Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics Volume II: Applications

Editors

Krassimir T. Atanassov
Michał Baczyński
Józef Drewniak
Janusz Kacprzyk
Maciej Krawczak
Eulalia Szmidt
Maciej Wygralak
Sławomir Zadrożny

Editors

Krassimir T. Atanassov
Władysław Homenda
Olgierd Hryniewicz
Janusz Kacprzyk
Maciej Krawczak
Zbigniew Nahorski
Eulalia Szmidt
Sławomir Zadrożny

SRI PAS IBS PAN

Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics Volume II: Applications

Systems Research Institute
Polish Academy of Sciences

Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics
Volume II: Applications

Editors

Krassimir T. Atanassov
Władysław Homenda
Olgierd Hryniewicz
Janusz Kacprzyk
Maciej Krawczak
Zbigniew Nahorski
Eulalia Szmidt
Sławomir Zadrożny

IBS PAN **SRI PAS**

Systems Research Institute
Polish Academy of Sciences
Newelska 6, 01-447 Warsaw, Poland
www.ibspan.waw.pl

ISBN 9788389475305

IFSs theory in the modelling of trust and distrust

Anna Stachowiak
Adam Mickiewicz University
Faculty of Mathematics and Computer Science
Umultowska 87, 61-614 Poznań, Poland
aniap@amu.edu.pl

Abstract

In big systems, such as social networks or data integration systems, a matter of trust or reliability is significant when assesing the value of information that one source is receiving from other sources. As it was disscused in a literature, the most natural way to model trust is to model it in a dual and gradual form of trust and distrust degrees. Such duality can be represented in a convienient way by a theory of Attanassov's intuitionistic fuzzy sets (IFSs theory) and this representation is discussed in the present article. Moreover, the article deals with a problem of trust propagation and presents a new way of propagation based on relative scalar cardinality of IFSs.

Keywords: Atanassov's intuitionistic fuzzy sets, IFS, relative scalar cardinality of IFS, trust networks, trust propagation.

1 Introduction

Gathering and integrating information from many outside sources, especially through the Internet, has became a common method of acquiring and enhancing knowledge in today's world. The value of this information depends on a level of reliability of the source that provides the information. This reliability can be objective (e.g. a source has a quality certificate) or subjective (results from a degree of trust that it was granted by others).

Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume II: Applications *(K.T. Atanassow, W. Homenda, O. Hryniewicz, J. Kacprzyk, M. Krawczak, Z. Nahorski E. Szmidt, S. Zadrożny, Eds.), IBS PAN - SRI PAS, Warsaw, 2009.*

The first issue considered in this paper is how to model subjective trust degree in a network in a convienient, flexible and realistic way. The second one is how to propagate this trust along network. The latter question arises from the fact that such networks are usually very big, so it is impossible for a source to express a level of trust in *all* other sources. Instead, we are looking for a mechanism of trust propagation that enables to compute trust in sources not connected directly.

In the following section we recall a definition of a trust network and briefly describe some of the approaches to trust modelling, with special attention to IFS model. Next we introduce a definition of relative scalar cardinality of IFS, and explain how to use it to calculate trust propagation. At the end we present some examples and conclusions.

2 Modelling of trust and distrust

We take under consideration a big network of connected sources, such as social networks (e.g. facebook.com, epinions.com), auctions systems (e.g. ebay.com) or integrating systems. Each source expresses its trust degree $R(x, y)$ in some other sources; a model of such a network is depicted on a Fig.1.

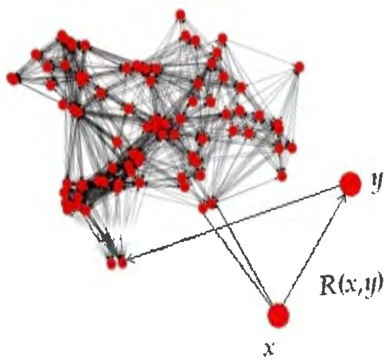

Figure 1: An example of trust network

Trust may be represented in many different ways:

- the simplest is a binar approach: 1 for trust, 0 for lack of trust; this is however very restricted and not realistic;
- a gradual approach based e.g. on fuzzy sets - a trust degree is a value from the interval [0,1], e.g. 0.9 - for 'trust somebody very much' or 0.4 - 'rather

not trust'; however, in this representation we don't have possibility to express a lack of knowledge;

- dual approach which is of special interest of this paper; in the literature we can find a probabilistic approach (e.g. [6]) or gradual approaches ([2], [5], [8], [12]). The main idea of this approaches is that trust should always be accompanied with distrust to avoid limitations of the previous model; among others, this approach can be based on IFS theory.

A trust network where trust is modelled in a dual way, through trust and distrust degrees, is formally defined as follows.

Definition 1 *A trust network is a couple (A, R) such that A is a set of sources and R is an* $A \times A \to [0,1]^2$ *mapping. For every* x *and* y *in A, we have*

$$R(x,y) = (R^+(x,y), R^-(x,y))$$

where

- $R(x,y)$ *is called the trust score of* x *in* y*;*
- $R^+(x,y)$ *is called the trust degree of* x *in* y*;*
- $R^-(x,y)$ *is called the distrust degree of* x *in* y*.*

The trust network can change dynamicly so the above definition applies to a certain moment in time.

Let us now look closer at the representation proposed by De Cock et al. in [2]. It was noticed that a suitable method to model trust in such dual form of both trust and distrust degrees is IFS theory ([1]). An IFS $\mathcal{E}$ is a pair of fuzzy sets:

$$\mathcal{E} = (A^+, A^-),$$

where A^+ is a fuzzy set of elements that belong to $\mathcal{E}$, and A^- is a fuzzy set of elements that do not belong to $\mathcal{E}$. Moreover, in contrast with fuzzy set theory:

$$A^- \subset (A^+)^c$$

where a complement of fuzzy set A is defined as $\forall_{x \in A}\, A^c(x) = 1 - A(x)$. Therefore, a value $1 - A^+(x) - A^-(x)$ reflects uncertainty or hesitation about membership of an element x in IFS $\mathcal{E}$.

Similarly, in a trust network from Definition 1 $R^+(x,y)$ is modelled by fuzzy set A^+, $R^-(x,y)$ is modelled by fuzzy set A^-, and a value $1 - R^+(x,y) - R^-(x,y)$ is called a hesitation or uncertainty degree when specyfying trust and distrust degrees. This model lets us thus express diverse cases like:

- (0, 0) - represents total ignorance (lack of knowledge) about trust and distrust; hesitation is maximal and equals 1;
- (0.5, 0.5) - there is as many evidence to trust someone as not to trust (a situation of conflicting opinions); this is the case with a maximal entropy;
- (0.6, 0) - there is some evidence to trust someone, but we don't have enough knowledge to express it clearly; the missing knowledge (uncertainty, hesitation) equals 0.4;
- (1, 0) and (0, 1) - represents full trust and full distrust, respectively; (as already mentioned in [6], such situations practically never take place, as people are not willing to trust someone without any doubts).

3 Relative scalar cardinality of IFS

To introduce the notion of relative scalar cardinality of IFS let us first recall some necessary definitions.

Definition 2 *(Scalar cardinality of fuzzy set, [11]). A function $\sigma : X \to [0, \infty)$ is a scalar cardinality of fuzzy set iff there exists a non-decreasing function f : $[0, 1] \to [0, 1]$ such that $f(0) = 0$, $f(1) = 1$ and*

$$\forall A \in X : \sigma(A) = \sum_{x \in supp(A)} f(A(x)).$$

Function f is called a cardinality pattern.

Definition 3 *(Scalar cardinality of IFS, [7]). A scalar cardinality of IFS $\mathcal{E} = (A^+, A^-)$ is $\sigma_I : X \to [0, \infty)^2$ defined as follows*

$$\forall \mathcal{E} \in X : \sigma_I(\mathcal{E}) = [\sigma(A^+), \sigma((A^-)^c)].$$

A scalar cardinality of IFS is thus an interval representing minimum and maximum number of elements that the set may contain. The lengh of this interval corresponds to the uncertainty (hesitation) marigin of IFS's elements.

Definition 4 *(Relative scalar cardinality of fuzzy set). A relative scalar cardinality of fuzzy set A (relative to a fuzzy set B) is a number:*

$$\sigma(A|B) = \frac{\sigma(A \cap_t B)}{\sigma(B)}.$$

A relative cardinality is a proportion of elements of fuzzy set A that are also elements of fuzzy set B. It is thus a number from the interval [0, 1]. The intersection $A \cap_{\boldsymbol{t}} B$ is defined as $A(x)\boldsymbol{t}B(x)$, where $\boldsymbol{t}$ is a triangular norm (a t-norm).

Similarily we define a relative scalar cardinality of IFS.

Definition 5 *(Relative scalar cardinality of IFS). A relative scalar cardinality of IFS $\mathcal{E} = (A^+, A^-)$ (relative to IFS $\mathcal{F} = (B^+, B^-)$) is defined as:*

$$\sigma_I(\mathcal{E}|\mathcal{F}) = \frac{\sigma_I(\mathcal{E} \cap_{\boldsymbol{t},\boldsymbol{s}} \mathcal{F})}{\sigma_I(\mathcal{F})}.$$

The intersection $\mathcal{E} \cap_{\boldsymbol{t},\boldsymbol{s}} \mathcal{F}$ is defined as $(A^+ \cap_{\boldsymbol{t}} B^+, A^- \cup_{\boldsymbol{s}} B^-)$, where $\boldsymbol{t}$ is a t-norm and $\boldsymbol{s}$ is a t-conorm.

Relative scalar cadinality of IFS is a proportion of two intervals. To compute it correctly we have to take into account some of the properties of IFS and interval arithmetic. It can be proved (see [10]) that the following theorem is true.

Theorem 1 *A relative scalar cardinality of IFS $\mathcal{E} = (A^+, A^-)$ (relative to IFS $\mathcal{F} = (B^+, B^-)$) is an interval from $[0, 1]$ defined as:*

$$\begin{aligned}\sigma_I(\mathcal{E}|\mathcal{F}) = \big[&min\left(\sigma(A^+|B^+), \sigma(A^+|(B^-)^c)\right), \\ &max\left(\sigma((A^-)^c|(B^-)^c), \sigma((A^-)^c|B^+)\right)\big].\end{aligned}$$

4 Relative scalar cardinality of IFS in trust propagation

The problem of trust propagation can be ilustrated by the simple network shown on Fig. 2.

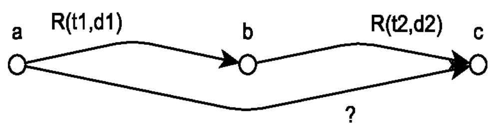

Figure 2: Trust propagation

In its simplest, atomic form, this problem can be formulated as follows: if source a trusts source b to some degree and source b trusts source c to some degree, then how can we estimate a trust degree of a in c? In the paper we mentioned, [2], three propagation operators were defined:

$$Prop1((t1, d1), (t2, d2)) = (\boldsymbol{t}(t1, t2), \boldsymbol{t}(t1, d2))$$
$$Prop2((t1, d1), (t2, d2)) = (\boldsymbol{s}(\boldsymbol{t}(t1, t2), \boldsymbol{t}(d1, d2)), \boldsymbol{s}(\boldsymbol{t}(t1, d2), \boldsymbol{t}(d1, t2)))$$
$$Prop3((t1, d1), (t2, d2)) = (\boldsymbol{t}(t1, t2), \boldsymbol{t}(1 - (d1), d2)).$$

In this paper we would like to consider a more complex case, ilustrated on Fig.3, with more then one intermediate source. This case seems to be more realistic when thinking about real networks with a lot of connections between sources. Now we want to estimate a trust score of x in y knowing all trust scores of x in a_i and a_i in y.

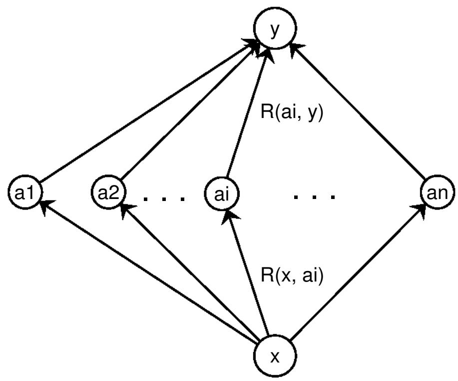

Figure 3: A model of trust network

The method that we propose resembles a behavior of a person that forms his opinion about somebody by asking "How many other people trust this person?", or "How many people that I trust, trust this person?"

Adopting this idea to a network shown on Fig.3 we ask:

"How many sources trust y?" (Q1)

To be more certain about the answer (e.g. not to take into account opinions of fraud sources) we may better ask:

"How many sources trusted by x trust y?" (Q2)

We construct two IFSs: "sources trusted by x" and "sources that trust y". Using notation from Fig.3. we define those sets as:

- $S_x : \{\frac{R(x,a_i)}{a_i} | i = 1, ..., n\}$ - a set of sources $a_1, ..., a_n$ trusted by x to a degree $R(x, a_1), ..., R(x, a_n)$;
- $S_y : \{\frac{R(a_i,y)}{a_i} | i = 1, ..., n\}$ - a set of sources $a_1, ..., a_n$ that trust y to a degree $R(a_1, y), ..., R(a_n, y)$.

Therefore, the question Q2 is in fact a question about relative scalar cardinality of IFS:

$$\sigma_I(S_y|S_x) = [\underline{s}, \overline{s}].$$

It is an interval from $[0, 1]$ showing the (approximate) proportion of number of sources that trust y and are trusted by x to the number of all sources trusted by x. This proportion is given by lower $\underline{s}$ and upper $\overline{s}$ bound. It was shown e.g. in [3] that such interval representation is equivalent to IFS defined as $(\underline{s}, 1 - \overline{s})$. The trust score we are looking for is thus determined as:

$$R(x, y) = (\underline{s}, 1 - \overline{s}). \qquad (*)$$

Some further discussion, examples and properties of proposed method are the subject of the next section.

5 Examples

Let us ilustrate the idea contained in previous section with a couple of the following examples. All of them are based on t-norms:

- minimum: $a\boldsymbol{t}b = a \wedge b$
- algebraic: $a\boldsymbol{t}_a b = a * b$
- Łukasiewicz: $a\boldsymbol{t}_Ł b = 0 \vee (a + b - 1)$

and cardinality patterns:

- identity: $f_{id}(x) = x$
- s-type:

$$f_{p,q}(x) = \begin{cases} 0, & \text{if } x \le p, \\ \frac{x-p}{q-p}, & \text{if } x \in (p, q), \\ 1, & \text{if } x \ge q \end{cases}$$

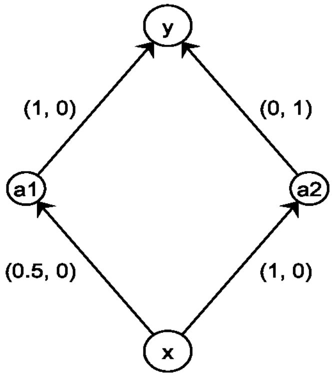

Figure 4: Trust network - Example 1

Example 1

As depicted in Fig. 4, the source x expresses its trust in two sources: a_1 and a_2. It partially trusts in a_1 but has no knowledge about distrust degree, and it fully trusts in a_2. To estimate a trust score of x in y, x examines how many of its trusted neighbours trust y:

$$\sigma_I\left(\left\{\frac{R(a_1,y)}{a_1},\frac{R(a_2,y)}{a_2}\right\} \mid \left\{\frac{R(x,a_1)}{a_1},\frac{R(x,a_2)}{a_2}\right\}\right).$$

.

According to Theorem 1 and (*), a trust score $R(x,y) = (R^+(x,y), R^-(x,y))$ is equal to:

$$R^+(x,y) =$$
$$= min \left(\sigma\left(\left\{\frac{R^+(a_1,y)}{a_1},\frac{R^+(a_2,y)}{a_2}\right\} \mid \left\{\frac{R^+(x,a_1)}{a_1},\frac{R^+(x,a_2)}{a_2}\right\}\right),\right.$$
$$\left.\sigma\left(\left\{\frac{R^+(a_1,y)}{a_1},\frac{R^+(a_2,y)}{a_2}\right\} \mid \left\{\frac{R^-(x,a_1)}{a_1},\frac{R^-(x,a_2)}{a_2}\right\}^c\right)\right)$$

$$R^-(x,y) =$$
$$= 1 - max \left(\sigma\left(\left\{\frac{R^-(a_1,y)}{a_1},\frac{R^-(a_2,y)}{a_2}\right\}^c \mid \left\{\frac{R^-(x,a_1)}{a_1},\frac{R^-(x,a_2)}{a_2}\right\}^c\right),\right.$$
$$\left.\sigma\left(\left\{\frac{R^-(a_1,y)}{a_1},\frac{R^-(a_2,y)}{a_2}\right\}^c \mid \left\{\frac{R^+(x,a_1)}{a_1},\frac{R^+(x,a_2)}{a_2}\right\}\right)\right)$$

We use an algebraic t-norm $\boldsymbol{t}_a$ and a cardinality pattern $f = f_{id}$. Then we get:

$$\begin{aligned} R^+(x,y) &= min\left(\frac{1\boldsymbol{t}_a 0.5 + 0\boldsymbol{t}_a 1}{0.5+1}, \frac{1\boldsymbol{t}_a 1 + 0\boldsymbol{t}_a 1}{1+1}\right) \\ &= min\left(\frac{0.5}{1.5}, \frac{1}{2}\right) = 0.33 \end{aligned}$$

$$\begin{aligned} R^-(x,y) &= 1 - max\left(\frac{1\boldsymbol{t}_a 1 + 0\boldsymbol{t}_a 1}{1+1}, \frac{1\boldsymbol{t}_a 0.5 + 0\boldsymbol{t}_a 1}{0.5+1}\right) \\ &= 1 - max\left(\frac{1}{2}, \frac{0.5}{1.5}\right) = 0.5 \end{aligned}$$

Thus $R(x,y) = (0.33, 0.5)$. A source x is more willing not to trust y as a more trusted source (a_2) doesn't trust it. A marigin of uncertainy was also propageted.

Example 2

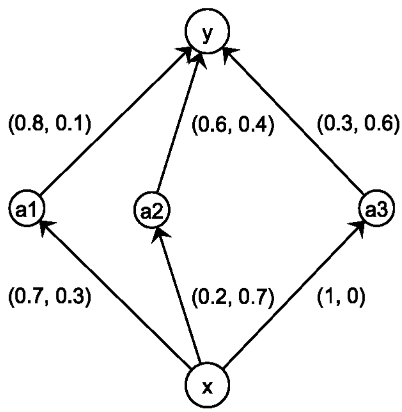

Figure 5: Trust network - Example 2

Let us now consider the trust network depicted in Fig. 5. The source x partially trusts in a_1, fully trusts in a_3 and rather doesn't trust a_2 but with some hesitation. The values of trust score of x in y $R(x,y)$ for different t-norms (minimum, algebraic and Łukasiewicz) and cardinality patterns (f_{id} and $f_{p,q}$) are gathered in a Table 1.

Table 1: Example 2 - A value of trust score $R(x, y)$

	$f = f_{id}$	$f = f_{p,q}$
$\boldsymbol{t} = \wedge$	(0.63, 0.3)	(0.44, 0.44)
$\boldsymbol{t} = \boldsymbol{t}_a$	(0.52, 0.39)	(0.29, 0.52)
$\boldsymbol{t} = \boldsymbol{t}_Ł$	(0.4, 0.47)	(0.22, 0.55)

As it can be seen in this example, by choosing different t-norms we can be more or less restrictive when evaluating final trust score. Moreover, by using an appropriate cardinality pattern we can eliminate an opinion of low-trusted sources (in this case, a_2), for example fraud sources that want to influance our opinion dishonestly. After this elimination a trust degree $R^+(x, y)$ has decreased because we now longer base our opinion on the opinion of source a_2.

Example 3

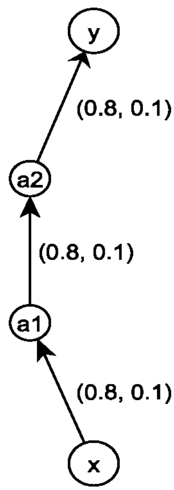

Figure 6: Trust network - Example 3

This simple example ilustrates the case of longer chain of trust dependences shown on Fig.6. Intuitively, the longer the chain, the smaller should be the final trust score. Indeed, when using Łukasiewicz t-norm, we obtain such effect, as shown in Table 2. For comparision, the table contains results also for other t-norms and different cardinality patterns. For $f = f_{p,q}$ information about distrust was not propagated as its value was too small.

Table 2: Example 3 - A value of trust score $R(x, y)$

	$f = f_{id}$	$f = f_{p,q}$
$\boldsymbol{t} = \wedge$	(0.8, 0.0)	(1.0, 0.0)
$\boldsymbol{t} = \boldsymbol{t}_a$	(0.8, 0.1)	(0.64, 0.0)
$\boldsymbol{t} = \boldsymbol{t}_Ł$	(0.74, 0.11)	(0.34, 0.0)

6 Conclusions

The method of trust propagation, proposed in this article, seems to be interesting as it reflects human intuition about trust and trust propagation between people. We concidered few different t-norms and cardinality pattern for calculating relative scalar cardinality of IFSs used in this method. It seems that the simplest t-norm, minimum, is too rigid to give interesting results, and that an algebraic t-norm and Łukasiewicz t-norm are more appropriate. However, this method needs much more attention and analysis.

Acknowledgment

The research has been partially supported by the Ministry of Science and Higher Education Grant N N519 384936.

References

[1] Atanassov K., *Intuitionistic Fuzzy Sets. Theory and Applications*, Physica-Verlag, Heidelberg New York, 1999.

[2] De Cock M., Pinheiro da Silva P., *A Many Valued Representation and Propagation of Trust and Distrust*, in: Lecture Notes in Computer Science, Vol.3849, Springer, Berlin, 2006, pp.108113.

[3] Cornelis C. , Atanassov K. , Kerre E. , *Intuitionistic fuzzy sets and interval-valued fuzzy sets: a critical comparison*, Proc. Third European Conf. on Fuzzy Logic and Technol. (Eusflat03), Zittau, Germany, 2003, pp. 159163.

[4] Deschrijver G., Kerre E. E., *A generalization of operators on intuitionistic fuzzy sets using triangular norms and conorms*, Notes on IFS 8, 2002, pp. 19-27.

[5] Guha R., Kumar R., Raghavan P., Tomkins A., *Propagation of Trust and Distrust*, in: Proc.WWW2004,2004,pp.403412.

[6] Josang A., Knapskog S.J., *A Metric for Trusted Systems*, in:Proc.NIST-NCSC,1998,pp.1629.

[7] Kral P., *Scalar Cardinalities of IF Sets*, Acta Univ. M. Belii Math. No. 12(2005), pp. 67-77

[8] Morzy M., Wierzbicki A. *Eksploracja aukcji internetowych w poszukiwaniu pozytywnej i negatywnej reputacji sprzedawcw*, II Krajowa Konferencja Naukowa Technologie Przetwarzania Danych TPD'2007, 2007, Pozna

[9] Pankowska A., Wygralak M. *General IF-sets with triangular norms and their applications to group decision making*, in: Information Sciences, Volume 176, Issue 18 , (2006) p. 2713-2754.

[10] Stachowiak A. *Trust Propagation - Cardinality-Based Approach*, Proceedings of IMCSIT 2009, p. 125-129.

[11] Wygralak M., *Cardinalities of Fuzzy Sets*, Springer-Verlag, Berlin Heidelberg 2003.

[12] Victor P., Cornelis C., De Cock M., Pinheiro da Silva P., *Gradual trust and distrust in recommender systems*, in: Fuzzy Sets and Systems Volume 160, Issue 10, 16 May 2009, Pages 1367-1382

[13] Zadeh L. A. , *Fuzzy sets*, Inform. and Control **8**, (1965) pp. 338-353.

The papers presented in this Volume 2 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) organized in Warsaw on October 16, 2009 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Centre for Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT – Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bistrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

http://www.ibspan.waw.pl/ifs2009

The Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) has been meant to commence a new series of scientific events primarily focused on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Moreover, other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems are discussed.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

ISBN-13 9788389475305
ISBN 838947530-8
9 788389 475305