New Trends in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics
Volume I: Foundations

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Some notes about boundaries on IF-sets

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Abstract

In the first part of the paper the basic definition and operations defined on IF-sets are recalled. In the second part the IF functions, their properties like continuity and boundaries are studied. In the third part there are proved some properties of bounded IF-sets.

Keywords: IF-set, IF function, continuity and boundaries of IF function.

1 Introduction

In this paper we would like to study some properties of functions which are defined on IF-sets. We will start with basic definitions in the first part of article and then we will studied the properties of boundaries.
By an IF-set we consider a pair $A = (\mu_A, \nu_A)$ of functions $\mu_A, \nu_A : \Omega \to [0, 1]$ such that

$$\mu_A + \nu_A \leq 1.$$

The function μ_A is called a membership function of A, ν_A a nonmembership function of A. If $(\Omega, \mathcal{S})$ is a measurable space and μ_A, ν_A are $\mathcal{S}$-measurable, then A is called an IF-event. Denote by $\mathcal{F}$ the family of all IF-events.
In the paper [4] there was proved that we could construct such ℓ-group G that $\mathcal{F}$

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can be embedded into G.
Consider the set $A = (\mu_A, \nu_A)$, where $\mu_A, \nu_A : \Omega \to R$. Denote by G the set of all pairs $A = (\mu_A, \nu_A)$ and for any $A, B \in G$ define the following operation

$$A + B = (\mu_A + \mu_B, \nu_A + \nu_B - 1) =$$

$$= (\mu_A + \mu_B, 1 - (1 - \nu_A) + (1 - \nu_B))$$

and the relation

$$A \leq B \Longleftrightarrow \mu_A \leq \mu_B, \nu_A \geq \nu_B.$$

Then $G = (G, +, \leq)$ is the mentioned ℓ-group.
On the set G there are defined some operations and relations. We will need following of them

$$A - B = (\mu_A - \mu_B, 1 + \nu_A - \nu_B).$$

The function on the set G is defined by the following way

$$\tilde{f}(A) = (f(\mu_A), 1 - f(1 - \nu_A)),$$

where $f : R \to R$. Then for example for any natural number n it holds

$$n.A = (n.\mu_A, 1 - n.(1 - \nu_A)),$$

$$A^n = ((\mu_A)^n, 1 - (1 - \nu_A)^n).$$

2 Boundary of the function defined on IF set

The motivation for this article were the papers [2], [3], [4], [5] where the authors defined and studied some functions which are important in classical calculus. We would like to restrict these results.
In the first step let us look better on the domain of the function $\tilde{f}$. We know, that

$$\tilde{f}(A) = (f(\mu_A), 1 - f(1 - \nu_A))$$

where $f : R \to R$. Therefore if the domain of the function $\tilde{f}$ is an interval $[A, B]$ then it must hold:

$$A \leq B \Longleftrightarrow \mu_A \leq \mu_B \text{ and } \nu_A \geq \nu_B$$

and therefore

$$[\mu_A, \mu_B] \cup [\nu_B, \nu_A] \subset Domf.$$

In the paper [5] there were given following notations:

Definition 1 *Let $A \in G$. Then $|A| = (|\mu_A|, 1 - |1 - \nu_A|)$ is the absolute value of A.*

Proposition 1 *Let $A, B \in G$ and let $\tilde{\delta} = (\delta, 1 - \delta), \delta > 0$. Then*

$$|A - B| < \tilde{\delta} \Longleftrightarrow A - \tilde{\delta} < B < A + \tilde{\delta}.$$

Definition 2 *Let $A_0, A, \tilde{\delta} = (\delta, 1 - \delta)$ be from the ℓ-group G. A point A is in the $\tilde{\delta}$-neighborhood of a point A_0 if it holds*

$$|A - A_0| < \tilde{\delta}.$$

In our paper we will use the definition of the neighborhood of the point a lot of times therefore it is useful denote it by $\tilde{\mathcal{U}}(A_0)$. Then the notation $A \in \tilde{\mathcal{U}}(A_0)$ means that $A \in (A_0 - \tilde{\delta}, A_0 + \tilde{\delta})$.

Proposition 2 *The element $X = (\mu_X, \nu_X)$ belongs to $\tilde{\mathcal{U}}(X_0)$ if and only if*

$$\mu_X \in (\mu_{X_0} - \delta, \mu_{X_0} + \delta)$$

and at the same time

$$\nu_X \in (\nu_{X_0} - \delta, \nu_{X_0} + \delta).$$

Proof.
Since

$$\tilde{\mathcal{U}}(X_0) = (X_0 - \tilde{\delta}, X_0 + \tilde{\delta})$$

then

$$X_0 - \tilde{\delta} = (\mu_{X_0} - \delta, 1 + \nu_{X_0} - (1 - \delta)) = (\mu_{X_0} - \delta, \nu_{X_0} + \delta)$$

and on the other hand

$$X_0 + \tilde{\delta} = (\mu_{X_0} + \delta, 1 + \nu_{X_0} - (1 + \delta)) = (\mu_{X_0} + \delta, \nu_{X_0} - \delta).$$

Therefore

$$X = (\mu_X, \nu_X) \in \tilde{\mathcal{U}}(X_0) \Longleftrightarrow X_0 - \tilde{\delta} < X < X_0 + \tilde{\delta}.$$

From the previous inequality it follows

$$\mu_{X_0} - \delta < \mu_X < \mu_{X_0} + \delta$$

$$\nu_{X_0} + \delta > \nu_X > \nu_{X_0} - \delta.$$

Therefore

$$\mu_X \in (\mu_{X_0} - \delta, \mu_{X_0} + \delta)$$

and

$$\nu_X \in (\nu_{X_0} - \delta, \nu_{X_0} + \delta).$$

In the paper [5] there was given the definition of the limit of the function in a point A_0. Outgoing for this definition let us define the continuous function on interval $[A, B]$.

Definition 3 *Function $\tilde{f}$ is continuous on interval $[A, B]$ if and only if to each point $X_0 \in [A, B]$ and for each $\tilde{\varepsilon} = (\varepsilon, 1 - \varepsilon), \varepsilon > 0$ there exist such $\tilde{\delta} = (\delta, 1 - \delta), \delta > 0$ that for each $X \in (X_0 - \tilde{\delta}, X_0 + \tilde{\delta}) = \tilde{\mathcal{U}}(X_0)$, $X \in [A, B]$ it holds $\tilde{f}(X) \in (\tilde{f}(X_0) - \tilde{\varepsilon}, \tilde{f}(X_0) + \tilde{\varepsilon}) = \tilde{\mathcal{V}}(\tilde{f}(X_0))$.*

Theorem 1 *Let $f : R \to R$ and $\tilde{f}(X) = (f(\mu_X), 1 - f(1 - \nu_X))$ be the function defined on the set G. Function $\tilde{f}$ is continuous on the interval $[A, B]$ if and only if f is continuous on the interval $[\mu_A, \mu_B]$ and also on the interval $[\nu_B, \nu_A]$.*

Proof.
The real function f is continuous on the interval $[a, b]$ if and only if for each $x_0 \in [a, b]$ and for each $\varepsilon > 0$ there exists such $\delta > 0$ that for each $x \in \mathcal{U}(x_0)$ it holds $f(x) \in \mathcal{V}(f(x_0))$.
In our case f is continuous on the interval $[\mu_A, \mu_B]$ if and only if for each $\mu_{X_0} \in [\mu_A, \mu_B]$ and for each $\varepsilon > 0$ there exists such $\delta > 0$ that for each $\mu_X \in [\mu_A, \mu_B]$ it holds

$$\mu_X \in (\mu_{X_0} - \delta, \mu_{X_0} + \delta) \Rightarrow f(\mu_X) \in (f(\mu_{X_0}) - \varepsilon, f(\mu_{X_0}) + \varepsilon).$$

Similarly f is continuous on the interval $[\nu_A, \nu_B]$ if and only if for each $\nu_{X_0} \in [\nu_B, \nu_A]$ and for each $\varepsilon > 0$ there exists such $\delta > 0$ that for each $\nu_X \in [\nu_B, \nu_A]$ it holds

$$\nu_X \in (\nu_{X_0} - \delta, \nu_{X_0} + \delta) \Rightarrow f(1 - \nu_X) \in (f(1 - \nu_{X_0}) - \varepsilon, f(1 - \nu_{X_0}) + \varepsilon).$$

Let $X_0 = (\mu_{X_0}, \nu_{X_0})$, $X = (\mu_X, \nu_X)$, $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B)$, $\tilde{\delta} = (\delta, 1 - \delta), \delta > 0$, $\tilde{\varepsilon} = (\varepsilon, 1 - \varepsilon), \varepsilon > 0$. From the previous definition function $\tilde{f}$ is continuous on interval $[A, B]$ if and only if to each point $X_0 \in [A, B]$ and for each $\tilde{\varepsilon}$ there exist such $\tilde{\delta}$ that for each $X \in (X_0 - \tilde{\delta}, X_0 + \tilde{\delta})$, $X \in [A, B]$ it holds $\tilde{f}(X) \in (\tilde{f}(X_0) - \tilde{\varepsilon}, \tilde{f}(X_0) + \tilde{\varepsilon})$.
From the previous proposition it follows

$$X \in (X_0 - \tilde{\delta}, X_0 + \tilde{\delta}) \Longleftrightarrow$$

$$\Longleftrightarrow \mu_X \in (\mu_{X_0} - \delta, \mu_{X_0} + \delta) \wedge \nu_X \in (\nu_{X_0} - \delta, \nu_{X_0} + \delta)$$

Then for any $X \in [A, B]$ and for the function $\tilde{f}(X) = (f(\mu_X), 1 - f(1 - \nu_X))$ could be derived following properties

$$\tilde{f}(X_0) - \tilde{\varepsilon} = (f(\mu_{X_0}), 1 - f(1 - \nu_{X_0})) - (\varepsilon, 1 - \varepsilon) =$$

$$= (f(\mu_{X_0}) - \varepsilon, 1 - f(1 - \nu_{X_0}) + \varepsilon),$$

$$\tilde{f}(X_0) + \tilde{\varepsilon} = (f(\mu_{X_0}), 1 - f(1 - \nu_{X_0})) + (\varepsilon, 1 - \varepsilon) =$$

$$= (f(\mu_{X_0}) + \varepsilon, 1 - f(1 - \nu_{X_0}) - \varepsilon)$$

and

$$\tilde{f}(X) \in (\tilde{f}(X_0) - \tilde{\varepsilon}, \tilde{f}(X_0) + \tilde{\varepsilon}) \Longleftrightarrow$$

$$\Longleftrightarrow f(\mu_X) \in (f(\mu_{X_0}) - \varepsilon, f(\mu_{X_0}) + \varepsilon) \wedge$$

$$\wedge f(1 - \nu_X) \in (f(1 - \nu_{X_0}) - \varepsilon, f(1 - \nu_{X_0}) + \varepsilon).$$

Therefore f is a function continuous on the intervals $[\mu_A, \mu_B]$ and $[\nu_A, \nu_B]$.

Theorem 2 *If function $\tilde{f}$ is continuous on an interval $[A, B]$ then $\tilde{f}$ is also bounded on interval the $[A, B]$.*

Proof.
Let take some fix point $X_0 \in [A, B]$. Let $X = (\mu_X, \nu_X) \in \tilde{\mathcal{U}}(X_0)$ then from the previous definitions $\tilde{f}(X) = (f(\mu_X), 1 - f(1 - \nu_X)) \in \tilde{\mathcal{V}}(\tilde{f}(X_0))$. Therefore $f(\mu_X) \in (f(\mu_{X_0}) - \varepsilon, f(\mu_{X_0}) + \varepsilon)$ but $(f(\mu_{X_0}) - \varepsilon, f(\mu_{X_0}) + \varepsilon) = |f(\mu_{X_0}) - \varepsilon| = K_0$ and therefore

$$|f(\mu_X)| \leq K_0.$$

Similarly $f(1 - \nu_X) \in (f(1 - \nu_{X_0}) - \varepsilon, f(1 - \nu_{X_0}) + \varepsilon)$ but $(f(1 - \nu_{X_0}) - \varepsilon, f(1 - \nu_{X_0}) + \varepsilon) = |f(1 - \nu_{X_0}) - \varepsilon| = L_0$. Therefore

$$|f(1 - \nu_X)| \leq L_0.$$

Since

$$|\tilde{f}(X)| = (|f(\mu_X)|, 1 - |1 - (1 - f(1 - \nu_X))|) =$$

$$= (|f(\mu_X)|, 1 - |f(1 - \nu_X)|) \leq (K_0, 1 - L_0).$$

This relations hold for each $X_0 \in [A, B]$. Therefore we could take such X_1, $X_2, ..., X_k$ that it hold

$$\bigcup_{i=1}^{k} (\mu_{X_i} - \delta, \mu_{X_i} + \delta) \supset [\mu_A, \mu_B]$$

and

$$\bigcup_{i=1}^{k}(\nu_{X_i}-\delta,\nu_{X_i}+\delta)\supset[\nu_B,\nu_A].$$

Then for any $X\in[A,B]$ it holds

$$|f(\mu_X)|\leq\max\{K_{X_1},K_{X_2},...,K_{X_k}\}=K$$

and similarly

$$|f(1-\nu_X)|\leq\max\{L_{X_1},L_{X_2},...,L_{X_k}\}=L$$

where $K_{X_i},L_{X_i},i=1,2,...,k$ are the values appertaining to X_i. Therefore there exist such $K,L\in R$ that for each $X\in[A,B]$ it hold

$$|\tilde{f}(X)|=(|f(\mu_X)|,1-|f(1-\nu_X)|)\leq(K,1-L).$$

i.e. $\tilde{f}$ is bounded on interval $[A,B]$.

3 Some properties of bounded IF-sets

Definition 4 *Let $\mathcal{A}$ be the set defined on the set G. Then this set is bounded if and only if there exist such $(c,d)\in G$ that for each element $A\in\mathcal{A}$ it holds*

$$A=(\mu_A,\nu_A)\leq(c,d).$$

Theorem 3 *Let $\mathcal{A}\subset G$ be a bounded set. Then there exist the supremum and the infimum of the set $\mathcal{A}$.*

Proof.
We will show that if the assumptions are fulfilled then there exists the supremum of the set $\mathcal{A}$. The existence of the infimum could be proved by the same way.
Let $\mathcal{A}$ be a bounded set, then there exist such pair $(c,d)\in G$ that for each $A\in\mathcal{A}$ it holds

$$A=(\mu_A,\nu_A)\leq(c,d).$$

Denote by $\{\mu_A;A\in\mathcal{A}\}$ the set of all membership functions of the elements which belong to $\mathcal{A}$. Then this set is bounded and therefore there exists such α that

$$\alpha=\sup\{\mu_A;A\in\mathcal{A}\}.$$

Similarly for the set $\{\nu_A; A \in \mathcal{A}\}$ of all nonmembership functions of the elements which belong to $\mathcal{A}$ holds that this set is bounded and there exists such β that

$$\beta = \inf\{\nu_A; A \in \mathcal{A}\}.$$

Put $S = (\alpha, \beta)$ then for each $A \in \mathcal{A}$ it holds

$$\mu_A \leq \alpha,\ \nu_A \geq \beta$$

and

$$A = (\mu_A, \nu_A) \leq (\alpha, \beta) = S.$$

On the other hand let there exists such $C = (\mu_C, \nu_C) \in G$ that for each $A \in \mathcal{A}$ it holds $A \leq C$, i.e. $\mu_A \leq \mu_C$ and $\nu_A \geq \nu_C$. Then

$$\alpha = \sup \mu_A \leq \mu_C$$

$$\beta = \inf \nu_A \geq \nu_C$$

and therefore

$$S = (\alpha, \beta) \leq (\mu_C, \nu_C) = C.$$

Hence $S = (\alpha, \beta)$ is the wanted supremum of the set $\mathcal{A}$.

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Eleventh International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2012) organized in Warsaw on October 12, 2012 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, Prof. Asen Zlatarov University, Burgas, Bulgaria, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Eleventh International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2012) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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