Modern Approaches in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics
Volume I: Foundations

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A general point of view to inclusion - exclusion property

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Abstrakt

Two binary operations on the real line are given satisfying some conditions. The IE - property is proved with regard to the operations and with respect to a state on IF-sets. The main instrument for the proof are IE-property theorem from [9, 10] and IF-state representation theorem from [5, 6].

Keywords:

1 Introduction

The classical inclusion exclusion property says that

$$m(A \cup B) = m(A) + m(B) - m(A \cap B),$$

whenever the domain of M is closed under the union $A \cup B$, the intersection $A \cap B$, and the difference $A \setminus B$ of any two sets A, B, and m is additive on this domain. Of course, the property can be extended to any three sets A, B, C, (+)

$$m(A \cup B \cup C) = m(A) + m(B) + m(C) - m(A \cap B) -$$

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$$-m(A \cap C) - m(B \cap C) + m(A \cap B \cap C),$$

to any four sets A, B, C, D

(++)

$$m(A \cup B \cup C \cup D) = m(A) + m(B) + m(C) + m(D) - m(A \cap B) -$$

$$-m(A \cap C) - m(A \cap D) - m(B \cap C) - m(B \cap D) - m(C \cap D) + m(A \cap B \cap C) +$$

$$+m(A \cap B \cap D) + m(A \cap C \cap D) + m(B \cap C \cap D) - m(A \cap B \cap C \cap D),$$

etc.This property was generalized for fuzzy sets, first probably in [8]. It was realized actually for IF-sets, i.e. such pairs

$$A = (\mu_A, \nu_A)$$

of functions $\mu_A, \nu_A : \Omega \to [0, 1]$ such that

$$\mu_A + \nu_A \leq 1.$$

The function $\mu_A : \Omega \to [0, 1]$ is called the membership function of A, the function $\nu_A : \Omega \to [0, 1]$ is called the non - membership function of A. The fuzzy set is a special case of IF-set, where $\nu_A = 1 - \nu_A$.

The paper consists of three parts. In the first part we present the Kelemenová IE - theorem. The theorem works with a mapping $m : F \to H$, where $(H, +)$ is a semigroup. There are given two operations $\square, \triangle$ on H satistfying the following properties:

$$(1) m(a \sqcup b) + m(\sqcap b) = m(a) + m(b),$$

$$(2) m((a \sqcup b) \sqcap c) + m(a \sqcap b \sqcap c) = m(a \sqcap c) + m(b \sqcap c).$$

As a consequence of the Kelemenová theorem a special case is considered where $(H, +)$ is a commutative group.

The second part is dedicated to the states on IF - sets. Using the Cignoli representation theorem the assumptions (10) and (2) stated above are proved.

Finally in the third part thye interval valued states are considerede and the IE-property is obtained for them.

2 The Kelemenová inclusion - exclusion theorem

In [9, 10] a simple but original idea is used. E. g. instead of (+) to use the equality

$$m(A \cup B \cup C) + m(A \cap B) + m(A \cap C) + m(B \cap C) =$$

$$= m(A) + m(B) + m(C) + m(A \cap B \cap C),$$

instead of (++) the equality

$$m(A \cup B \cup C \cup D) + m(A \cap B) + m(A \cap C) + m(A \cap D) +$$

$$+ m(B \cap C) + m(B \cap D) + m(C \cap D) + m(A \cap B \cap C \cap D) =$$

$$= m(A) + m(B) + m(C) + m(D) + m(A \cap B \cap C) +$$

$$+ m(A \cap B \cap D) + m(A \cap C \cap D) + m(B \cap C \cap D).$$

Let us to present the Kelemenovâ theorem.

Theorem 1. Let $(G, \sqcup, \sqcap)$ be an algebraic system, where $\sqcup, \sqcap$ are binary operations, $\sqcup$ being commutative and associative $\sqcap$ being associative. Let $)H, +)$ be a commutative subgroup. Let $m : G \to H$ be a mapping satisfying the following two conditions:

$$(1) m(a \sqcup b) + m(\sqcap b) = m(a) + m(b),$$

$$(2) m((a \sqcup b) \sqcap c) + m(a \sqcap b \sqcap c) = m(a \sqcap c) + m(b \sqcap c).$$

Then for every n there holds

$$(3) m(\bigsqcup_{k=1}^{n} a_k) + \Sigma_{k \leq n, k-even} \Sigma_{1 \leq i_1 < i_2 < \ldots < \imath_k \leq n} m(a_{i_1} \sqcap a_{i_1} \sqcap \ldots \sqcap a_{i_1}) =$$

$$= \Sigma_{k \leq n, k-odd} \Sigma_{1 \leq i_1 < i_2 < \ldots < \imath_k \leq n} m(a_{i_1} \sqcap a_{i_1} \sqcap \ldots \sqcap a_{i_1}).$$

Proof. See [10], Theorem 2.3.

Of course, if $(H, +$ is a group, we can return again to the naturaql operationb $-$ and to present (3) in the usuaql form. as a corrolary of Theorem 1 we obtain the following assertion.

Theorem 2. Let $(G, \sqcup, \sqcap)$ be an algebraic system, where $\sqcup, \sqcap$ are binary operations, $\sqcup$ being commutative and associative $\sqcap$ being associative. Let $)H, +)$ be a commutative group. Let $m : G \to H$ be a mapping satisfying the following two conditions:

$$(1) m(a \sqcup b) + m(\sqcap b) = m(a) + m(b),$$

$$(2) m((a \sqcup b) \sqcap c) + m(a \sqcap b \sqcap c) = m(a \sqcap c) + m(b \sqcap c).$$

Then for every n there holds

$$(4) m(\bigsqcup_{k=1}^{n} a_k) = \Sigma_{i=1}^{n} m(a_i) - \Sigma_{i<j} m(a_i \sqcap a_j) + \Sigma_{i<j<k} m(a_i \sqcap a_j \sqcap a_k) + ...+$$

$$+\Sigma_{1 \leq i_1 < i_2 < ... < i_k \leq n} (-1)^k m(a_{i_k} \sqcap a_{i_2} \sqcap ... \sqcap a_{i_k}) + ... + (-1)^{n+1} m(a_1 \sqcap a_2 \sqcap ... \sqcap a_n).$$

Proof. Using the group operations we can express the element

$$m(a_1 \sqcup a_2 \sqcup ... \sqcup a_n)$$

as the sum of all sums

$$\Sigma_{1 \leq i_1 < ... < i_k \leq n} m(a_{i_1} \sqcap ... \sqcap a_{i_k})$$

with k odd minus the sum

$$\Sigma_{1 \leq i_1 < ... < i_k \leq n} m(a_{i_1} \sqcap ... \sqcap a_{i_k})$$

with k even. So at the end of the sequence of the sums we obtain

$$m(a_1 \sqcap a_2 \sqcap ... \sqcap a_n)$$

with the sign $+$ if n is odd, or sign $-$, if n is even. Therefore the last element in the sequence is

$$(-1)^{n+1} m(a_1 \sqcap a_2 \sqcap ... \sqcap a_n).$$

3 Cignoli representation

Let X be a non-empty set, $\mathcal{A}$ be the σ-algebra of subsets of X. An IF-vent is a pair

$$A = (\mu_A, \nu_A)$$

of Borel measurable functions

$$\mu_A, \nu_A : X \to [0, 1]$$

such that

$$\mu_A + \nu_A \leq 1.$$

Let $\mathcal{F}$ be the set of all IF-events. We shall use use the Lukasiewicz operations on $\mathcal{F}$:

$$A \oplus B = ((\mu_A + \mu_B) \wedge 1, (\nu_A + \nu_B - 1) \vee 0),$$
$$A \odot B = ((\mu_A + \mu_B - 1) \vee 0, ((\nu_A + \nu_B) \wedge 1).$$

Definition 1. A mapping $m : \mathcal{F} \to [0, 1]$ is an IF-state if the following properties are satisfied:

(i) $m((1_X, 0_X)) = 1, m((0_X, 1_X)) = 0,$

(ii) $m(A \oplus B) = m(A) + m(B) - m(A \odot B),$

(iii) $A_n \nearrow A \Longrightarrow m(A_n) \nearrow m(A).$

The main instrument in our investigations is the following representation theorem.

Theorem 3. Let $m : \mathcal{F} \to [0, 1]$ be an IF-state. Then there exist probability measures $P, Q : \mathcal{A} \to [0, 1]$ and $\alpha \in R$ such that

$$m(A) = \int_X \mu_A dP + \alpha(1 - \int_X (\mu_A + \nu_A) dQ)$$

for all $A \in \mathcal{F}$.

Proof. See [5, 6, 16].

Now let us return to our general binary operations $\sqcup, \sqcap$ on R. We shall say that $\sqcup, \sqcap$ forms an IF-pair, if the following identities are satisfied:

$$a \sqcup b = a + b - a \sqcap b,$$

$$(a \sqcup b) \sqcap c = a \sqcap c + b \sqcap c - a \sqcap b \sqcap c.$$

We define the corresponding operations on $\mathcal{F}$:

$$A \sqcup B = (\mu_A \sqcup \mu_B, 1 - (1 - \nu_A) \sqcup (1 - \nu_B)),$$

$$A \sqcap B = (\mu_A \sqcap \mu_B, 1 - (1 - \nu_A) \sqcap (1 - \nu_B)).$$

Of course, we assume that $A \sqcup B \in \mathcal{F}, A \sqcap B \in \mathcal{F}$ whenever $A, B \in \mathcal{F}$. It is satisfied if $\sqcup, \sqcap$ are monotone, i.e.

$$a \leq b \Longrightarrow a \sqcup b \leq a \sqcup c, a \sqcap b \leq a \sqcap c.$$

Indeed, since $\mu_A + \nu_A \leq 1, \mu_B + \nu_B \leq 1$, and

$$A \sqcup B = (\mu_A \sqcup \mu_B, 1 - (1 - \nu_A) \sqcup (1 - \nu_B)),$$

we obtain

$$\mu_A \sqcup \mu_B + 1 - (1 - \nu_A) \sqcup (1 - \nu_B) \leq$$

$$\leq (1 - \nu_A) \sqcup (1 - \nu_B) + 1 - (1 - \nu_A) \sqcup (1 - \nu_B) = 1,$$

hence

$$A, B \in \mathcal{F} \Longrightarrow A \sqcup B \in \mathcal{F}.$$

Similarly it can be proved that

$$A, B \in \mathcal{F} \Longrightarrow A \sqcap B \in \mathcal{F}.$$

Theorem 4. Let $(\sqcup, \sqcap)$ be an IF-pair of operations on R, $m : \mathcal{F} \to [0,1]$ be an IF-state. Then

$$(*) m(A \sqcup B) + m(A \sqcap B) = m(A) + m(B),$$

$$(**) m((A \sqcup B) \sqcap C) + m(A \sqcap B \sqcap C) = m(A \sqcap B) + m(A \sqcap C).$$

Proof. The main instrument is Theorem 3:

$$m(A) = \int \mu_A dP + \alpha(1 - \int (\mu_A + \nu_A) dQ,$$

$$m(B) = \int \mu_B dP + \alpha(1 - \int (\mu_B + \nu_B) dQ,$$

$$m(A \sqcup B) = \int \mu_{A \sqcup B} dP + \alpha(1 - \int (\mu_{A \sqcup B} + \nu_{A \sqcup B}) dQ,$$

$$m(A \sqcap B) = \int \mu_{A \sqcap B} dP + \alpha(1 - \int (\mu_{A \sqcap B} + \nu_{A \sqcap B}) dQ,$$

Of curse,

$$\mu_{A \sqcup B} = \mu_A \sqcup \mu_B, \mu_{A \sqcap B} = \mu_A \sqcap \mu_B,$$

and therefore

$$\mu_A + \mu_B = \mu_A \sqcup \mu_B + \mu_A \sqcap \mu_B = \mu_{A \sqcup B} + \mu_{A \sqcap B},$$

hence

$$\int \mu_A dP + \int \mu_B dP = \int \mu_{A \sqcup B} dP + \int \mu_{A \sqcap B} dP,$$

$$\int \mu_A dQ + \int \mu_B dQ = \int \mu_{A \sqcup B} dQ + \int \mu_{A \sqcap B} dQ.$$

On the oher hand

$$\nu_{A \sqcup B} + \nu_{A \sqcap B} = 1 - (1 - \nu_A) \sqcup (1 - \nu_B) + 1 - (1 - \nu_A) \sqcap (1 - \nu_B) =$$

$$= 2 - (1 - \nu_A + 1 - \nu_B) = \nu_A + \nu_B,$$

hence also

$$\int \nu_A dQ + \int \nu_B dQ = \int \nu_{A \sqcup B} dQ + \int \nu_{A \sqcap B} dQ.$$

Summarizing all the equalities we obtain

$$m(A) + m(B) = m(A \sqcup B) + m(A \sqcap B).$$

Similarly the identity (**) can be proved.

As a consequence of Theorem 2 and Theorem 4 we obtain the following result.

Theorem 5. Let $(\sqcup, \sqcap)$ be an IE-pair of binary operations on R, $m : \mathcal{F} \to [0,1]$ be an IF-state. Then for any $n \in N$ and any $Ai \in \mathcal{F}(i = 1, 2., , , , n)$

$$m(\bigsqcup_{i=1}^{n} A_i) = \Sigma_{k=1}^{n}(-1)^{k+1}\Sigma_{1 \le i_1 < i_2 < \ldots < i_k \le n}(-1)^k m(A_{i_k} \sqcap A_{i_2} \sqcap \ldots \sqcap A_{i_k})$$

Of courese, one can choose some special IE - operations on R.

Theorem 6. Put $a \vee b = \max(a, b), a \wedge b = \min(a, b)$ for any $a, b \in R$. Let $m : \mathcal{F} \to [0,1]$ be an IF-state. Then

$$m(\bigvee_{i=1}^{n} A_i) = \Sigma_{k=1}^{n}(-1)^{k+1}\Sigma_{1 \le i_1 < i_2 < \ldots < i_k \le n}(-1)^k m(A_{i_k} \wedge A_{i_2} \wedge \ldots \wedge A_{i_k})$$

for any $n \in N$ and any $A_1, \ldots, A_n \in \mathcal{F}$.

Proof. Evidently

$$a \vee b + a \wedge b = a + b$$

and

$$(a \vee b) \wedge c + a \wedge b \wedge c = a \wedge c + b \wedge c,$$

hence $(\vee, \wedge)$ is an IE-pair.

Theorem 7. Put $a\sigma b = a + b - a.b, a\pi b = a.b$ for any $a, b \in R$. Let $m : \mathcal{F} \to [0,1]$ be an IF-state. Then

$$m(\sigma_{i=1}^{n} A_i) = \Sigma_{k=1}^{n}(-1)^{k+1}\Sigma_{1 \le i_1 < i_2 < \ldots < i_k \le n}(-1)^k m(A_{i_k}\pi A_{i_2}\pi \ldots \pi A_{i_k}).$$

for any $n \in N$ and any $A_1, \ldots, A_n \in \mathcal{F}$.

Proof. Evidently

$$a\sigma b + a\pi b = a + b - a.b + a.b = a + b,$$

and

$$(a\sigma b)\pi c + a\pi b\pi c = (a + b - a.b).c + a.b.c = a.c + b.c = a\pi c + b\pi c,$$

hence (σ, π) is an IE-pair.

4 Grzegorzewski's concept of IF - probability

P. Grzegorzewski defined ([7]) the probability of an IF-event $A = (\mu_A, \nu_A)$ as a compact interval

$$\mathcal{P}(A) = [\int_X \mu_A dP, 1 - \int_X \nu_A dP].$$

Axiomatically the probability was defined in [13] by the following way:

Definition 2. A mapping $\mathcal{P} : \mathcal{F} \to \mathcal{J}$, where $\mathcal{J} = \{[a,b]; a, b \in R, a \leq b\}$ is IF-probability, if the following conditions are satisfied:

1. $\mathcal{P}((1,0)) = [1,1], \mathcal{P}((0,1)) = [0,0]$,
2. $A \odot B = (0,1) \Longrightarrow \mathcal{P}(A \oplus B) = \mathcal{P}(A) + \mathcal{P}(B)$,
3. $An \nearrow A \Longrightarrow \mathcal{P}(A_n) \nearrow \mathcal{P}(A)$.

Recall that $[a,b] + [c,d] = [a+c, b+d]$, and $[a_n, b_n] \nearrow [a,b]$ means $a_n \nearrow a, b_n \nearrow b$. On the other hand $A_n = (a_n.b_n) \nearrow A = (a,b)$ means $\mu_{An} \nearrow \mu_A, \nu_{An} \searrow \nu_A$.

Theorem 8. Let $\mathcal{P} : \mathcal{F} \to \mathcal{J}$ be a probability. Denote $\mathcal{P}(A) = [\mathcal{P}_1(A), \mathcal{P}_2(A)]$. Then $\mathcal{P}$ is an IF-probability if and only if $\mathcal{P}_1, \mathcal{P}_2$ are states.

The proof is straightforward.

Theorem 9. Let $(\sqcup, \sqcap)$ be an IE-pair of binary operations on R. Let $\mathcal{P} : \mathcal{F} \to \mathcal{J}$ be an IF-probability. Then

$$\mathcal{P}(\bigsqcup_{i=1}^{n} A_i) = \Sigma_{k=1}^{n}(-1)^{k+1}\Sigma_{1 \leq i_1 < i_2 < \ldots < i_k \leq n}(-1)^k \mathcal{P}(A_{i_k} \sqcap A_{i_2} \sqcap \ldots \sqcap A_{i_k}).$$

Proof. It follows by Theorem 8 and Theorem 5 if we use the formula $[a,b] - [c,d] = [a-c, b-d]$.

5 Conclusion

In the paper the Kelemenová inclusion exclusion theorem ([10]) is applied to the Atanassov intuitionistic fuzzy system ([1]). Similarly as in [4] two binary operations $\sqcup, \sqcap$ on the family of all IF-events are considered satisfying the identity

$$(**) m((A \sqcup B) \sqcap C) + m(A \sqcap B \sqcap C) = m(A \sqcap B) + m(A \sqcap C).$$

Recently in [13] it was proved that the identity

$$(A \sqcup B) \sqcap C = A \sqcap B \sqcap C + A \sqcap B + A \sqcap C - A \sqcap B \sqcap C$$

implies

$$\bigsqcup_{i=1}^{n} A_i = \Sigma_{k=1}^{n}(-1)^{k+1}\Sigma_{1\leq i_1<i_2<\ldots<i_k\leq n}(-1)^k A_{i_k}\sqcap A_{i_2}\sqcap\ldots\sqcap A_{i_k}.$$

for every t-norm $\sqcap$ and every t-conorm $\sqcup$. Therefore using Butnariu - Klement representation theorem ([2, 3]) the inclusion - exclusion principle is proved for fuzzy events. It would be interesting to use the Cignoli representation theorem for proving the principle for IF-events. Moreover, recall that the assumption of the Kelemenová theorem ((1) and (2) in Theorem 1) are weaker that in [13].

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.

It may be viewed as a result of fruitful discussions held during the Twelfth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2013) organized in Warsaw on October 11, 2013 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, Prof. Asen Zlatarov University, Burgas, Bulgaria, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Twelfth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2013) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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