper is to exhibit similarities and differences in a family of coordination methods for complex systems, namely in the price coordination methods.

The complex system subject to coordinated control can be depicted as in Fig 1. We may distinguish:

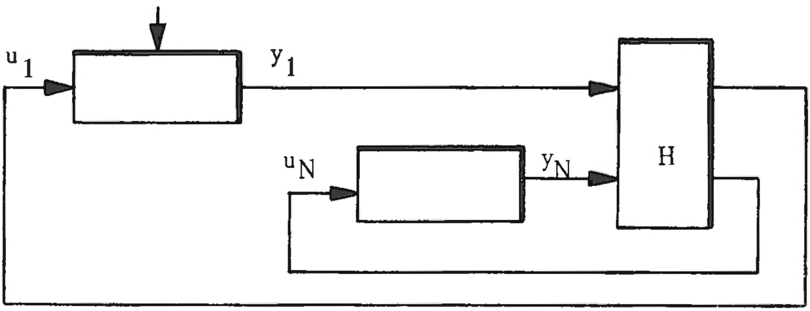

Fig. 1

system elements (subsystems), having input vectors $u_i \in U_i$, output vectors $y_i \in Y_i$ and control vectors $c_i \in C_i$. Mappings $F_i = C_i \times U_i \rightarrow Y_i$ exist, so that

$$y_i = F_i(c_i, u_i), \quad i \in \overline{1, N} \tag{1}$$

system interconnections, consisting in each of the input vector component u_{ik} being equal to a specified component of an output vector y_{jl}. This is described for input u_i and for the whole system, respectively,

$$u_i = H_i y, \quad \text{and} \quad u = Hy \tag{2}$$

where $u = (u_1, \ldots, u_N)$, $y = (y_1, \ldots, y_N)$, H — interconnection matrix.

system resource constraint

$$\sum_{i=1}^{N} r_i(c_i, u_i) \leqslant r \tag{3}$$

where $r_i(.)$ is the amount of common system resources used by subsystem No. i.

subsystem (local) constraints

$$\text{t}c_i, u_i) \in CU_i, \quad i \in 1, N \quad \overline{1, N} \tag{4}$$

We assume that a **local performance index** $Q_i(c_i, u_i)$ is associated with each subsystem and the local decision maker would tend to extremize it.

The task of coordination will be to influence the local decision makers in such a way, that:

— system constraints will be preserved,

— overall optimum will be achieved.

Price coordination consists in letting the coordinator to prescribe prices on inputs, outputs and resources and then permitting the local decision makers to define their own choices of the values of these variables. The system is **coordinated** when the local choices cause the interconnection equation (2) to be satisfied and the global constraint (3) to be non-violated. The prices which effect this state of coordination can be termed equilibrium prices, since satisfaction of (2) means equilibrium.

Price coordination brings about overall system optimum if the **global performance index** is a sum of local ones

$$Q = \sum_{i=1}^{N} Q_i$$

It is worth mentioning, that direct and penalty function coordination methods would allow more general form of global performance

$$Q = \psi(Q_1, \ldots, Q_N)$$

where ψ is a strictly order-preserving function.

The discussion of price coordination in this paper will omit the resource constraint (3), thus focusing on interconnections (2). Suitable extensions are possible.

2. INTERACTION BALANCE AND "TATÔNNEMENT" PROCEDURES

Price coordination has been known to economic systems for a long time. The prices were called upon to cause equilibrium of production and demand, that is equality of the coorresponding outputs and inputs. In terms of the system description in Section 1, the aim of price adjustment will be to provide for satistaction of the interconnection equation (2).

Let us look at this in some detail. The **local problems** i. e. problems associated with the individual system elements can be formulated

$$\text{minimise } Q_{i\,mod} = Q_i(c_i, u_i) + \langle p_i, u_i\rangle - \langle q_i, F_i(c_i, u_i)\rangle \tag{5}$$

subject to

$$(c_i, u_i) \in CU_i$$

with the results $\hat{c}_i(p)$, $\hat{u}_i(p)$, $\hat{y}_i(p) = F_i(\hat{c}_i(p), \hat{u}_i(p))$.

If (5) is related to a finite-dimensional problem, then the scalar product $\langle p_i, u_i\rangle$ means $\sum_j p_{ij} u_{ij}$.

In problem (5) we assumed coordination by a price vector p, composed of prices on inputs u in the whole system. Hence p_i are prices on u_i, and prices q_i on output y_i are defined as well by virtue of (2), namely

$$q_i = \sum_{j=1}^{N} H_{ji}^T p_j \tag{6}$$

It is therefore right to say that the results of (5) are all dependent on vector p. The "interaction balance" or "equilibrium" prices $\hat{p}$ will be defined as

$$\hat{p} : \hat{u}(p) - H\hat{y}(p) = 0 \tag{7}$$

where $\hat{y}(p) = HF(\hat{c}(p), \hat{u}(p))$.

Providing for the condition (7) to be satisfied is the task of the coordinator. Walras would assign this to his "tatônnement" procedure at the stock exchange: a person outside the negotiating parties should vary the price $\boldsymbol{p}$, listen to responses $\hat{u}(p)$ and $\hat{y}(p)$, and stop the procedure at $p = \hat{p}$.

Essentially the same idea has been brought up in the study of multilevel control systems [Mesarović 1970], [Malinowski 1974]. Several questions have been raised in those more formalised studies, for example:

— existence of $\hat{p}$ ("coordinability" of Mesarović),

— system optimality with control $\hat{c}(\hat{p})$ ("applicability" of Mesarović),

— procedures to obtain $\hat{p}$.

The exact answers can be found elsewhere [Malinowski, 1974, 1975].

They are based on discussion of the Lagrangian' function

$$\varphi(p) = \sum_{i=1}^{N} Q_i(\hat{c}_i(p), \hat{u}_i(p)) + \langle p, \hat{u}(p) - HF(\hat{c}(p), \hat{u}(p))\rangle \tag{8}$$

for which we require that it has a maximum at $p = \hat{p}$:

$$\varphi(\hat{p}) = \max_p \varphi(p)$$

If $\hat{p}$ so defined exists, its further use to determine control $\hat{c}$ is practically restricted to the case where $(\hat{c}, \hat{p})$ are single-valued functions of p. This requirement appears to be vital. Unfortunately we know sufficient conditions only:

$(\hat{c}, \hat{u})$ are single-valued if the functions $Q_i(.)$ are strictly convex and the mappings $F_i(.)$ are linear.

With $p=\hat{p}$ the solutions $(\hat{c}, u)$ are optimal.

A gradient procedure can be used in the search for $\hat{p}$ if $Q_i(.)$ are continuous and $F_i(.)$ are continuous; with the uniqueness of $(\hat{c}, \hat{p})$ for every p it then holds for the (weak) derivative of $\varphi(p)$:

$$\varphi(p) = \hat{u}(p) - HF(\hat{c}(p), \hat{u}(p))$$

The second derivative, $\nabla^2\varphi(p)$, does not exist generally.

Condition (7) as a goal in search for $\hat{p}$ can obviously be used on models only. In the already operating system the interconnection equation (2) is satisfied for any control $c=(c_1, \ldots, c_N)$. The optimal control $\hat{c}(\hat{p})$, if applied, should provide optimality — but, as mentioned above, this may fail if the models are inadequate.

Interaction balance coordination described in this section can be applied to both static and dynamic problems, since it is a purely open-loop concept.

The next sections will show some other ideas of price coordination, where feedback from the real operating system will be used to improve the control.

3. ON-LINE COORDINATION OF STEADY STATE WITH FEEDBACK TO THE COORDINATOR

In this section we shall consider the optimisation problem to be in the finite-dimensional space, i.e. to form a problem of non-linear programming. In terms of control it means control of steady-state in a complex system. Practically, steady-state control is an appropriate technique if the optimal state trajectory of a dynamic system is slow enough to assume that the value of state vector x is at any time related to control only, the state derivative $\dot{x}$ being so small as to be neglected.

The mappings F_i, Q_i are now functions in finite-dimensional space. We re-label them into f_i, f_{oi} so as to obtain the following model-based global problem:

$$\text{minimise } f_0 = \sum_{i=1}^{N} f_{0i}(c_i, u_i) \qquad (9)$$

subject to

$$y_i = f_i(c_i, u_i), \qquad i \in \overline{1, N} \qquad (10)$$

$$u = Hy \qquad (11)$$

$$(c_i, u_i) \in CU_i \qquad (12)$$

A solution to (9)—(12) yields **model-based control** $\boldsymbol{c}$. We shall in this section pay considerable attention to the difference between model and reality, let us therefore formulate the followibg **real problem:**

$$\text{minimise } f_0 = \sum_{i=1}^{N} f_{0i}(c_i, u_i) \tag{13}$$

subject to

$$y_i = f_{*i}(c_i, u_i), \qquad i \in \overline{1, N} \tag{14}$$

$$u = Hy \tag{15}$$

$$(c_i, u_i) \in CU_i \tag{16}$$

We should notice that the only difference between model and reality is assumed to exist in the system element equations, (14) versus (10). We shall indicate in the sequel some effective way to fight the consequences of this difference.

It must be stressed, however, that differences may exist also in (13), (16) compared to (9), (12). For example, if a performance function is explicitly

$$f_{oi}(c_i, u_i, y_i)$$

then it will reduce to some $f_{oi}(c_i, u_i)$ only by using equation (10), but this makes it model-based. The real $f_{o*i}(c_i, u_i)$ would be different. A similar reason may lead to the set CU_{*i} being different from CU_i.

The solution to (13)—(16) will be termed **real-optimal control** $\hat{c}_*$. It is not obtainable by definition since reality (14) is not known. We can only look for a structure which would yield control better than purely model-based $\hat{c}$; in principle it is bound to be inferior than $\hat{c}_*$.

One of the possible structures is price coordination with feedback to the coordinator, to which this section is devoted. It is shown schematically by Fig. 2.

The **local problems** are exactly the same as in the interaction balance method of section 2, it means:

$$\text{minimise } f_{oi}(c_i, u_i) + \langle p_i, u_i \rangle - \langle q_i, f_i(c_i, u_i) \rangle \tag{17}$$

subject to

$$(c_i, u_i) \in CU_i \tag{18}$$

The controls $\hat{c}_i(p)$ determined by (17) (18) for the current value of p are applied to the real system, resulting in some u_* and y_*. The coordination concept consists in the following upper-level problem:

$$\text{find } p = \tilde{p}\colon \ \hat{u}(p) - u_*(\hat{c}(p)) = 0 \tag{19}$$

Condition (19) is an equality of model-based solution for the inputs $\hat{u}(p)$ and the inputs u_x, measured in the real system and caused by control $\hat{c}(p)$.

The properties of control based on cdonition (19) have been studied quite extensively [Mesarović 1970], [Malinowski 1974, 1975]. [Malinowski, Ruszczyński 1975]. The usual questions on existence of $\tilde{p}$, system optiminality with control $\hat{c}(\tilde{p})$ and procedures to obtain $\tilde{p}$ have been asked in those studies. The answers are in principle as follows.

Solution $\tilde{p}$ exists, if solution $\hat{p}$ of the usual interaction balance method exists for all s-shifted systems

$$u = Hf(c, u)+s$$

where $s \in S$, and S is the set of all possilbe values of the model-reality difference

$$Hf_*(c, u)-Hf(c, u) = s$$

with $(c, u) \in CU = CU_1 \times \ldots \times CU_N$.

When the models do not differ from reality, that is (10) is identical to (14), $\hat{c}(\tilde{p})$ is strictly optimal control and $\tilde{p}$ equals equilibrium prices $\hat{p}$ which would be obtained by solving problem (9)—(12) by the interaction balance method of section 2. When models differ from reality, control based on (19) is in the first approximation always non-inferior to the one based on open-loop value $\hat{p}$. In the particular case where

$$f_{*i}(c_i, u_i) = f_i(c_i, u_i)+\beta_i, \quad i \in \overline{1, N}$$

that is the model-reality difference consists in a shift, control based on (19) is strictly real-optimal. The open-loop control would of course in this case be much inferior.

A most important feature of control based upon (19) is its property to keep to constraints (12) in the real system. This happens because real c_* equals model c for any p, and for $p=\tilde{p}$ also $u_* = \hat{u}$. Since the model will keep $(\hat{c}_i, \hat{u}_i) \in CU_i$, $i \in \overline{1, N}$, the same will be kept in the real system, at $p=\tilde{p}$. Note that the open-loop control $\hat{c}(\hat{p})$ may violate these constraints in the real system, because at $p=\hat{p}$ it will in general be $u_* \neq \hat{u}$.

As far as the procedures to find $\tilde{p}$ are concerned, iterations have to be done at a rate acceptable by the real system, i.e. permitting new values u_* to establish themselves after a change of p. Unfortunately, the expression

$$F(p) = \hat{u}(p)-u_*(\hat{c}(p)) \tag{20}$$

which has to be brought to zero is not a derivative of any function, as it was in the case of (8). The value $\tilde{p}$ has to be found by equation-solving methods, $F(p)=0$. It should be stressed that if there are inequality constraints in the local problems, $F(p)$ will in general be non-differentiable. Suitable numerical methods to find $\tilde{p}$ have been proposed [Ruszczyński 1976].

Let us finish this section by a remark justifying discussion of steady-state control here as opposed to more general problem formulation in section 2. The reason is the practical field of application of coordination principle (19):

it must be iteratively done on the real system. This can be performed in steady-state optimisation, but not in a dynamical one. The only exception would be iterative optimisation of batch or cyclic processes, the iteration in time-function space being performed from one batch to another. For that particular case all considerations of this section can be appropriately generalised. It has in fact been done in the papers cited.

4. DECENTRALISED CONTROL OF STEADY-STATE WITH PRICE COORDINATION

The structure of Fig. 2, however proved to be effective and superior to open-loop model-based control, may be criticised: the information about real system u_* is made available to the coordinator only. The local problems base on models and calculate their imaginative $\hat{u}$ for each p, "knowing" that reality is different. The scheme of Fig. 2 is therefore a structure suitable for a mechanistic control system, but does not reflect the situation which would be established if the local problems were confined to some free decision makers.

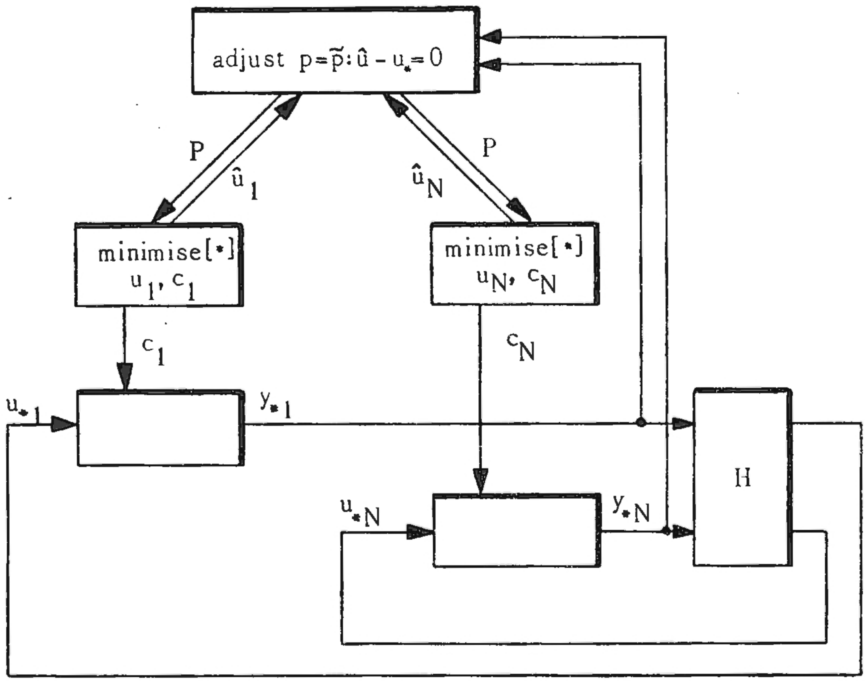

Fig. 2

In that case the local decision maker would tend to use real value u_{*i} in his problem, that is he would perform

$$\text{minimise } f_{oi}(c_i, u_{*i}) + \langle p_i, u_{*i}\rangle - \langle q_i, f_i(c_i, u_{*i})\rangle \tag{21}$$

subject to

$$(c_i, u_{*i}) \in CU_i \tag{22}$$

Schematically this is presented in Fig. 3 as feeding u_{*i} to the corresponding local problem. Even with fixed p the control exercised by local decision makers on the system as a whole remains to some extent coordinated, since the value of p will influence the control set by solving (21) (22). However, since u_{*i} are used locally, we call the structure of Fig. 3 **decentralised control.**

A problem for itself is system stability or the convergence of iterations made by local optimisers while trying to achieve their goals (21). It is obvious that all the iteration loops in the system are interdependent, since any u_{*i} will depend on all decisions $c=(c_1, \dots, c_N)$ in the previous stage.

If the iterations converge, steady values $\hat{c}(p)$, $\hat{u}_*(p)$ and $\hat{y}_*(p)$ will be obtained for the given price vector p.

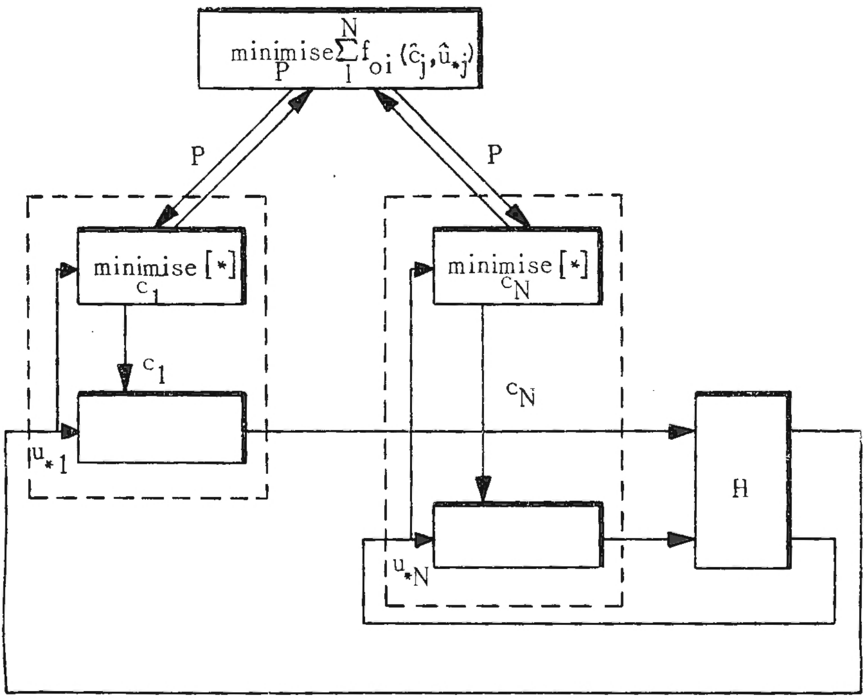

Fig. 3

It may be predicted that if this p would happen to be $\tilde{p}$ from the previous section, the result of decentralised control would, also be the same as in the previous structure. This does not say that we should aim at it, since the results obtained with $\tilde{p}$ are not real-optimal and a better value of p may exist.

We must therefore look for a way to iterate on prices p in the system of Fig. 3. A possibility is indicated in Figure 3:

$$\text{minimise } f_o = \sum_{i=1}^{N} f_{oi}(\hat{c}_i(p), \hat{u}_{*i}(p)) \tag{23}$$

which simply means to find a price p such, that the overall result of local controls be optimised.

The theoretical properties of problem (23) are not known.

Two of those properties seem predictable. If the models are adequate, and all iterations converge, they will converge to strict overall optimum for the system. If the models differ from reality, then the constraints CU_i will be secured (like in the structure Fig. 2), but the overall result will be suboptimal. This suboptimality is due to the fact that in performing (21) we continue to have an inadequate value of the output y_i.

5. ON-LINE DYNAMIC PRICE COORDINATION

System coordination exercised by putting prises on the inputs and outputs should be extendable to dynamic problems. In that case we expect of course the price vector p to be a time function. The extension can be based on the following.

Assume the global problem:

$$\text{minimise } Q = \sum_{i=1}^{N} \int_{0}^{t_f} f_{oi}(x_i, m_i, u_i)\,\mathrm{d}t \tag{24}$$

subject to

$\dot{x}_i = f_i(x_i, m_i, u_i), \quad i \in \overline{1, N}$ — state equations

$y_i = g_i(x_i, m_i, u_i), \quad i \in \overline{1, N}$ — output equations

$u = Hy$ — interconnection equation

with $\boldsymbol{x}(0)$ given, $\boldsymbol{x}(t_f)$ free or specified.

Assume also that the problem (24) is being solved using the Lagrangian

$$L = \sum_{i=1}^{N} \int_{0}^{t_f} f_{oi}(x_i, m_i, u_i)\,\mathrm{d}t + \int_{0}^{t_f} \langle p, u - Hy \rangle\,\mathrm{d}t \tag{25}$$

where the notation $\langle p, u—Hy \rangle$ has the same meaning as in (5) in its final--dimensional case.

Solving the global problem we would get:

$\hat{x}_1, i \in \overline{1, N}$ optimal state trajectories

$\hat{m}_i, i \in \overline{1, N}$ optimal controls

$\hat{u}_i, i \in \overline{1, N}$ optimal inputs

$\hat{y}_i, i \in \overline{1, N}$ optimal outputs

$\hat{p}$ optimal prices

The Lagrangian (25) can be split into additive parts, thus allowing to form a kind of local problems

$$\text{minimise } Q_i = \int_0^{t'_f} [f_{oi}(x_i, m_i, u_i) + \langle \hat{p}_i, u_i \rangle - \langle \hat{q}_i, y_i \rangle] \, dt \tag{26}$$

where $y_i = g_i(x_i, m_i, u_i)$ and optimisation is subject to

$\dot{x}_i = f_i(x_i, m_i, u_i)$

$x_i(0)$ given, $x_i(t_f)$ free or specified.

In (26) the price vectors p_i and q_i derived from p by formula (6) had to be set at optimal values obtained by solving the global problem; thanks to that the solutions to (26) would be strictly optimal.

There is little sense, however, in solving (26) if the global problem was solved before, because the global solution provides not only $\hat{p}$ but $\hat{x}, \hat{m}$ as well for the whole system. To make the thing priactical let us take the following steps.

(i) shorten the horizon for the local problems, so that (26) becomes

$$\text{minimise } Q_i = \int_0^{t'_f} [f_{oi}(x_i, m_i, u_i) + \langle \hat{p}_i, u_i \rangle - \langle \hat{q}_i, y_i \rangle] \, dt \tag{27}$$

with $x_i(0)$ given as before, but $x_i(t'_f) = \hat{x}_i(t'_f)$ that is must be specified by the long-horizon solution:

(ii) introduce feedback at local level, consisting in repeating (27) at some intervals $T_1 < t'_f$ and using the actual value of state $x_{*i}(kT_1)$ at each repetition of the optimisation problem.

Step (ii) in particular brings a new quality: we now have a truly on-line control structure and can expect, in appropriate cases, to get results better than the accuracy of the models. We also have now a gain from decomposition of the global problem into local ones: the repeated solutions concern problems (27), of low dimensionality and short horizon.

The feedback just introduced cannot compensate for the errors done by the coordination level in setting the prices $\hat{p}$. Another repetitive feedback can be introduced to overcome this shortage, for example bringing to the coordinator actual values x_{*i} at times t'_f, $2\,t'_f$, ... and asking the global problem to be re-solved for each new initial value. The structure of control proposed so far is presented in Fig. 4.

We should very well note that feeding back the actual values of state achieved makes sense if the models used in computation differ from reality, for example because of disturbances. Otherwise the actual state is exactly equal to what the models have predicted and the feedback information is irrelevant.

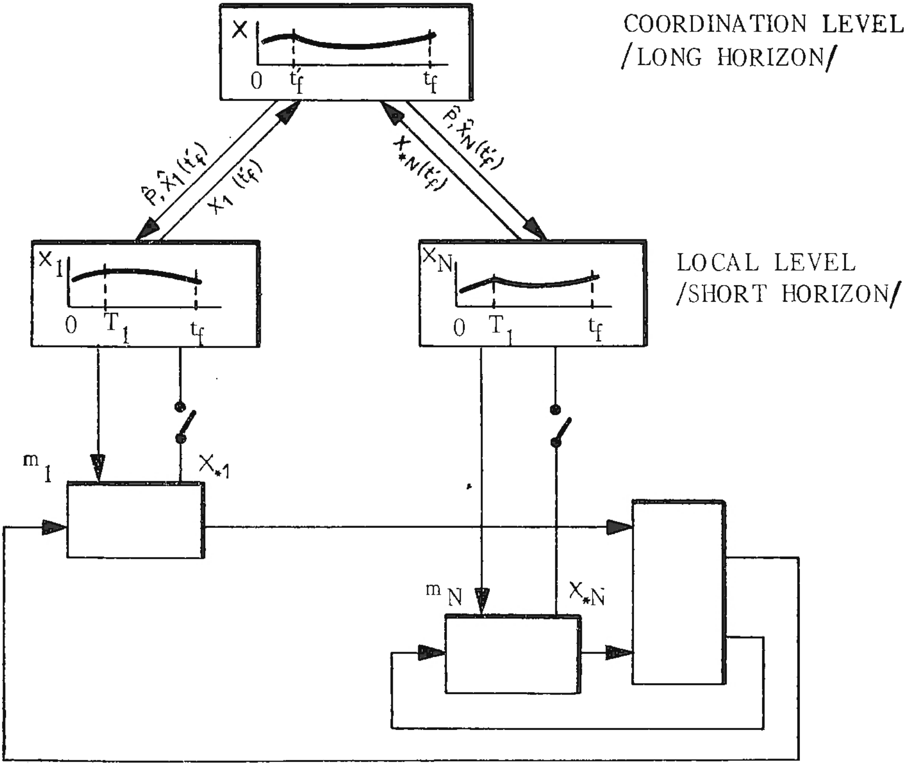

Fig. 4

A doubt may exist whether the feedback to the coordinator shown in Fig. 4 makes sense, because the lower level problems have to achieve $x_i(t'_f) = \hat{x}_i(t'_f)$ as their goal and already use feedback to secure it. It should be remembered, however, that the model-based target value $x_i(t'_f)$ is not optimal for the real system and asking the local decision making to achieve exactly $x_{*i}(t'_f) = \hat{x}_i(t'_f)$ may be not advisable or even not feasible.

In the described structure of on-line dynamic coordination we made no use of the possibility of having a simplified model in the global problem, solved at the coordination level at times 0, t'_f, $2\,t'_f$ etc.

The global problem may be simplified for at least two reasons: the solution of the full problem may be too expensive to be done, and the data on the real system, in particular prediction of disturbances, may be too inaccurate to justify an exact model computation.

Simplification may concern dimension of state vector (indroduce aggregated x^c instead of x), dimension of control vector (m^c instead of m) and dimensions of inputs and outputs ($u^c = H^c y^c$ instead of $u = Hy$).

The global problem Lagrangian will now be, instead of (25):

$$L = \sum_{i=1}^{N} \int_0^{t_f} f_{oi}^c(x_i^c, m_i^c, u_i^c)\,\mathrm{d}t + \int_0^{t_f} \langle p^c, u^c - H^c y^c \rangle\,\mathrm{d}t \tag{28}$$

The simplified solution will yield optimal state trajectory $\hat{x}^c = (\hat{x}_1^c, \hat{x}_2^c, \dots, \hat{x}_N^c)$ and optimal price function $\hat{p}^c$. The linking of those values to the local problems cannot be done directly, because they cont nue to have the full vectors x_i, u_i and y_i.

We have to change the previous requirement $x_i(t_f') = \hat{x}_i(t_f')$ into a new one

$$\gamma_i(x_i(t_f')) = \hat{x}_i^c(t_f') \tag{29}$$

which incidentally is a more flexible constraint, and we also have to generate a full price vector $\hat{p}$:

$$\hat{p} = R\hat{p}^c \tag{30}$$

where R is an appropriate "price proportion matrix". The prices composing the aggregated p^c may be termed "group prices".

We should note that functions γ_i and matrix R have to be appropriately chosen. The choice may be made by model considerations, but even with the best possible choice optimality of overall solution will be affected, except for some special cases.

The system interconnections in problem (24) were stiff, that is an output was assumed to be connected to an input in a permanent way. The dynamic problem formulation gives an opportunity to consider another type of interconnection, a "soft" constraint of integral type:

$$\int_{kt_b}^{(k+1)t_b} (u_{ij} - y_{lr})\,\mathrm{d}t = 0$$

which corresponds to taking input u_{ij} from a store, with an output y_{lr} connected to the same store and causing its filling. Asking for the integral over $[kt_b,(k+1)t_b']$ to be zero means that supply and drain have to be in balance over each balancing period t_b.

A store may be supplied by several outputs and drained by more than one subsystem input. There may also be many stores, for example for different products. If we assume the balancing period for all of them the integral constraint becomes

$$\int_{kt_b}^{(k+1)t_b} (Pu_w - My_w)\,\mathrm{d}t = 0 \tag{31}$$

where u_w, y_w are parts of u, y connected to the stores (the stiffly interconnected parts will be termed u_s, y_s). Matrices P, M show the way by which u_w, y_w

are connected to various stores. The number of stores is of course $\dim My_w = \dim Pu_w$. A state vector w of the inventories can also be introduced

$$w(kt_b+t) = w(kt_b) + \int_{kt_b}^{kt_b+t} (Pu_w - My_w)\,dt \tag{32}$$

With both stiff and soft interconnections present in the system, the global Lagrangian becomes

$$L = \sum_{i=1}^{N} \int_0^{t_f} f_{oi}(x_i, m_i, u_i)\,dt + \int_0^{t_f} \langle p, u_s - Hy_s \rangle\,dt +$$

$$+ \sum_{k=0}^{k=\frac{t_f}{t_b}-1} \langle \psi^k, \int_{kt_b}^{(k+1)t_b} (Pu_w - My_w)\,dt \rangle \tag{33}$$

and we of course continue to have

$\dot{x}_i = f_i(x_i, m_i, u_i), \quad i \in \overline{1, N}$

$y_i = g_i(x_i, m_i, u_i), \quad i \in \overline{1, N}$

$x_i(0)$ given, $x_i(t_f)$ free or specified, $i \in \overline{1, N}$

In comparison with the Lagrangian (15) a new term has appeared in (33), reflecting the constraint (31). Note that prices ψ^k associated with this integral constraint are constant over periods t_b. Note also, that if t_b will tend to zero, the integral constraint gets similar to the stiff one and the stepwise changing ψ will change continuously, like p does.

With two kinds of interconnections the local problems also change correspondingly and they become

$$\text{minimise } Q_i = \int_0^{t_f} [f_{oi}(x_i, m_i, u_i) + \langle \hat{p}_i, u_{si} \rangle - \langle \hat{q}_i, y_{si} \rangle]\,dt +$$

$$+ \sum_{k=0}^{k=\frac{t_f}{t_b}-1} \langle \psi^k, \int_0^{t_b} (P_i u_{wi} - M_i y_{wi})\,dt \rangle \tag{34}$$

where $y_{si} = g_{si}(x_i, m_i, u_i)$, $y_{wi} = g_{wi}(x_1, m_i, u_i)$ and optimisation is subject to

$\dot{x}_i = f_i(x_i, m_i, u_i)$

$x_i(0)$ given, $x_i(t_f)$ free or specified.

A new quality has appeared in problem (34) in comparison with (26): the inputs u_{wi} taken from the stores are now free control variables and can be shaped by the local decision maker, who previously had only m_i in his hand. The local decisions will be under the influence of prices $\hat{p}$ and $\hat{\psi} = (\hat{\psi}^0, \hat{\psi}^1, \ldots)$, where both $\hat{p}$ and $\hat{\psi}$ have to be set by the solution of the global problem.

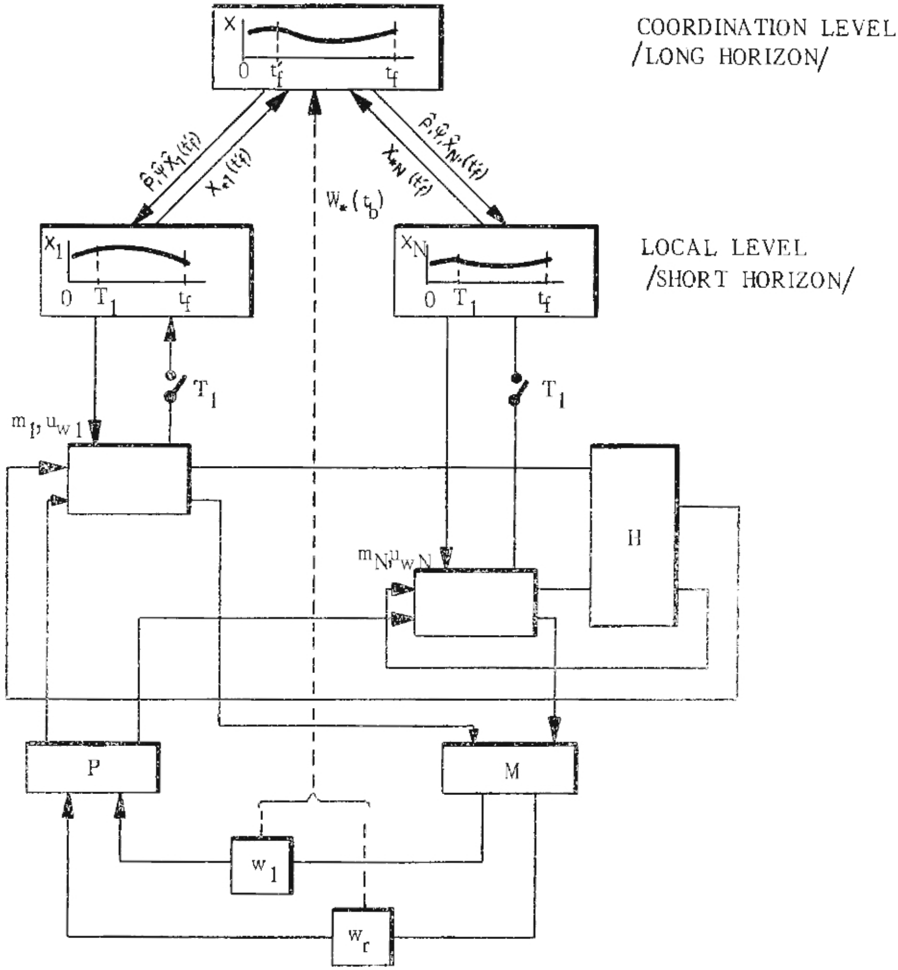

Fig. 5

The local problem (34) gas no practical meaning yet; it will make sense when we introduce local feedback and shortem the horizon, like it was in the previous stiff-interconnection case.

We shall omit the details and show it only as a control scheme, Fig. 5.

Thinking about how to improve action of the coordinator we made previously a proposal to feed actual $x_*(t'_f)$ to this level. We have now additional state variables, the inventories w defined by (32). If the price $\hat{\psi}^k$ is wrong, the stores will not balance over $[kt_b, (k+1)t_b]$. It is almost abvious that we can catch-up by influencing the price for the next period $\hat{\psi}^{k+1}$ and that we should condition the change on the difference $\hat{w}((k+1)t_b)-\hat{w}_*((k+1)t_b)$, where $w_*(.)$ is a value measured in the real system. This kind of feedback is also shown in Fig. 5.

6. CONCLUSIONS

Prices are a suitable and convenient instrument of coordination in various control structures.

In solving decomposable problems of mathematical programming prices are identical with lagrangian multipliers and applicable to both static and dynamic problems.

Iterative procedures of price adjustment in control structures using feedback from real system behaviour are possible and advisable, with results reaching beyond the exactness of the models, but in principle for steady-state or static problems only.

Sequential procedures are needed for on-line dynamic optimisation. Suitable structures using prices as coordination instruments are developed and apply to systems with stiff as well as with integral-type interconnections.

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SUMMARY

Coordination of complex systems activity by means of prices has been known to practice for a long time. Later on, lagrangian multipliers (identifiable with prices) have been used in coordination of decomposed mathematical programming problems. Almost in parellel, prices have appeared as one of the possible coordination instruments for on-line control structures, using feedback from the reality. The paper presents a survey or cross-section of all those applications of price instruments: the "tatônnement" procedures and open loop control for static and dynamic systems; price coordination with feedback to the coordinator; price coordination with feedback to the lower-level decision maker (partly or fully decentralised control); dynamic coordination by price method.

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