Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics Volume II: Applications

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Statistics with fuzzy data - a short overview

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Abstract

Statistical data do not necessarily come from exact measurements. In many practical cases they are imprecise or even vague. Thus, they are both random and fuzzy. In the paper we present basic notions and methods which have been proposed to cope with that kind of data. It is assumed that fuzzy observations are the realizations of a fuzzy random variable. In this paper we focus our attention on statistical methods when data are considered as fuzzy perceptions of original non-fuzzy random data.

Keywords: fuzzy data, fuzzy random variables, fuzzy statistics

1 Introduction

When statistical methods are used for the analysys of data it is usually assumed that these data precisely defined and come from precise measurements. Even if we know that there exist measurements errors we often assume that such errors do not influence the results of statistical analysis. The sources of statistical data may be of different nature. Usually the data are collected from observations of random experiments, i.e. experiments whose performance leads to an outcome which cannot be predicted in advance with one hundred percent sureness. When the knowledge about the nature of these random events, described by a stochastic mathematical model, is available we can use the methods of mathematical statistics and draw conclusions about the source of available data. Uncertainty being

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intrinsic to random outcomes/events is properly described by using the formalism of the mathematical theory of probability, and generally it is attributed to *future* observations of these outcomes/events.

Traditionally, statistical experimental data are described by real numbers or by vectors whose components are real numbers. These numbers are either observed directly as results of measurements (e.g. height and weight of a person) or represent observed counts of certain categories representing labelled events (e.g. a gender of that person). However, in real life applications the results of measurements, as we have already mentioned above, are never precise.

If the measurement error cannot be neglected statistical analysis of interval data is recommended by specialists in metrology. However, when statistical data come from observations made by human beings without using precise measurement devices we need more sophisticated methods for the description of their lack of precision. Therefore, there is a practical need to generalize traditional statistical methods in order to make them applicable for handling imprecise data, e.g. the data described in commonly spoken language (the so-called "linguistic data").

There is a common agreement that uncertainty characterized by randomness shall be described using the language of the theory of probability. Thus, traditional mathematical statistics is sufficient and proper tool for dealing with data generated by random experiments and described by precisely defined numbers. However, there also exist other types of uncertainty which are related to vagueness, imprecision, existence of only partial information about experimental outcomes/events of interest, etc. In contrast to randomness, uncertainties of such types are attributed rather to *current* perceptions/observations.

It has to be noted that the mathematical modelling of all these types of uncertainty which are different from simple randomness is still the subject of controversies. There exist different sources of such uncertainty. For example, when the result of a measurement has been obtained without the usage of any meter (e.g., when we visually evaluate the distance between two points), or when we deal with retrospective data recalled by human beings etc., the statistical data consist of *imprecise perceptions* of actual real values. Thus, the description of such uncertainty in terms of the theory of probability seems to be questionable.

Till now we have considered situations when actual values of measured quantities exist, but they are imprecisely perceived. Hisdal [8] attributes this type of uncertainty to anticipation of errors of observations under nonexact conditions. There exist, however, situations when we have to analyze statistical data that represent *imprecisely defined concepts*. In such cases we could try to collect precise statistical data, e.g., by either asking a respondent to a questionnaire to indicate exactly one choice or coding it numerically. However, it seems to be more prudent, natural and informative to expect imprecise answers to questions pertained to vague notions. If we do so, we may face imprecise statistical data for further analysis. In all considered cases the lack of precision can be appropriately described by using fuzzy sets introduced by Lotfi A. Zadeh (Zadeh [15]).

2 Fuzzy random variables

When fuzzy vagueness or lack of precision is mixed with stochastic randomness, either in the sense that available fuzzy data are supposed to come from the perception of real- or vectorial-valued data generated by a random mechanism, or in the sense of they are directly generated by a random mechanism, a convenient tool to use is that of the notion of a fuzzy random variable. Historically, the first widely accepted definition of the fuzzy random variable was proposed in (Kwakernaak [12],[13]). Kruse [10] proposed an interpretation of this notion, and according to this interpretation a fuzzy random variable $\mathcal{Z}$ may be considered as a fuzzy perception of an unknown true real-valued random variable Z_0 associated with a random experiment, and referred to as 'the original' of $\mathcal{Z}$.

Definition 1 (Kruse and Meyer [11]). *Let $(\Omega, \mathcal{A}, P)$ be a probability space, where Ω is the set of all possible outcomes of the random experiment, $\mathcal{A}$ is a σ-field of subsets of Ω (the set of all possible events of interest), and P is a probability measure associated with $(\Omega, \mathcal{A})$. A mapping $\mathcal{X} : \Omega \to \mathcal{F}_c(\mathbb{R})$, where $\mathcal{F}_c(\mathbb{R})$ is the space of all fuzzy numbers, is called a* ***fuzzy random variable*** *if it satisfies the following properties:*

i) $\{\mathcal{X}_\alpha(\omega) : \alpha \in [0,1]\}$, *where* $\mathcal{X}_\alpha(\omega) = (\mathcal{X}(\omega))_\alpha$ *is a set representation of* $\mathcal{X}(\omega)$ *for all* $\omega \in \Omega$*;*

ii) for each $\alpha \in [0,1]$ *both* $\mathcal{X}_\alpha^L : \Omega \to \mathbb{R}$ *and* $\mathcal{X}_\alpha^U : \Omega \to \mathbb{R}$*, with* $\mathcal{X}_\alpha^L(\omega) = \inf \mathcal{X}_\alpha(\omega)$ *and* $\mathcal{X}_\alpha^U(\omega) = \sup \mathcal{X}_\alpha(\omega)$*, are usual real-valued random variables associated with* $(\Omega, \mathcal{A}, P)$.

The fuzzy random variable defined according to this definition has been introduced to model fuzzy perceptions of existing real-valued values of a traditional random variable (the values of the unknown original). Puri and Ralescu [14] introduced another concept of the fuzzy random variable as a generalization of the concept of random set or set-valued random element (and hence, as a generalization also of the concept of the random variable). According to this definition a fuzzy random variable $\mathcal{Z}$ may be considered as a random element associating with each experimental outcome a value which is intrinsically fuzzy.

Definition 2 (Puri and Ralescu [14]). *Given a probability space* $(\Omega, \mathcal{A}, P)$*, a mapping* $\mathcal{X} : \Omega \to \mathcal{F}_c(\mathbb{R})$ *is said to be a fuzzy random variable (also referred to as random fuzzy set) if for each* $\alpha \in [0,1]$ *the set-valued mapping* $X_\alpha : \Omega \to \mathcal{K}_c(\mathbb{R})$*, where* $\mathcal{K}_c(\mathbb{R})$ *is the class of the nonempty compact intervals and* $X_\alpha(\omega) = (X(\omega))_\alpha$ *for all* $\omega \in \Omega$*, is a compact convex random sets (that is, a Borel-measurable mapping with respect to the Borel* σ*-field generated by the topology associated with the Haussdorf metric on* $\mathcal{K}_c(\mathbb{R})$*).*

Although motivation to introduce fuzzy random variables was different in the approaches by Kwakernaak/Kruse and Meyer and by Puri and Ralescu, one can prove that the notion introduced in the second definition implies the notion introduced in the first one. As a consequence, probabilistic ideas and results for the notion described in the second definition (or for its even more general version) apply to the notion introduced in the first one, and the same happens when we apply them in the statistical analysis of fuzzy random data. However, many probabilistic conclusions, and most of the statistical procedures introduced according to the definition by Kwakernaak/Kruse and Meyer are based on the assumption of having an unknown but existing original, and considering Zadeh's extension principle, so that these conclusions and procedures are not usually applicable to deal with data coming from fuzzy random variables defined by Puri and Ralescu.

3 Basic statistical methods for fuzzy perceptions

When imprecise statistical data correspond to fuzzy perceptions of unobserved/unknown precise (i.e. crisp) statistical data we can treat them as observed values of fuzzy random variables in the sense of Kwakernaak/Kruse and Meyer. In such a case we can analyze imprecise data in terms of probability distributions of their unobserved originals in a similar way as in the case of precise statistical data, i.e. using methods of traditional mathematical statistics. The only difference stems from the fact that having imprecise input information in the form of fuzzy data we cannot precisely evaluate the characteristics of the underlying probability distribution. Therefore, instead of finding precise values of the estimators of the 'parameters' describing the underlying probability distribution (the one of the original), it seems more coherent to find their imprecise fuzzy perceptions.

Assume that we observe a fuzzy random sample $\mathcal{X}_1, \ldots, \mathcal{X}_n$ which is viewed as a fuzzy perception of an unobserved random sample $X_1, \ldots, X_n$ generated by the original random variable X. Let $F(x;\theta)$ be the cumulative probability function of the original random variable X characterized by a crisp parameter $\theta \in \Theta$. Suppose now that an estimator of θ is given by a statistic $\hat{\theta} = \phi(X_1, \ldots, X_n)$. By using Zadeh's extension principle we can consider as a *fuzzy estimator* of θ

based on $\mathcal{X}_1, \ldots, \mathcal{X}_n$ (or, alternatively, as a fuzzy estimator of the induced fuzzy parameter $\vartheta = \theta(\mathcal{X})$ with $\theta(\mathcal{X})(t) = \sup_{X \in \mathcal{E}(\Omega,\mathcal{A},P) \,|\, \theta(X)=t} \inf_{\omega\in\Omega} \mu_{\mathcal{X}(\omega)} X(\omega)$ and $\mathcal{E}(\Omega, \mathcal{A}, P)$ being the class of all possible originals of $\mathcal{X}$) the one associating with each fuzzy sample information $(\widetilde{x}_1, \ldots, \widetilde{x}_n)$ the *fuzzy estimate* $\phi(\widetilde{x}_1, \ldots, \widetilde{x}_n)$ given by

$$\mu_{\phi(\widetilde{x}_1,\ldots,\widetilde{x}_n)}(t) = \\ = \begin{cases} \sup\limits_{(x_1,\ldots,x_n) \,|\, t=\phi(x_1,\ldots,x_n)} \min\{\mu_{\widetilde{x}_1}(x_1), \ldots, \mu_{\widetilde{x}_n}(x_n)\} & \text{if } t \in \mathrm{Im}(\phi(X_1,\ldots,X_n) \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

In many practical cases, when $\phi(x_1, \ldots, x_n)$ has a simple form, the calculation of (1) is straightforward. For example, when $\phi(x_1, \ldots, x_n)$ describes the sample mean expressed as $(x_1 + \ldots + x_n)/n$, the α-levels (α-cuts) of the estimate of its expected value θ are expressed as

$$(\phi(\widetilde{x}_1, \ldots, \widetilde{x}_n))_\alpha = \left[\frac{1}{n}\sum_{i=1}^{n}(\widetilde{x}_i)_L(\alpha), \frac{1}{n}\sum_{i=1}^{n}(\widetilde{x}_i)_U(\alpha)\right], \tag{2}$$

where $(\widetilde{x}_i)_L(\alpha), (\widetilde{x}_i)_U(\alpha)$ are, respectively, the lower and upper limits of the α-cuts of observed fuzzy data. Analogously, whenever the estimator of θ is given by a continuous function or an injective function, the calculation of α-levels becomes also rather simple. In other cases, the limits of the α-levels should often be found by solving nonlinear mathematical programming problems defined previously. Similar approach may be applied when we try to construct fuzzy versions of other statistics such as e.g. the confidence intervals of the unknown parameter θ.

4 Statistical tests for fuzzy perceptions

Testing statistical hypotheses is the second main branch of mathematical statistics. Tests of statistical hypotheses have to be applied if we want to make decisions based on the analysis of random data. When our decisions depend on the values of the parameters of probability distributions that describe observed statistical data we use parametric statistical methods. In such a case we test statistical hypotheses about the values of the parameters of probability distributions utilizing a well known equivalence between the set of values of the considered probability distribution parameter for which the null hypothesis is accepted and a certain confidence interval for this parameter. Kruse and Meyer [11] have shown that the same equivalence exists in the case of statistical tests with fuzzy data.

Let $\mathcal{X}_1, \ldots, \mathcal{X}_n$ denote a fuzzy sample, i.e. a fuzzy perception of the usual random sample $X_1, \ldots, X_n$, from the population with the distribution P_θ. Let δ be a given number from the interval $(0, 1)$. In (Grzegorzewski [7]) one can find the following definition of the *fuzzy test* for vague data:

Definition 3 *A function $\varphi : \left(\mathcal{F}_c(\mathbb{R})\right)^n \to \mathcal{F}(\{0,1\})$ is called a fuzzy test for the hypothesis H, on the significance level δ, if*

$$\sup_{\alpha \in [0,1]} P\left\{\omega \in \Omega : \varphi_\alpha(\mathcal{X}_1(\omega), \ldots, \mathcal{X}_n(\omega)) \subseteq \{1\} \,|H\right\} \le \delta, \tag{3}$$

where φ_α is the $\alpha-$level set (α-cut) of $\varphi(\mathcal{X}_1, \ldots, \mathcal{X}_n)$.

The fuzzy test defined above can be regarded as a family of classical tests $\{\varphi_\alpha : \alpha \in (0, 1]\}$ for which the significance level is given as the upper bound of type I error for the whole family $\{\varphi_\alpha : \alpha \in (0, 1]\}$.

Grzegorzewski [7] has shown that the membership function of the fuzzy test for the hypothesis H against K is given by

$$\begin{aligned} \mu_\varphi(t) &= \mu_\Pi(\theta_0) I_{\{0\}}(t) + \mu_{\neg\Pi}(\theta_0) I_{\{1\}}(t) \\ &= \mu_\Pi(\theta_0) I_{\{0\}}(t) + (1 - \mu_\Pi(\theta_0)) I_{\{1\}}(t), \qquad t \in \{0, 1\}, \end{aligned} \tag{4}$$

where Π is a fuzzy acceptance region depending on the considered hypotheses. Thus, the fuzzy fuzzy test defined above, contrary to the classical crisp test, does not lead to the binary decision – to accept or to reject the null hypothesis – but to a fuzzy decision.

One may get $\varphi = 1/0 + 0/1$ which indicates that we should accept H, or $\varphi = 0/0 + 1/1$ which means the rejection of H. However, one may also get $\varphi = \mu_0/0 + (1-\mu_0)/1$, where $\mu_0 \in (0, 1)$, which can be interpreted as a degree of conviction that we should accept (μ_0) or reject ($1-\mu_0$) the hypothesis H. Thus, in situation when μ_0 is neither 0 nor 1, a user must decide using other criteria whether to reject or to accept the considered hypothesis. There exist several approaches that are suitable for solving this problem.

One of these approaches which is formulated in the language of the possibility theory has been proposed by Hryniewicz [9] who used the results of Dubois *et al.* [3] who proposed to use statistical confidence intervals of parameters of probability distributions for the construction of possibility distributions of these parameters. According to their approach, the family of two-sided confidence intervals

$$\left[\pi_L\left(x_1, \ldots, x_n; 1 - \delta/2\right), \pi_U\left(x_1, \ldots, x_n; 1 - \delta/2\right)\right], \delta \in (0, 1) \tag{5}$$

forms the *possibility distribution* $\tilde{\vartheta}$ of the estimated value of the unknown parameter ϑ. In a similar way it is possible to construct one-sided possibility distributions based on one-sided nested confidence intervals. Hryniewicz [9] proposed to compare this possibility distribution with a hypothetical value of the tested parameter.

For this purpose he proposed to use the necessity of strict dominance measure (NSD index) defined in (Dubois and Prade [2]) as

$$NSD = Ness\left(\tilde{A} \succ \tilde{B}\right) = 1 - \sup_{x,y;x \leq y} \min\left\{\mu_A\left(x\right), \mu_B\left(y\right)\right\}. \tag{6}$$

for measuring the necessity of the strict dominance relation $\tilde{A} \succ \tilde{B}$, where $\tilde{A}$ and $\tilde{B}$ are fuzzy sets described, respectively, by their membership functions μ_A and μ_B.

Hryniewicz [9] has shown that in the classical case of precise statistical data and precisely defined statistical hypotheses the value of the NSD index is equal to the p-value of the test which is commonly used by statisticians for decision making.

In case of fuzzy data the confidence intervals used for the construction of the possibility distribution of the estimated parameter θ can be replaced by their fuzzy equivalents presented in the previous sections of this article. In his paper Hryniewicz [9] assumes that the value of the significance level of the corresponding statistical test δ is equal to the possibility degree α that defines the respective α-cut of the possibility distribution of $\tilde{\theta}$. He also assumes that in the possibilistic analysis of statistical tests on the significance level δ we should take into account only those possible values of the fuzzy sample whose possibility is not smaller than δ. Thus, the α-cuts of the membership function $\mu_F\left(\theta\right)$ denoted by $\left[\mu_{F,L}^{(\alpha)}, \mu_{F,U}^{(\alpha)}\right]$ are equivalent to the α-cuts of the respective fuzzy confidence intervals on a confidence level $1 - \alpha$. Having the possibility distribution of the test statistic we can use the NSD index for the calculation of the degree on necessity that the considered statistical hypothesis has to be accepted. When we set a critical value for this characteristic we arrive at unequivocal (crisp) decisions. It is worthy to note that this approach has been generalized in [9] to the case of testing imprecisely defined hypothesis using fuzzy statistical data.

5 Statistical methods for genuine fuzzy data

When imprecise statistical data correspond to observed fuzzy-valued data we can treat them as observed values of fuzzy random variables in the sense of Puri and Ralescu. In the literature these data are often treated as categorical/ordinal/interval-valued ones. It should be emphasized that the model given by fuzzy random variables allows us to describe and handle these data in a more expressive scale and way (in contrast to just ranking or stating simply the interval support of the values). Thus, many statistical developments for real-valued data are based on distances/deviations between values rather than on the diversity of these values. The use of the fuzzy scale allows to consider metrics with a meaning similar to that for the real-valued case (i.e., distinguishing not only the ranks of variable values w.r.t. a certain criterion, but a physical distance between them).

The distance which is often considered in this context is the one stated by Bertoluzza *et al.* [1], so that if $A, B \in \mathcal{F}_c(\mathbb{R})$

$$D_W^{\varphi}(A,B) = \sqrt{\int_{[0,1]} \Big[\int_{[0,1]} [\mathrm{f}_A(\alpha,\lambda) - \mathrm{f}_B(\alpha,\lambda)]^2 \, dW(\lambda) \Big] d\varphi(\alpha)} \tag{7}$$

with $\mathrm{f}_A(\alpha,\lambda) = \lambda \cdot A_U(\alpha) + (1-\lambda) \cdot A_L(\alpha)$, where

- W and φ are normalized weighted measures on $[0,1]$ formalized as probability measures on $([0,1], \mathcal{B}_{[0,1]})$,
- W is associated with a non-degenerate distribution,
- φ is associated with a strictly increasing distribution function on $[0,1]$.

Several developments have been made in connection with both estimation and testing of fuzzy- and real-valued parameters associated with the distribution of a fuzzy random variable in Puri and Ralescu's sense. More information on statistical methods for dealing with genuine fuzzy data can be found in overview papers such as [4] and [6].

However, nearly all of them are related to the problem of the analysis of a *mean value*. Moreover, statistical properties of estimators and tests in such cases can be usually obtained only via bootstrap-type Monte-Carlo simulations (see [5]).

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References

[1] Bertoluzza, C., Corral, N., Salas, A., *On a new class of distances between fuzzy numbers*, Mathware and Soft Computing, vol. 2 (1995), 71–84.

[2] Dubois D., Prade H., *Ranking fuzzy numbers in the setting of possibility theory*, Information Sciences, vol. 30 (1983), 184–244.

[3] Dubois D., Foulloy L., Mauris G., Prade H., *Probability-possibility transformations, triangular fuzzy-sets and probabilistic inequalities*, Proc. of the Ninth International Conference IPMU, Annecy (2002), 1077–1083.

[4] Gil M.A., Hryniewicz O., *Statistics with Imprecise Data*, In: Robert A. Meyers (Ed.): *Encyclopedia of Complexity and Systems Science*. Springer, Heidelberg 2009, 8679–8690.

[5] González-Rodríguez, G., Montenegro, M., Colubi, A., Gil, M.A., *Bootstrap techniques and fuzzy random variables: synergy in hypothesis testing with fuzzy data*, Fuzzy Sets and Systems, vol.157 (2006), 2608–2613.

[6] Gil M.A., Lopez-Diaz M., Ralescu D.A., *Overview on the development of fuzzy random variables*, Fuzzy Sets and Systems, vol.157 (2006), 2546–2557.

[7] Grzegorzewski P., *Testing statistical hypotheses with vague data*, Fuzzy Sets and Systems, vol. 112 (2000), 501–510.

[8] Hisdal E., *Are grades of membership probabilities?*, Fuzzy Sets and Systems, vol.25 (1988), 325–348.

[9] Hryniewicz O., *Possibilistic decisions and fuzzy statistical tests*, Fuzzy Sets and Systems, vol. 157 (2006), 2665–2673.

[10] Kruse R., *The strong law of large numbers for fuzzy random variables*, Information Sciences, vol.28 (1982), 233–241.

[11] Kruse R., Meyer K.D., *Statistics with Vague Data*, D. Riedel Publishing Company, 1987.

[12] Kwakernaak H., *Fuzzy Random Variables*, Part I: *Definitions and Theorems*, Information Science, vol. 15 (1978), 1–15;

[13] Kwakernaak H., *Fuzzy Random Variables*, Part II: *Algorithms and Examples for the Discrete Case*, Information Science, vol. 17 (1979), 253–278.

[14] Puri M.L., Ralescu D.A., *Fuzzy Random Variables*, J. Math. Anal. Appl., vol. 114 (1986), 409–422.

[15] Zadeh L.A. (1965) Fuzzy sets. Information and Control, 8, 338–353.

The papers presented in this Volume 2 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.

It may be viewed as a result of fruitful discussions held during the Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) organized in Warsaw on October 16, 2009 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Centre for Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT – Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bistrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) has been meant to commence a new series of scientific events primarily focused on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Moreover, other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems are discussed.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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