Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

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Systems Research Institute
Polish Academy of Sciences
Newelska 6, 01-447 Warsaw, Poland
www.ibspan.waw.pl

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On intuitionistic fuzzy multi-dimensional sets. Part 3

Krassimir Atanassov[1], **Eulalia Szmidt**[2] **and Janusz Kacprzyk**[2]
[1] Centre for Biomedical Engineering – Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Bl. 105, Sofia-1113, Bulgaria
krat@bas.bg
[2] Systems Research Institute, Polish Academy of Sciences
ul. Newelska 6, 01–447 Warsaw, Poland
and
Warsaw School of Information Technology
ul. Newelska 6, 01-447 Warsaw, Poland
{szmidt, kacprzyk}@ibspan.waw.pl

Abstract

A concept of intuitionistic fuzzy multi-dimensional set (IFMDS) is developed. New properties of IFMDSs are discussed.

Keywords: intuitionistic fuzzy sets, intuitionistic fuzzy multi-dimensional sets.

1 Introduction

The Intuitionistic Fuzzy Sets (IFSs, see [1]) were defined as extensions of the ordinary fuzzy sets. After this, they also were extended. The later in time of these extensions are Intuitionistic Fuzzy Multi-Dimensional Sets (IFMDSs, see [5, 6]).

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2 Short remarks on intuitionistic fuzzy multi-dimensional sets

Let a set E be fixed. An IFS A in E is an object of the following form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Let for every $x \in E$:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x).$$

Therefore, function π determines the degree of uncertainty.

For every two IFSs A and B a lot of relations and operations are defined (see, e.g. [1]), the most important of which are:

$$\begin{array}{lll} A \subset B & iff & (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \& \nu_A(x) \geq \nu_B(x)); \\ A \supset B & iff & B \subset A; \\ A = B & iff & (\forall x \in E)(\mu_A(x) = \mu_B(x) \& \nu_A(x) = \nu_B(x)); \end{array}$$

$$\begin{array}{lll} \neg A & = & \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}; \\ A \cap B & = & \{\langle x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x))\rangle | x \in E\}; \\ A \cup B & = & \{\langle x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x))\rangle | x \in E\}; \\ A@B & = & \{\langle x, (\frac{\mu_A(x) + \mu_B(x))}{2}, \frac{(\nu_A(x) + \nu_B(x))}{2}\rangle | x \in E\}. \end{array}$$

Two analogies of the topological operators can be defined over the IFSs, too: operator *"closure"* C and operator *"interior"* I:

$$C(A) = \{\langle x, \sup_{y \in E} \mu_A(y), \inf_{y \in E} \nu_A(y)\rangle | x \in E\},$$

$$I(A) = \{\langle x, \inf_{y \in E} \mu_A(y), \sup_{y \in E} \nu_A(y)\rangle | x \in E\}.$$

These operators are modified to the following four operators (see [2]):

$$\begin{array}{lll} C_\mu(A) & = & \{\langle x, K, \min(1 - K, \nu_A(x))\rangle | x \in E\}; \\ C_\nu(A) & = & \{\langle x, \mu_A(x), L\rangle | x \in E\}; \\ I_\mu(A) & = & \{\langle x, k, \nu_A(x)\rangle | x \in E\}; \\ I_\nu(A) & = & \{\langle x, \min(1 - l, \mu_A(x)), l\rangle | x \in E\}, \end{array}$$

where K, L, k, l are as defined above.

The weight operator W is also defined over IFSs (see [3]) by

$$W(A) = \{\langle x, \frac{\sum\limits_{y \in E} \mu_A(y)}{card(E)}, \frac{\sum\limits_{y \in E} \nu_A(y)}{card(E)}\rangle | x \in E\},$$

where $card(E)$ is the number of the elements of the (finite) set E.

Now, following [5], we shall introduce the concept of IFMDSs.

Let sets $Z_1, Z_2, ..., Z_n$ be fixed and let for each $i(1 \leq i \leq n) : z_i \in Z_i$.

Let set E be fixed.

An IFS A in E is an object of the IFMDS A in $E, Z_1, Z_2, ..., Z_n$ is an object of the form

$$A(Z_1, Z_2, ..., Z_n) = \{\langle x, \mu_A(x, z_1, z_2, ..., z_n), \nu_A(x, z_1, z_2, ..., z_n)\rangle |$$

$$\langle x, z_1, z_2, ..., z_n\rangle \in E \times Z_1 \times Z_2 \times ... \times Z_n\},$$

where:

- **(a)** $\mu_A(x, z_1, z_2, ..., z_n) + \nu_A(x, z_1, z_2, ..., z_n) \leq 1$ for every $\langle x, z_1, z_2, ..., z_n\rangle \in E \times Z_1 \times Z_2 \times ... \times Z_n$,
- **(b)** $\mu_A(x, z_1, z_2, ..., z_n)$ and $\nu_A(x, z_1, z_2, ..., z_n)$ are the degrees of membership and non-membership, respectively, of the element $x \in E$ and $\langle z_1, z_2, ..., z_n\rangle \in Z_1 \times Z_2 \times ... \times Z_n$.

For every two IFSs A and B over sets $E, Z_1, Z_2, ..., Z_n$ all above relations, operations and operators can be defined by an analogous way. For example, here we can define the following topological operators:

Now, we shall extend the above defined operators C^* and I^* to the forms:

$$C_i^*(A(Z_1, Z_2, ..., Z_n))$$

$$= \{\langle x, \sup_{t_i \in Z_i} \mu_{A(Z_1, Z_2, ..., Z_n)}(x, z_1, ..., z_{i-1}, t_i, z_{i+1}, ..., z_n),$$

$$\inf_{t_i \in Z_i} \nu_{A(Z_1, Z_2, ..., Z_n)}(x, z_1, ..., z_{i-1}, t_i, z_{i+1}, ..., z_n)\rangle | x \in E\},$$

$$I_i^*(A(Z_1, Z_2, ..., Z_n))$$

$$= \{\langle x, \inf_{t_i \in Z_i} \mu_{A(Z_1, Z_2, ..., Z_n)}(x, z_1, ..., z_{i-1}, t_i, z_{i+1}, ..., z_n),$$

$$\sup_{t_i \in Z_i} \nu_{A(Z_1, Z_2, ..., Z_n)}(x, z_1, ..., z_{i-1}, t_i, z_{i+1}, ..., z_n)\rangle | x \in E\}.$$

Their properties are discussed in [5].

3 Main result

In [4] some convex hull algorithms are discussed. Let, for a given IFS A, the region obtained by one of these algorithm is marked by $R(A)$. Therefore, $R(A)$ is the minimal convex region containing IFS A.

First, following [4], we shall introduce four operators acting in IF-interpretation triangle. Let the point $\langle\mu,\nu\rangle$ be given. Then

$$\begin{aligned}\gamma_{L,\nu}(\mu) &= \mu',\\ \gamma_{R,\nu}(\mu) &= \mu'',\\ \gamma_{U,\mu}(\nu) &= \nu',\\ \gamma_{D,\mu}(\nu) &= \nu'',\end{aligned}$$

where μ' and μ'' are the left and the right intersection points of the line

$$l : y = \nu$$

and $R(A)$; ν' and ν'' are the upper and the down intersection points of the line

$$l : x = \mu$$

and $R(A)$.

For ordinary IFS A the new operators have the forms:

$$\begin{aligned}\Gamma_L(A) &= \{\langle x, \gamma_{L,\nu}(\mu), \nu\rangle | x \in E\},\\ \Gamma_R(A) &= \{\langle x, \gamma_{R,\nu}(\mu), \nu\rangle | x \in E\},\\ \Gamma_U(A) &= \{\langle x, \mu, \gamma_{U,\mu}(\nu)\rangle | x \in E\},\\ \Gamma_D(A) &= \{\langle x, \mu, \gamma_{D,\mu}(\nu)\rangle | x \in E\}.\end{aligned}$$

In [6] for the fixed set A, using operators C_i^* and I_i^*, sequentially for each i ($1 \leq i \leq n$) we constructed the following points of the interpretation triangle:

$$\begin{aligned}\langle\mu_i^C, \nu_i^C\rangle = \langle &\sup_{t_i\in Z_i} \mu_{A(Z_1,Z_2,...,Z_n)}(x, z_1, ..., z_{i-1}, t_i, z_{i+1}, ..., z_n),\\ &\inf_{t_i\in Z_i} \nu_{A(Z_1,Z_2,...,Z_n)}(x, z_1, ..., z_{i-1}, t_i, z_{i+1}, ..., z_n)\rangle\\ \langle\mu_i^I, \nu_i^I\rangle = \langle &\inf_{t_i\in Z_i} \mu_{A(Z_1,Z_2,...,Z_n)}(x, z_1, ..., z_{i-1}, t_i, z_{i+1}, ..., z_n),\\ &\sup_{t_i\in Z_i} \nu_{A(Z_1,Z_2,...,Z_n)}(x, z_1, ..., z_{i-1}, t_i, z_{i+1}, ..., z_n)\rangle\end{aligned}$$

Therefore, the sets

$$\overline{C}(A) = \{\langle i^C, \mu_i^C, \nu_i^C\rangle | 1 \leq i \leq n\}$$

and

$$\overline{I}(A) = \{\langle i^I, \mu_i^I, \nu_i^I\rangle | 1 \leq i \leq n\}$$

are ordinary IFSs. Here, we use notation "i^C" and "i^I" to denote the fact that number i ($1 \leq i \leq n$) are components of the elements of the first and of the second above sets, respectively, but this is not necessary and we will not use this notation below.

Now, we shall construct other type of points that have the form

$$\langle \mu_i^{Av}, \nu_i^{Av}\rangle = \langle \frac{1}{card(Z_i)} \sum_{t_i \in Z_i} \mu_{A(Z_1, Z_2, \dots, Z_n)}(x, z_1, \dots, z_{i-1}, t_i, z_{i+1}, \dots, z_n),$$
$$\frac{1}{card(Z_i)} \sum_{t_i \in Z_i} \nu_{A(Z_1, Z_2, \dots, Z_n)}(x, z_1, \dots, z_{i-1}, t_i, z_{i+1}, \dots, z_n)\rangle$$

Therefore, the set

$$\overline{Av}(A) = \{\langle i, \mu_i^{Av}, \nu_i^{Av}\rangle | 1 \leq i \leq n\}$$

is an ordinary IFS.

Now, we can construct the sets

$$\begin{aligned}
\overline{Av}_L(A) &= \Gamma_L(\overline{Av}(A)),\\
\overline{Av}_R(A) &= \Gamma_R(\overline{Av}(A)),\\
\overline{Av}_U(A) &= \Gamma_U(\overline{Av}(A)),\\
\overline{Av}_D(A) &= \Gamma_D(\overline{Av}(A)),
\end{aligned}$$

The following assertion is valid.

Theorem 1: For each two IFMDSs A and B:

$$\neg W(\neg \overline{Av}(A)) = \neg W(\overline{Av}(\neg A)) = W(\neg \overline{Av}(\neg A)) = W(\overline{Av}(A)).$$

Proof: Let the IFMDS A be given. Then we obtain sequentially

$$\neg W(\neg \overline{Av}(A)) = \neg W(\neg \{\langle i, \mu_i^{Av}, \nu_i^{Av}\rangle | 1 \leq i \leq n\})$$
$$= \neg W(\{\langle i, \nu_i^{Av}, \mu_i^{Av}\rangle | 1 \leq i \leq n\})$$

$$= \neg\{\langle i, \frac{1}{n}\sum_{j=1}^{n}\nu_j^{Av}, \frac{1}{n}\sum_{j=1}^{n}\mu_j^{Av}\rangle | 1 \leq i \leq n\})$$

$$= \{\langle i, \frac{1}{n}\sum_{j=1}^{n}\mu_j^{Av}, \frac{1}{n}\sum_{j=1}^{n}\nu_j^{Av}\rangle | 1 \leq i \leq n\})$$

$$= W(\overline{Av}(A)).$$

The other equalities are checked analogously.

Theorem 2: For any IFS A:

$$\overline{I}(A) \subset \overline{Av}(A) \subset \overline{C}(A).$$

Theorem 3: For any IFS A:

$$\left.\begin{array}{l} \overline{Av}_L(A) \\ \overline{Av}_U(A) \end{array}\right\} \subset \overline{Av}(A) \subset \left\{\begin{array}{l} \overline{Av}_R(A) \\ \overline{Av}_D(A) \end{array}\right. .$$

The proofs of these assertions are analogous as the above one.

4 Conclusion

The so constructed IFMDSs can have different applications. For example, they can be used in procedures for decision making. Now, we can have a universe of objects for estimation E and sets of values of the estimations - $Z_1, Z_2, ..., Z_n$ given by the different experts. They will generate a space of values that can be interpreted by an IFMDS in which, using, e.g. the new topological operators, we can find the extremal values.

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) organized in Warsaw on October 16, 2009 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Centre for Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT – Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bistrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) has been meant to commence a new series of scientific events primarily focused on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Moreover, other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems are discussed.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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