Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

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On Intuitionistic Fuzzy Subtractions $-'_{19}$ and $-''_{19}$

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Abstract

An intuitionistic fuzzy subtraction, generated by the intuitionistic fuzzy negation $\neg_{19}$ is constructed. Some of its basic properties are studied.

1 Introduction

In a series of papers, more than 150 different intuitionistic fuzzy implications were defined by K. Atanassov, D. Dimitrov, B. Kolev, T. Trifonov and others.

As it was discussed in [1, 2], in intuitionistic fuzzy propositional calculus, if x is a variable then its truth-value is represented by the ordered couple

$$V(x) = \langle a, b \rangle,$$

so that $a, b, a+b \in [0, 1]$, where a and b are degrees of validity and of non-validity of x.

Below we shall assume that for the three variables x, y and z the equalities: $V(x) = \langle a, b \rangle, V(y) = \langle c, d \rangle, V(z) = \langle e, f \rangle$ $(a, b, c, d, e, f, a+b, c+d, e+f \in [0, 1])$ hold.

For the needs of the discussion below, we shall use the notion of Intuitionistic Fuzzy Tautology (IFT), defined in [1] as:

$$x \text{ is an IFT if and only if for } V(x) = \langle a, b \rangle \text{ it holds: } a \geq b,$$

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while x will be a tautology iff $a = 1$ and $b = 0$. As in the case of ordinary logics, x is a tautology, if $V(x) = \langle 1, 0\rangle$.

For two variables x and y the operation "conjunction" (&) is defined (see [1]) by:

$$V(x\&y) = \langle \min(a, c), \max(b, d)\rangle.$$

2 Main results

In [3] different intuitionistic fuzzy forms of operation "subtraction" are introduced. They are based on the intuitionistic fuzzy forms of operation "negation". One of these negations - $\neg_{19}$ was introduced by the author in [4, 5]. It has the form

$$\neg_{19}\langle a, b\rangle = \langle b.\text{sg}(a), 0\rangle,$$

where functions sg and $\overline{\text{sg}}$ are defined by:

$$\text{sg}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}, \qquad \overline{\text{sg}}(x) = \begin{cases} 0 & \text{if } x > 0 \\ 1 & \text{if } x \leq 0 \end{cases}$$

Now, we will transform these results to the case of Intuitionistic Fuzzy Sets (IFSs, see [2]). Let the IFSs

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}$$

and

$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in E\}$$

be given. Then, we can define for them:

$$\neg_{19}A = \{\langle x, \nu_A(x).\text{sg}(\mu_A(x)), 0\rangle | x \in E\}.$$

Therefore,

$$\neg_{19}\neg_{19}A = \{\langle x, 0, 0\rangle | x \in E\}.$$

Now, operation "subtraction" will have two different forms:

$$A -'_{19} B = \{\langle x, \min(\mu_A(x), \nu_B(x).\text{sg}(\mu_B(x))), \nu_A(x)\rangle | x \in E\}$$

and

$$A -''_{19} B = \{\langle x, 0, \nu_A(x)\rangle | x \in E\}.$$

We can check that as a result of the operations "$-'_{19}$" and "$-''_{19}$" we obtain IFSs. Really, for two given IFSs A and B and for each $x \in E$ we obtain that

$$0 \leq \min(\mu_A(x), \nu_B(x).\text{sg}(\mu_B(x))) + \nu_A(x) \leq \mu_A(x) + \nu_A(x) \leq 1.$$

The check for the second subtraction is similar.

Let us make use of the *empty IFS,* the *totally uncertain IFS,* and the *unit IFS* (as defined in [2]) by:

$$O^* = \{\langle x, 0, 1\rangle | x \in E\},$$
$$U^* = \{\langle x, 0, 0\rangle | x \in E\},$$
$$E^* = \{\langle x, 1, 0\rangle | x \in E\}.$$

Let

$$A^* = \{\langle x, 0, \nu_A(x)\rangle | x \in E\}$$

The validity of the following assertions is directly checked

Theorem 1: For every IFS A:
(a) $A -'_{19} E^* = A^*$,
(b) $A -'_{19} O^* = A^*$,
(c) $A -'_{19} U^* = A^*$,
(d) $E^* -'_{19} A = \neg_{19} A$,
(e) $O^* -'_{19} A = O^*$,
(f) $U^* -'_{19} A = U^*$.

Theorem 2: For every two IFSs A and B:
(a) $(A -'_{19} B) \cap C = (A \cap C) -'_{19} B = A \cap (C -'_{19} B)$,
(b) $(A \cap B) -'_{19} C = (A -'_{19} C) \cap (B -'_{19} C)$,
(c) $(A -'_{19} B) -'_{19} C = (A -'_{19} C) -'_{19} B$.

Theorem 3: The following equalities hold:
(a) $O^* -'_{19} U^* = O^*$,
(b) $O^* -'_{19} E^* = O^*$,
(c) $O^* -'_{19} O^* = O^*$,
(d) $U^* -'_{19} U^* = U^*$,
(e) $U^* -'_{19} E^* = U^*$,
(f) $U^* -'_{19} O^* = U^*$,
(g) $E^* -'_{19} U^* = U^*$,

(h) $E^* -'_{19} E^* = U^*$,

(i) $E^* -'_{19} O^* = U^*$.

Theorem 4: For every two IFSs A and B:
$A -''_{19} B = A^*$.

Theorem 5: For every IFS A:

(d) $E^* -''_{19} A = U^*$,

(e) $O^* -'_{19} A = O^*$,

(f) $U^* -'_{19} A = U^*$.

Theorem 6: For every two IFSs A and B:

(a) $(A -''_{19} B) \cap C = (A \cap C) -''_{19} B = A \cap (C -''_{19} B)$,

(b) $(A \cap B) -''_{19} C = (A -''_{19} C) \cap (B -''_{19} C)$,

(c) $(A -''_{19} B) -''_{19} C = (A -''_{19} C) -''_{19} B$.

Theorem 7: The following equalities hold:

(a) $O^* -''_{19} U^* = O^*$,

(b) $O^* -''_{19} E^* = O^*$,

(c) $O^* -''_{19} O^* = O^*$,

(d) $U^* -''_{19} U^* = U^*$,

(e) $U^* -''_{19} E^* = U^*$,

(f) $U^* -''_{19} O^* = U^*$,

(g) $E^* -''_{19} U^* = U^*$,

(h) $E^* -''_{19} E^* = U^*$,

(i) $E^* -''_{19} O^* = U^*$.

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) organized in Warsaw on October 16, 2009 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Centre for Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT – Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bistrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) has been meant to commence a new series of scientific events primarily focused on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Moreover, other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems are discussed.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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