POLISH ACADEMY OF SCIENCES
SYSTEMS RESEARCH INSTITUTE

STRATEGIC REGIONAL POLICY

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PART II

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Paradigms, Methods, Issues and Case Studies

A. Straszak and J.W. Owsiński
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PART II

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VII. SOFTWARE

INCOMPLETE MODELLING APPROACH TO ANALYZING ECONOMIC STRUCTURAL CHANGES

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1. Introduction

The chief measure of the quality of a mathematical model is its degree of correspondence to the modelled object, i.e. how accurately the model reflects all the features which essentially determine the behavior of the object. There are two reasons why a model may not be considered acceptable by the users. Firstly, the mathematical description may have been made in the absence of adequate information. Secondly, it might not be possible to formalize all of the essential features of the object by mathematical means, or these features may not be known at all. Therefore we may call a mathematical model containing a formal description (with an acceptable level of accuracy) *of not all* the essential features of the object under consideration an *incomplete mathematical model*.

It is clear that any developer of mathematical models wants to make them as complete as possible. And most of the mathematical tools developed to analyze these models are based on the assumption that they are complete. Nevertheless, in practice this assumption of completeness is often invalid, which means that such models cannot be used *to generate a forecast* or *to find an optimal solution*. The user of an incomplete mathematical model should try to improve it by increasing the level of completeness; otherwise, he/she should restate the problem to be solved to avoid contradictions which arise from the incompleteness of the model.

This paper is concerned with the correct use of incomplete mathematical models.

2. Statement of Problems for Incomplete Mathematical Models

We define an incomplete mathematical model as a set of formalized descriptions which have been made with an acceptable level of accuracy, but which do not reflect all essential features (such as links, constraints, etc.) influencing the behavior of the modeled object.*

To obtain results of practical value it is necessary to take into consideration both formalized and nonformalized features of the object. The formalized features may be presented in the form of an incomplete mathematical model, but for the latter we must engage the *model* user in the process of decisionmaking. The main aim of this approach is to *combine the ability of the user to extract acceptable states of the model from the set of feasible solutions with the computer's ability to generate this set for a given incomplete model.*

Two definitions should be given here. A state of the model is *feasible* if it satisfies all formalized constraints included in the description of the incomplete model; and a state of the model is *acceptable* if the user has no objection to this state. It is obvious that the set of feasible states of the model includes the set of

* An incomplete model may be augmented by including new variables, constraints and so on, but not by changing the existing ones, otherwise it should be considered a different incomplete model.

acceptable solutions, but not vice versa. As there is no formalized way to extract acceptable solutions from the set of feasible ones, the decisionmaker cannot use the computer to verify sufficient conditions of acceptability. He/she can only check (by means of formal tools) whether the necessary conditions of acceptability are valid, i.e. whether feasible solutions exist or not. This is why no optimization or forecasting problems can be solved using incomplete mathematical models. These models may help us to find out 'what will not happen', but not 'what will happen'.

The following scheme is suggested for seeking acceptable solutions, combining the abilities of human decisionmaking and formal computer analysis. As a first step the computer generates the set of feasible solutions for a given incomplete model, or determines that such solutions do not exist. Because it is practically impossible for the user to manipulate a whole set of solutions, the decisionmaker analyzes only one of them. If the solution is not acceptable, the user introduces additional constraints into the incomplete model, trying to eliminate unacceptable features of the solution. The computer corrects the feasible set of solutions in accordance with these new constraints and generates a new solution, the acceptability of which is to be tested by the user. The process is repeated until an acceptable solution is found.

This scheme is not concrete enough for one to make conclusions about its convergency from a purely formal viewpoint. In practice a decisionmaker will usually find a solution. The existence of the solution (or set of solutions) depends on the problem, but is not a property of the described scheme.

In spite of the theoretical simplicity of the approach, its practical use has been found to be difficult. In the next sections we will discuss in detail the problems that arize in the case of finite-dimensional mathematical models, describe the software for linear flow models, and give an example of the practical application of the approach.

3. The Case of Finite-Dimensional Models

Let a state of the mathematical model considered be described by an n-dimensional vector x, the components of which are $x_1, x_2, \cdots, x_n$. We will assume that the relations

$$y_s(x) \begin{bmatrix} \geq \\ = \\ \leq \end{bmatrix} 0, \quad s = [1,m], \tag{1}$$

are expressions of the only essential features of the modelled object which can be formalized at an acceptable level of accuracy. We will also assume that all $y_s(x)$ are convex functions of components of x defined for a nonempty domain $\Omega \subset E^n$.

Suppose now that the set of all x satisfying the system (1) is not empty, i.e. that there exists at least one x^* which is a feasible state of the model. The decisionmaker verifies whether x^* is an acceptable solution as well. If it is found to be acceptable, the procedure is finished. Otherwise, the user can insert additional constraints

$$g_t(x) \begin{bmatrix} \geq \\ = \\ \leq \end{bmatrix} 0, \quad t = [1,l], \tag{2}$$

where functions $g_t(x)$ have the same properties as the functions $y_s(x)$.

The main purpose of these new constraints is to convert the feasible solution x^* to an acceptable solution. The difference between functions $g_t(x)$ and $y_s(x)$ is that the first ones may be unknown to the user before analysis of the feasible solution x^*, whereas $y_s(x)$ are known *a priori*. Together systems (1) and (2) are the *conditions of feasibility*.

This correcting procedure may be repeated several times until an acceptable solution is found. At each step new constraints are included in the system (1)-(2) which, generally speaking, make tbe domain Ω more narrow.

A difficulty which may arise at some step of the procedure is the infeasibility of the system (1)-(2). It is suggested that the following special procedure is used to avoid this situation. Let the set of constraints

$$g_{t^r}(x) \begin{bmatrix} \geq \\ = \\ \leq \end{bmatrix} 0, \quad t^r = [1, l^r] \tag{3}$$

cause the state of infeasibility. This means that the system (1)-(2)-(3) has an internal contradiction and all the conditions cannot be satisfied simultaneously. In this case it is possible to remove conditions (3) from the set of necessary conditions of the model and to start considering them only as 'desirable' conditions. But, on the other hand, this 'desirability' means that these constraints should be satisfied as exactly as possible. We can use the lack of uniqueness of the solution of the system (1)-(2) by choosing that solution which satisfies the new constraints (3) in the best way.

We may, for example, introduce a metric

$$\rho(x) = \max_{t^r \in [1, l^r]} \text{abs} \frac{g_{t^r} - g^*_{t^r}}{N^*_{t^r}}, \tag{4}$$

where $g^*_{t^r}$ are *reference values** for the 'desirable' constraints and $N^*_{t^r}$ are suitable normalizations.

The metric (4) has a disadvantage, namely that the minimization of $\rho(x)$ may not uniquely define all components of x. To avoid this we may repeat the minimization several times, fixing all the components of x which were defined uniquely during the previous steps. Technical details of this procedure, called *sequential fixation*, as well as choosing the reference values and normalization, will be discussed in the next section.

The last problem to be mentioned here is the possible infeasibility of the original system (1). If this is the case, parametric analysis is recommended to reconstruct the initial description of the incomplete model. A number of suitable algorithms and methods are known. One of them, called the *compact modelling approach*, was sucessfully tested in practice [Umnov, 1984].

4. Linear Flow Models

The ideas described in the previous sections are too general for a conclusion to be made about their practical effectiveness. Therefore it seems reasonable to move to a more concrete case: that of standard linear flow models.

Let us consider a mathematical model consisting of a network consisting of N nodes which may be linked by means of K component flows. Each of the nodes may be a source, a sink, or both. Generally speaking, the graph of the network may not be connected.

* We use the term 'reference value' following Wierzbicki at al. [1984], because of the technical similarity, but the described approach is opposite to optimization in general (and to multiobjective optimization in particular) owing to the main assumption about the incompleteness of the considered mathematical model. The reference values are formal parameters of the procedure and have no practical interpretation.

Let the value of the flow from the ith node to the jth of the kth type be x_{ij}^k. A state of the model is described by the set of variables $\{ x_{ij}^k, i,j = [1,N], k = [1,K] \}$.* For the convenience of the decisionmaker additional variables are introduced which make it possible to operate with the sums of the original variables over different groups of indices. For example, the additional variable S_{i+}^k is defined as

$$S_{i+}^k = \sum_{j=1}^{N} p_{ij}^k x_{ij}^k ,$$

where p_{ij}^k are coefficients permitting summation of the different kinds of flows in common units. Variables S_{+j}^k, S_{ij}^+, S_{++}^k, S_{i+}^+, S_{+j}^+, S_{++}^+ are defined in an analogous way.

The conditions of feasibility (1) are described in terms of a system of constraints, each of which is an equality or inequality imposed on both absolute and relative values of the variables. The decisionmaker may use the constraints

$$\begin{aligned} \underline{a}_{ij}^k \le x_{ij}^k \le \overline{a}_{ij}^k \\ A_j^k \ge S_{i+}^k \\ x_{ij}^k = b_{ij}^k S_{+j}^+, \end{aligned} \tag{5}$$

and the like. The values of the parameters $\underline{a}_{ij}^k$, $\overline{a}_{ij}^k$, A_j^k, b_{ij}^k, $\cdots$ are to be defined by the user.

To simplify the procedure of decisionmaking, a special subset of the 'soft' constraints (3) was used for the linear flow model. These constraints are to be equalities defining values of the primary variables x_{ij}^k. This means that the metric (4) should have the following form :

*** Here we give a short description of the '*fma.* 12'-software system developed by the Regional Issues Group of IIASA in 1983, Lenko [1985].**

$$\rho(x) = \max_{i,j,k} w_{ij}^k \,\text{abs}\, \frac{x_{ij}^k - x_{ij}^{k*}}{x_{ij}^{k*}}, \tag{6}$$

where nonnegative numbers w_{ij}^k are weight coefficients and x_{ij}^{k*} are the components of the reference point expressed in terms of primary variables.

The procedure of sequential fixation is essential here because the metric (6) may not define uniquely all components of the vector $\bar{x}$, which is the minimum point for the function (6). For each step of the procedure all the components of x which have nonzero dual values are fixed. The procedure is finished when all the components have been fixed or the minimum of (6) becomes zero. The obtained sequence of optimal values ρ $\{ \rho_1, \rho_2, \cdots, \rho_P \}$ may be very useful for the decisionmaker because they rank the set of components of vector x, measuring the minimal relative change necessary to transfer the reference point x^* to a feasible solution.

The importance of sequential fixation is also demonstrated by the fact that in the case of a complex system (1) the maximum element from the set $\{ \rho_t, t = [1,P] \}$ may not give the correct description of model properties. For example, Figure 1 presents the dependences of the maximal ρ and an average $\bar{\rho}$ on the value of a parameter of the model described in Umnov [1984]. The average $\bar{\rho}$ was calculated using

$$\bar{\rho} = \sum_{t=1}^{P} \alpha_t \rho_t,$$

where α_t is the ratio of the sum of the flows fixed on the tth step of the sequential fixation to the sum of all the flows, and P is the number of steps of the procedure.

Finally, it should be noted that the weight coefficients w^k_{ij} may be used for the following purposes. Firstly, the decisionmaker can give zero weight to those flows which do not exist or are zero at the reference point. Sometimes this trick permits one to avoid an infeasibility *a priori*. Secondly, using very large weights, it is possible to find maximum or minimum values of the corresponding components of vector x. The decisionmaker should be careful to have maximum or minimum values for these components only at the reference point. If the decisionmaker introduces simultaneously a set of criteria and their trends are contradictory, then, as can be easily checked, a semi-effective equilibrium on the Pareto set is achieved.

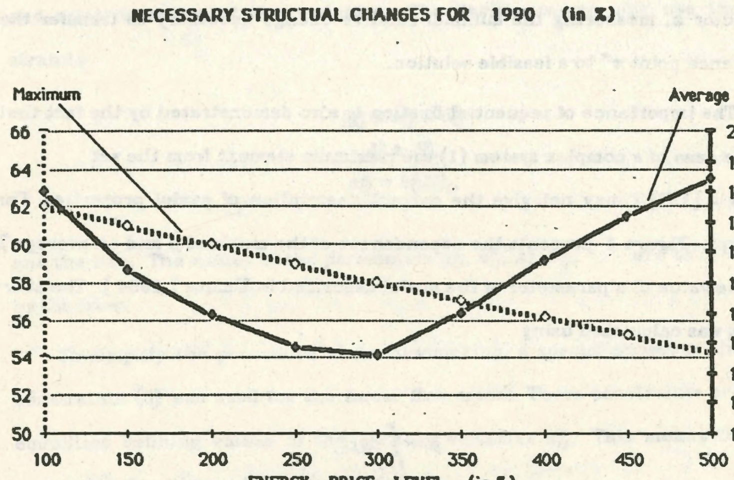

Figure 1.

5. Analysis of the Dynamics of the Energy Production-Consumption Structure for CMEA Countries

The approach described was applied to investigate trends of development of the energy production and consumption structures of the member countries of the Council for Mutual Economic Assistance (CMEA) up to the year 2000. The basic, incomplete model was developed by the Energy systems Group of IIASA in 1983/84; see, for example, Golovin [1985].

The main purpose of this investigation was to analyze the feasibility of different versions of consumption structures and evaluations of the potential growth of energy production. The following were taken into consideration:

- ranges of consumption levels consistent with the planned rates of general economic growth;
- ranges of possible capacities for energy production;
- the requirement to achieve the target levels with the minimal structural changes in energetics.

The first two conditions are the conditions of feasibility. The third condition is an informal definition of metric (6). The *reference state* of the model is the initial situation. Roughly speaking, we would like to change nothing to achieve the desired targets.

In terms of the linear flow model the problem may be formulated as follows. We have a system of eight nodes (Table 1) linked by a set of four component

flows (Table 2).

Identifier	Country
BG	**Bulgaria**
HU	**Hungary**
GR	**GDR**
PL	**Poland**
RO	**Romania**
SU	**USSR**
CS	**Czechoslovakia**
RW	**Rest of the World (as a supplier-consumer for CMEA)**

Table 1.

No.	Energy product	Unit of measurement	Coefficient of equivalence
1	**Coal**	**mill. tce**	**1.000**
2	**Primary Electricity**	**bill. kWth**	**0.326**
3	**Crude Oil**	**mill. tons**	**1.454**
4	**Natural Gas**	**bill. cu. m.**	**1.188**

Table 2.

The state of the production-consumption market for CMEA countries in 1980 was taken as the initial state.

The necessary conditions for feasibility were defined not only for the final point (year 2000) but also for intermediate points : 1985, 1990 and 1995. These conditions are inequalities for absolute and relative values of production-

consumption volumes for different countries and different kinds of products (Tables 3, 4, 5 and 6). The hypothesis about the dynamics of the energy potential for CMEA countries predicts moderate growth of the coal industry, stabilization of crude oil production, and intensive development of both nuclear energy and natural gas production. The sources of information used to evaluate the potential volumes of energy production were the World Energy Conference [1983], Wilson [1983], the British Institutes Joint Energy Policy Programme [1983], Stern [1982], and the official statistical CMEA reports [1982, 1983]. Because of the essential differences between the forecast levels of energy consumption, two independent scenarios were considered. The first, called 'high consumption' scenario (Table 4), suggests that the planned 3% economic growth will be provided by an energy elasticity (relative to GNP) for the USSR ranging from 0.85 in 1985 to 0.65 in 2000, and for the other CMEA countries from 0.75 to 0.50, respectively. The 'low consumption' scenario (Table 5) is based on the assumption that the energy elasticity ranges from 0.50 to 0.25 for the USSR and from 0.30 to 0.10 for the other CMEA countries.

Table 6 contains the description of three possible structures of energy consumption. Structure A corresponds to the state just after 1980 and permits relatively narrow variations. Structure C differs essentially from A. The main differences are: a reduction in the share of crude oil and increases in the shares of primary electricity and natural gas. The coal dynamics depend on the policy of the individual country, but the average share is slightly decreased. Structure B is an intermediate variant between A and C.

Exporter	Energy Product	Reachable maximum levels of production			
		1985	1990	1995	2000
BG	Coal	17.2	18.0	19.0	20.0
	Electr.	13.8	19.4	35.0	54.0
	Oil	0.2	0.2	0.2	0.3
	Gas	0.2	0.2	0.2	0.2
HU	Coal	11.0	11.0	13.0	14.0
	Electr.	0.13	9.1	22.0	36.0
	Oil	2.0	1.8	1.6	1.5
	Gas	6.5	7.5	9.0	6.0
GR	Coal	80.0	80.0	80.0	80.0
	Electr.	14.6	20.6	39.0	58.0
	Oil	.	.	.	.
	Gas	2.8	2.8	2.9	3.0
PL	Coal	180.0	200.0	210.0	220.0
	Electr.	2.5	6.6	18.0	36.0
	Oil	0.3	0.2	0.1	0.3
	Gas	6.5	7.5	9.0	6.0
RO	Coal	22.0	30.0	40.0	55.0
	Electr.	12.5	16.6	23.0	33.0
	Oil	11.5	11.0	10.5	10.0
	Gas	30.0	30.0	30.0	33.0
SU	Coal	540.0	590.0	660.0	780.0
	Electr.	440.0	705.0	940.0	1200.0
	Oil	630.0	640.0	650.0	630.0
	Gas	630.0	780.0	880.0	1100.0
CS	Coal	65.0	65.0	65.0	65.0
	Electr.	18.9	23.6	31.0	48.0
	Oil	.	.	.	.
	Gas	0.5	0.5	0.5	0.5

Table 3. Reachable maximum levels of energy production

Importer	Necessary minimum levels of energy consumption			
	1985	1990	1995	2000
BG	57.5	66.0	74.0	80.9
HU	44.0	50.0	57.0	62.0
GR	138.0	145.0	148.0	152.0
PL	200.0	220.0	240.0	260.0
RO	125.0	143.0	161.0	176.0
SU	1985.0	2300.0	2600.0	2900.0
CS	115.5	132.0	149.0	162.0

Table 4. Necessary minimum levels of energy consumption : 'High' scenario (mill. tce)

Importer	Necessary minimum levels of energy consumption			
	1985	1990	1995	2000
BG	57.5	60.0	62.0	63.9
HU	44.0	46.0	48.0	50.0
GR	138.0	144.0	146.0	150.0
PL	200.0	209.0	220.0	230.0
RO	125.0	133.0	137.0	140.0
SU	1985.8	2150.0	2300.0	2400.0
CS	115.5	122.0	127.0	130.0

Table 5. Necessary minimum levels of energy consumption: 'Low' scenario (mill. tce)

Importer	Energy product	Possible structures of energy consumption (in % of total consumption)					
		Variant A		Variant B		Variant C	
		min	max	min	max	min	max
BG	Coal	39	41	36	40	34	35
	Electr.	8	12	10	19	20	24
	Oil	34	39	30	35	20	24
	Gas	10	14	13	19	15	22
HU	Coal	30	33	30	34	33	35
	Electr.	8	12	10	15	20	25
	Oil	30	33	26	30	20	25
	Gas	25	29	24	25	22	27
GR	Coal	62	65	58	52	54	56
	Electr.	3	5	5	10	11	14
	Oil	22	24	20	23	19	22
	Gas	8	10	8	12	9	14
PL	Coal	75	79	65	77	62	65
	Electr.	0.5	2	1	4	3	5
	Oil	11	14	12	15	14	17
	Gas	7	9	9	15	15	19
RO	Coal	21	25	25	30	35	38
	Electr.	3	4	4	5	5	7
	Oil	30	36	26	32	17	21
	Gas	37	40	36	40	35	39
SU	Coal	26	29	25	25	24	27
	Electr.	4	6	6	10	10	13
	Oil	37	40	32	35	25	30
	Gas	26	28	28	33	30	35
CS	Coal	57	60	50	55	45	48
	Electr.	2	5	4	9	10	14
	Oil	23	26	21	25	10	13
	Gas	11	16	16	24	30	36

Table 6. Possible structures of energy consumption for CMEA countries.

6. Analysis of Results

For the model described above two series of calculations have been performed. The first series was performed to investigate the feasibility of different combinations of consumption structures for the 'high' scenario and the second one for the 'low' scenario.

The calculations were made in the following way. As a first step a solution satisfying all necessary conditions of feasibility for 1985 was found, minimizing the 'distance' (6) between the states of 1980 and 1985. In the next step a solution was built which satisfied all constraints for 1990 and minimized the 'distance' between the states of 1985 and 1990, and so on, until the final point 2000 was reached or an infeasibility appeared.

Some additional constraints were introduced during the process. These are a constant or increasing the total consumption of primary electricity, maximization of crude oil exports, and so on. Sequential fixation was used during all calculations.

On the basis of the results obtained we may conclude that the up-to-date evaluations of the energy potential of the CMEA countries do not contradict the planned economic target up to the end of the century. There are enough energy resources not only to provide the 3% economic growth, but also to permit the sale of a considerable amount of energy outside the CMEA. But this can happen only if some changes are made in the structure of the energy consumption.

Structure A (Table 6) will be in contradiction with the plans for economic growth after 1990 for the 'high' scenario or after 1995 for the 'low' scenario. A condition for keeping structure A until the year 2000 is to increase imports of oil after 1995 ('low' scenario) or to increase imports of oil after 1990 and coal after 1995 ('high' scenario). Evaluations of the relevant import levels are given in Tables 7 and 8.

On the other hand, the combination of structures A A B B (for the years 1985, 1990, 1995 and 2000, respectively) would avoid the contradiction and hence, an increase in energy imports for the 'low' scenario. For the 'high' scenario the combination A B E C is found to be necessary. These results are presented in Table 8.

Year	Oil import from RW (mill. tons)	Coal import from RW (mill. tons)
1985	30.9	8.3
1990	1[illegible].0	15.2
1995	1[illegible].4	41.5
2000	2[illegible].7	55.[illegible]

Table 7. Dynamics of imports assuring feasibility for structure A : 'High' scenario

Year	Oil import from RW (mill. tons)
1985	30.9
1990	30.9
1995	3[illegible].7
2000	3[illegible].2

Table 8. Dynamics of imports assuring feasibility for structure A : 'Low' scenario

Finally, we would like to emphasize once again that these solutions may be unacceptable from the viewpoint of the decisionmaker, because the actual solving process has not been finished here. Our purpose here was only to demonstrate all the main principles of *incomplete modelling*, considering both the positive and the negative aspects of the approach.

Variant	Combinations of the structures used				Solution found	Possible alternative
	1985	1990	1995	2000		
1	A	A	A	A	Infeasible after 1990	Increase of imports of oil and coal
2	A	B	B	B	Infeasible after 1995	Increase of imports of oil
3	A	B	B	C	Feasible state	-

Table 9. Results of the analysis: 'High' scenario

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DISCUSSIONS

Paper by K.W. Kim

Discussion participants: K. Polenske, R. Espejo, K. Kim.

It was clarified in the discussion that it is possible to apply the approach outlined to interconnected systems, and that this solely depends upon the availability of appropriate data. With regard to centralization-distribution question it was stated that at the moment of presentation the software systems were still created and run in a centralized manner. The problem of distribution was at the time being solved, both on the theoretical and on the technical levels. The main issue was to provide adequate links in cases when models are run in different locations.

Paper by A. Umnov

Discussion participants: R. Espejo, J. Hołubiec, A. Umnov.

Certain technical and methodological aspects of the software were discussed, and in particular: the model was presented as being manipulated mostly on the output rather than input side, so that it is possible to change a desirable state of the system once a solution is obtained and its rationality is assessed. Furthermore, the constraints to which solutions are subject allow avoiding of not quite uncommon spatial bang-bang solutions, practically infeasible in some situations (e.g. full specialization in foreign trade).

Paper by L. Kruś and J. Sosnowski

No discussion was recorded - main exchange of opinions took place in an informal way during the game playing.

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