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Volume I: Foundations

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On the quantifiers of the intuitionistic fuzzy logic

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Abstract

Some new types of quantifiers are introduced in intuitionistic fuzzy logic and some of their properties are discussed.

Keywords: Intuitionistic fuzzy logic, Quantifier, Topological operator

1 Introduction

Intuitionistic Fuzzy Logic (IFL) is introduced in [1, 2, 3] on the basis of ideas from [6, 7].

In the research on IFL, two real numbers, $\mu(p)$ and $\nu(p)$, are assigned to the proposition p with the following constraint to hold:

$$\mu(p) + \nu(p) \leq 1.$$

They correspond to the "truth degree" and to the "falsity degree" of p. Let this assignment be provided by an evaluation function V, defined over a set of propositions S in such a way that:

$$V(p) = \langle \mu(p),\ \nu(p) \rangle.$$

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When values $V(p) = \langle a, b\rangle$ and $V(q) = \langle c, d\rangle$ of the proposition forms p and q are known, the evaluation function V can be extended also for the operations “negation”, “conjunction”, “disjunction”, “implication” and others. For example,

$$V(\neg p) = \langle b, a\rangle,$$

$$V(p\&q) = \langle \min(a, c), \max(b, d)\rangle,$$

$$V(p \vee q) = \langle \max(a, c), \min(b, d)\rangle.$$

The proposition p is called an intuitionistic fuzzy tautology if and only if $a \geq b$ and a tautology if and only if $a = 1, b = 0$.

Also, let for propositions p and q

$$V(p) \leq V(q) \text{ if and only if } a \leq c \text{ and } b \geq d.$$

Meantime, in Intuitionistic Fuzzy Set (IFS) theory (see [5]) firstly two, and after this – six other topological operators were introduced. The author saw that the Intuitionistic Fuzzy (IF) interpretations of the first two topological operators (“*closure*” and “*interior*”) coincide with the two logical quantifiers “*there exists*” and “*for all*”), respectively.

In the present research, we give definition of six new logical quantifiers, that are analogous to the six topological operators, mentioned above. Some of the properties of the new operators will be studied and some open problems will be formulated.

2 Main results

Let x be a variable, obtaining values in set E and let $P(x)$ be a predicate with a variable x. Let

$$V(P(x)) = \langle \mu(P(x)), \nu(P(x))\rangle.$$

The IF-interpretations of the quantifiers *for all* ($\forall$) and *there exists* ($\exists$) are introduced in [2] by

$$V(\forall x P(x)) = \langle \sup_{y\in E} \mu(P(y)), \inf_{y\in E} \nu(P(y))\rangle,$$

$$V(\exists x P(x)) = \langle \inf_{y\in E} \mu(P(y)), \sup_{y\in E} \nu(P(y))\rangle.$$

Their geometrical interpretations are illustrated in Figs. 1 and 2, respectively, where $x_1, ..., x_5$ are the possible values of the variable x and $V(x_1), ..., V(x_5)$, their IF-estimations.

The most important property of the two quantifiers is that each of them juxtaposes to predicate P a point (exactly one for each quantifier) in the IF-interpretational triangle.

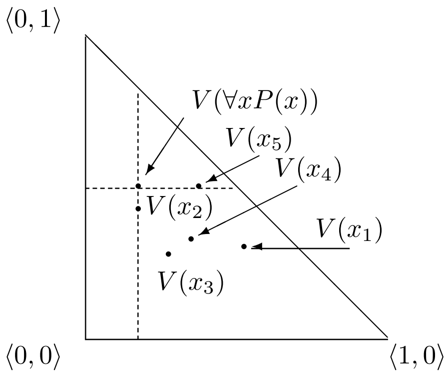

Fig. 1.

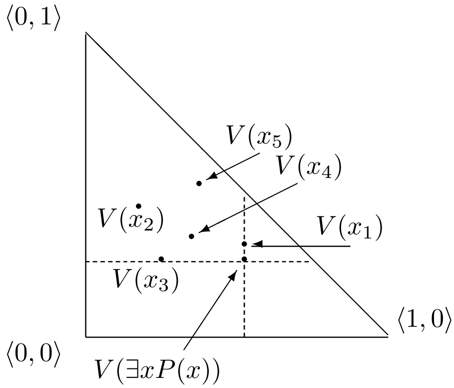

Fig. 2.

Now, we introduce the following six new quantifiers.

$$V(\forall_\mu x P(x)) = \{\langle x, \inf_{y\in E} \mu(P(y)), \nu(P(x))\rangle | x \in E\},$$

$$V(\forall_\nu x P(x)) = \{\langle x, \min(1 - \sup_{y\in E} \nu(P(y)), \mu(P(x)), \sup_{y\in E} \nu(P(y))\rangle | x \in E\},$$

$$V(\exists_\mu x P(x)) = \{\langle x, \sup_{y\in E} \mu(P(y)), \min(1 - \sup_{y\in E} \mu(P(y)), \nu(P(x))\rangle | x \in E\},$$

$$V(\exists_\nu x P(x)) = \{\langle x, \mu(P(x)), \inf_{y\in E} \nu(P(y))\rangle | x \in E\},$$

$$V(\forall_\nu^* x P(x)) = \{\langle x, \min(1 - \sup_{y \in E} \nu(P(y)), \mu(P(x)),$$
$$\min(\sup_{y \in E} \nu(P(y)), 1 - \mu(P(x))\rangle | x \in E\},$$
$$V(\exists_\mu^* x P(x)) = \{\langle x, \min(\sup_{y \in E} \mu(P(y)), 1 - \nu(P(x)),$$
$$\min(1 - \sup_{y \in E} \mu(P(y)), \nu(P((x))\rangle | x \in E\}.$$

Let the possible values of variable x be a, b, c and let their IF-estimations $V(a), V(b), V(c)$ be shown on Fig. 3. The geometrical interpretations of the new quantifiers are shown in Figs. 4-9.

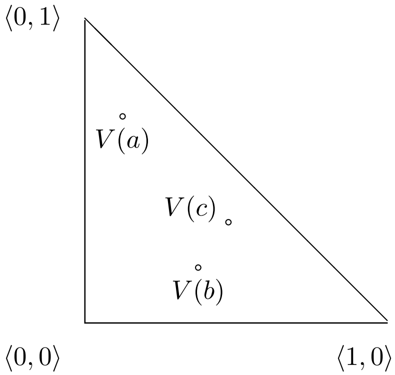

Fig. 3.

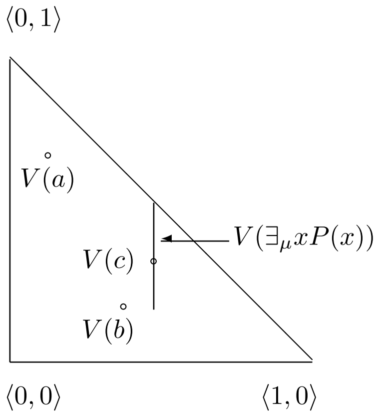

Fig. 4.

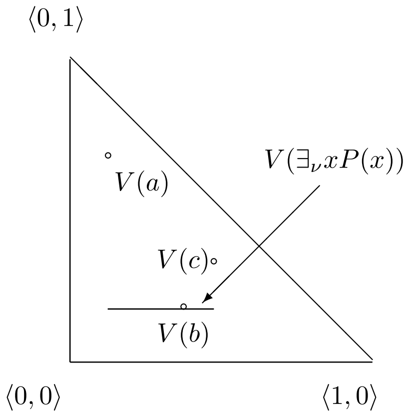

Fig. 5.

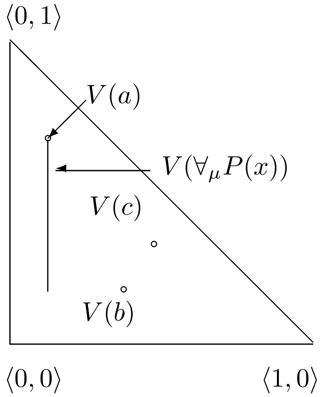

Fig. 6.

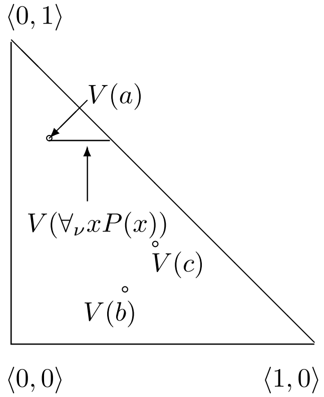

Fig. 7.

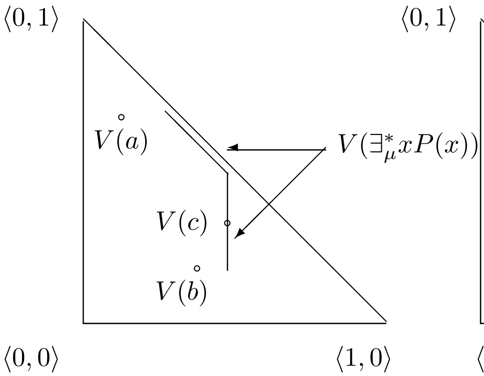

Fig. 8.

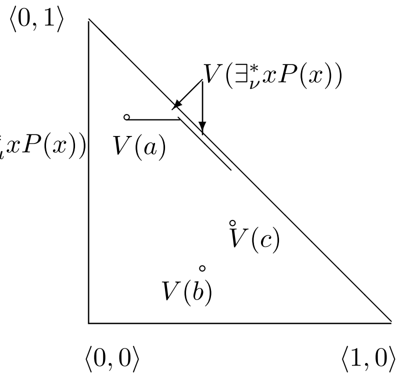

Fig. 9.

Therefore, we can change the forms of the first two quantifiers to the forms

$$V(\forall xP(x)) = \{\langle x, \inf_{y\in E} \mu(P(y)), \sup_{y\in E} \nu(P(y))\rangle | x \in E\},$$

$$V(\exists xP(x)) = \{\langle x, \sup_{y\in E} \mu(P(y)), \inf_{y\in E} \nu(P(y))\rangle | x \in E\}.$$

Obviously, for every predicate P,

$$V(\forall xP(x)) \subseteq V(\forall_\mu xP(x)) \subseteq V(\forall_\nu xP(x))$$

$$\subseteq V(\exists_\nu xP(x)) \subseteq V(\exists_\mu xP(x)) \subseteq V(\exists xP(x))$$

and

$$V(\forall x P(x)) \subseteq V(\forall_\nu x P(x)) \subseteq V(\forall_\nu^* x P(x))$$
$$\subseteq V(\exists_\mu^* x P(x)) \subseteq V(\exists_\mu x P(x)) \subseteq V(\exists x P(x)).$$

Theorem 1. For every predicate P,
(a) $V(\neg\exists_\mu x \neg P(x)) = V(\forall_\nu x P(x))$,

(b) $V(\neg\forall_\mu x \neg P(x)) = V(\exists_\nu x P(x))$,

(c) $V(\neg\exists_\nu x \neg P(x)) = V(\forall_\mu x P(x))$,

(d) $V(\neg\forall_\nu x \neg P(x)) = V(\exists_\mu x P(x))$,

(e) $V(\neg\exists_\mu^* x \neg P(x)) = V(\forall_\nu^* x P(x))$,

(f) $V(\neg\forall_\mu^* x \neg P(x)) = V(\exists_\nu^* x P(x))$,

Proof: (a) Let P be a predicate. Then

$$V(\neg\exists_\mu x \neg P(x))$$
$$= \neg\{\langle x, \sup_{y\in E} \nu(P(y)), \min(1 - \sup_{y\in E} \nu(P(y)), \mu(P(x))\rangle | x \in E\}$$
$$= \{\langle x, \min(1 - \sup_{y\in E} \nu(P(y))), \mu(P(x)), \sup_{y\in E} \nu(P(y))\rangle | x \in E\}$$
$$= V(\forall_\nu x P(x)).$$

The proofs of the remaining assertions are analogous.

Theorem 2. For every predicate P,
(a) $V(\exists_\mu x(\exists_\nu x P(x))) = V(\exists_\nu x(\exists_\mu x P(x))) = V(\exists x P(x))$,

(b) $V(\forall_\mu x(\forall_\nu x P(x))) = V(\forall_\nu x(\forall_\mu x P(x))) = V(\forall x P(x))$,

(c) $V(\exists_\mu x(\forall_\mu x P(x))) = V(\forall_\mu x(\exists_\mu x P(x)))$,

(d) $V(\exists_\nu x(\forall_\nu x P(x))) = V(\forall_\nu x(\exists_\nu x P(x)))$.

Theorem 3. For every two predicates P and Q,:

(a) $\exists_\mu(P(x) \vee Q(x)) = \exists_\mu P(x) \vee \exists_\mu Q(x)$,

(b) $\exists_\nu(P(x) \vee Q(x)) = \exists_\nu P(x) \vee \exists_\nu Q(x)$,

(c) $\forall_\mu(P(x) \vee Q(x)) = \forall_\mu P(x) \vee \forall_\mu Q(x)$,

(d) $\forall_\nu(P(x) \vee Q(x)) = \forall_\nu P(x) \vee \forall_\nu Q(x)$.

For difference with quantifiers *for all* and *there exists,* that juxtapose to all values of x exactly one value (specific for each one of the quantifiers), the herewith defined new quantifiers give us possibility to introduce more precise quantifiers, giving concrete values for the individual values of x. These precise quantifiers have the following forms for a value a of variable x:

$$V([\forall_\mu x P(x)](a)) = \langle \inf_{y \in E} \mu(P(y)), \nu(P(a)) \rangle,$$

$$V([\forall_\nu x P(x)](a)) = \langle \min(1 - \sup_{y \in E} \nu(P(y)), \mu(P(a)), \sup_{y \in E} \nu(P(y)) \rangle,$$

$$V([\exists_\mu x P(x)](a)) = \langle \sup_{y \in E} \mu(P(y)), \min(1 - \sup_{y \in E} \mu(P(y)), \nu(P(a)) \rangle,$$

$$V([\exists_\nu x P(x)](a)) = \langle \mu(P(a)), \inf_{y \in E} \nu(P(y)) \rangle,$$

$$V([\forall^*_\nu x P(x)](a)) = \langle \min(1 - \sup_{y \in E} \nu(P(y)),$$

$$\mu(P(a)), \min(\sup_{y \in E} \nu(P(y)), 1 - \mu(P(a)) \rangle,$$

$$V([\exists^*_\mu x P(x)](a)) = \langle \min(\sup_{y \in E} \mu(P(y)), 1 - \nu(P(a)),$$

$$\min(1 - \sup_{y \in E} \mu(P(y)), \nu(P((a)) \rangle.$$

3 Conclusion

The herewith defined quantifiers give answer to the Open Problem 11 from [5]. Now, the connections between the new quantifiers and the other operators and operations in IFL have to be studied.

It is interesting to mention the analogy between the first two (old) quantifiers $(\forall, \exists)$ and the definite integral ($\int\limits_a^b f(x)dx$) and the analogy between the new six quantifiers and the indefinite integral ($\int f(x)dx$).

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Eleventh International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2012) organized in Warsaw on October 12, 2012 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, Prof. Asen Zlatarov University, Burgas, Bulgaria, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Eleventh International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2012) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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