Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

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Systems Research Institute
Polish Academy of Sciences
Newelska 6, 01-447 Warsaw, Poland
www.ibspan.waw.pl

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S-implications in Atanassov's intuitionistic and interval-valued fuzzy set theory revisited

Michał Baczyński
Institute of Mathematics, University of Silesia
ul. Bankowa 14, 40-007 Katowice, Poland
michal.baczynski@us.edu.pl

Abstract

The aim of this paper it to present an overview of S-implications defined in Atanassov's intuitionistic and interval-valued fuzzy set theory. In particular we discuss their properties, characterizations and representations theorems.

Keywords: Atanassov's intuitionistic fuzzy set theory, interval-valued fuzzy set theory, fuzzy implications, S-implications.

1 Introduction

Recently we have presented the overview of (S,N)-implications defined on the unit interval [4]. In this paper we would like to continue these investigations but for the Atanassov's intuitionistic and the interval-valued fuzzy sets. To this end we describe in this paper the properties and known characterizations and representations of S-implications in both structures.

The paper is organized as follows. Firstly, in section 2, we recall the definitions of Atanassov's intuitionistic fuzzy set and the interval-valued fuzzy set. We also indicate that both Atanassov's intuitionistic fuzzy sets and interval-valued fuzzy sets are equivalent to L-fuzzy sets in the sense of Goguen w.r.t. a special

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lattices. Section 3 gives some preliminary results regarding negations and triangular conorms defined on both lattices. In the next section, which should be seen as a main overview, we recall the definition of the family of S-implications listing their main properties, characterization and representations theorems in the case of Atanassov's intuitionistic and interval-valued fuzzy set theory.

2 Preliminaries

Intuitionistic fuzzy set theory introduced by Atanassov [1] assign to each element of the universe not only a membership degree, but also a non-membership degree which is less than or equal to 1 minus the membership degree. Currently this theory is also called in the literature as the bipolar fuzzy set theory (see [5], [13], [14]).

Definition 1. An intuitionistic fuzzy set A on X is a set

$$A = \{(x, \mu_A(x), \nu_A(x)) \quad : \quad x \in X\},$$

where $\mu_A,\ \nu_A \colon X \to [0,1]$ are called, respectively, the membership function and the non-membership function. Moreover they satisfy the condition

$$\mu_A(x) + \nu_A(x) \le 1, \qquad x \in X.$$

We use in this paper the following notation, which is well accepted in the scientific literature

$$L^* = \{(x_1, x_2) \in [0,1]^2 \quad : \quad x_1 + x_2 \le 1\}, \tag{1}$$

$$(x_1, x_2) \le_{L^*} (y_1, y_2) \iff x_1 \le y_1 \wedge x_2 \ge y_2. \tag{2}$$

One can easily observe that $\mathcal{L}^* = (L^*, \le_{L^*})$ is a complete lattice with units $0_{L^*} = (0,1)$ and $1_{L^*} = (1,0)$. Moreover, an intuitionistic fuzzy set A on X can be represented by the $\mathcal{L}^*$-fuzzy set A given by

$$\begin{aligned} &A \colon X \to L^*, \\ &\qquad x \mapsto (\mu_A(x), \nu_A(x)), \qquad x \in X. \end{aligned}$$

Another extension of fuzzy set theory is interval-valued fuzzy set theory (see [17], [16]), in which to each element of the universe a closed subinterval of the unit interval is assigned which can be used as an approximation of the unknown membership degree. Let us define

$$L^I = \{(x_1, x_2) \in [0,1]^2 : x_1 \le x_2\}, \tag{3}$$

$$(x_1, x_2) \le_{L^I} (y_1, y_2) \iff x_1 \le y_1 \wedge x_2 \le y_2. \tag{4}$$

One can easily observe that $\mathcal{L}^I = (L^I, \leq_{L^I})$ is a complete lattice with units $0_{\mathcal{L}^I} = (0,0)$ and $1_{\mathcal{L}^I} = (1,1)$.

Definition 2. An interval-valued fuzzy set on X is a mapping $A \colon X \to L^I$.

In the sequel, if $x \in L^*$, then we denote it by $x = (x_1, x_2)$, while if $x \in L^I$, then we denote it by $x = [x_1, x_2]$.

It is important to notice that in [10] it is shown that intuitionistic fuzzy set theory is equivalent to interval-valued fuzzy set theory. In fact wen can see the point $(x_1, x_2) \in L^*$ as the interval $[x_1, 1 - x_2] \in L^I$. In this article we will show results in both notations, to indicate differences, in one side, but also to make the article more useful.

3 Basic connectives

Definition 3. Let $\mathcal{L} = (L, \leq_L)$ be a complete lattice. A function $\mathcal{N} \colon L \to L$ is called a fuzzy negation on $\mathcal{L}$ if it is decreasing and satisfies $\mathcal{N}(0_L) = 1_L$ and $\mathcal{N}(1_L) = 0_L$, where 0_L is the least element of L and 1_L is the greatest element of L. If, in addition, $\mathcal{N}$ is an involution, i.e.,

$$\mathcal{N}(\mathcal{N}(x)) = x, \qquad x \in L,$$

then N is called a strong (involutive) negation.

Example 1. The least fuzzy negation on a complete lattice $\mathcal{L}$ is the Gödel negation defined by

$$N_{\mathbf{G1}}(x) = \begin{cases} 1_L, & \text{if } x = 0_L \\ 0_L, & \text{if } x > 0_L \end{cases}, \qquad x \in L.$$

The greatest fuzzy negation on a complete lattice $\mathcal{L}$ is the dual Gödel negation defined by

$$N_{\mathbf{G2}}(x) = \begin{cases} 0_L, & \text{if } x = 1_L \\ 1_L, & \text{if } x < 1_L \end{cases}, \qquad x \in L.$$

We have the following characterizations of Atanassov's intuitionistic and interval-valued fuzzy negations.

Theorem 1 ([9], Theorem 3.6)**.** *For a function $\mathcal{N} \colon L^* \to L^*$ the following statements are equivalent:*

(i) $\mathcal{N}$ is a strong negation on $\mathcal{L}^*$.

(ii) There exists a strong negation N on $([0,1],\leq)$ such that

$$\mathcal{N}((x_1,x_2)) = (N(1-x_2), 1-N(x_1)), \qquad (x_1,x_2) \in L^*.$$

Theorem 2. *For a function $\mathcal{N}\colon L^I \to L^I$ the following statements are equivalent:*

(i) $\mathcal{N}$ is a strong negation on $\mathcal{L}^I$.

(ii) There exists a strong negation N on $([0,1],\leq)$ such that

$$\mathcal{N}([x_1,x_2]) = [N(x_2), N(x_1)], \qquad [x_1,x_2] \in L^I.$$

Proof. We will show only that $(i) \Longrightarrow (ii)$, sine the opposite implication is obvious. Additionally we will show the proof which is based on Theorem 1. Assume that $\mathcal{N}$ is a strong negation on $\mathcal{L}^I$. Then using connections between Atanassov's intuitionistic and interval valued fuzzy sets we get that there exists a strong negation $\mathcal{N}_1$ such that $\mathcal{N}([x_1,x_2]) = \mathcal{N}_1(x_1, 1-x_2)$. Therefore, for any $[x_1,x_2] \in L^I$, we get

$$\mathcal{N}([x_1,x_2]) = \mathcal{N}_1(x_1, 1-x_2) = (N(x_2), 1-N(x_1)) = (N(x_2), N(x_1)),$$

where N is a strong negation on the unit interval from Theorem 1. □

Example 2. The standard strong negation on $\mathcal{L}^*$ is $\mathcal{N}((x_1,x_2)) = (x_2,x_1)$. The standard strong negation on $\mathcal{L}^I$ is $\mathcal{N}([x_1,x_2]) = [1-x_2, 1-x_1]$. In both cases they are generated by the standard strong negation on the unit interval defined by $N(x) = 1-x$, for $x \in [0,1]$.

It should be noted that strong negations on the unit interval have the form $N(x) = \varphi^{-1}(1-\varphi(x))$, for $x \in [0,1]$, where φ is an increasing bijection of the unit interval. Therefore the characterizations and representations of strong negations in both lattices are well developed.

Definition 4. Let $\mathcal{L} = (L, \leq_L)$ be a complete lattice. An associative, commutative operation $\mathcal{T}\colon L^2 \to L$ is called a t-norm if it is increasing and 1_L (the greatest element of L) is the neutral element of $\mathcal{T}$.

Definition 5. Let $\mathcal{L} = (L, \leq_L)$ be a complete lattice. An associative, commutative operation $\mathcal{S}\colon L^2 \to L$ is called a t-conorm if it is increasing and O_L (the least element of L) is the neutral element of $\mathcal{S}$.

For a t-conorms on $\mathcal{L}^*$ and $\mathcal{L}^I$ we can consider the following special subclasses (see [8]).

Definition 6. A t-conorm $\mathcal{S}$ on $\mathcal{L}^*$ is called t-representable if there exist a t-norm T and a t-conorms S on $([0,1], \leq)$ such that

$$T(x,y) \leq 1 - S(1-x, 1-y), \qquad x, y \in [0,1]$$

and

$$\mathcal{S}((x_1,x_2),(y_1,y_2)) = (S(x_1,y_1), T(x_2,y_2)), \qquad (x_1,x_2),(y_1,y_2) \in L^*.$$

Definition 7. A t-conorm $\mathcal{S}$ on $\mathcal{L}^I$ is called t-representable if there exist t-conorms S_1 and S_2 on $([0,1], \leq)$ such that

$$S_1(x,y) \leq S_2(x,y), \qquad x, y \in [0,1]$$

and

$$\mathcal{S}([x_1,x_2],[y_1,y_2]) = [S_1(x_1,y_1), S_2(x_2,y_2)], \qquad [x_1,x_2],[y_1,y_2] \in L^I.$$

It is interesting and important that not every t-norm and t-conorm on $\mathcal{L}^*$ and $\mathcal{L}^I$ have these representations. It is still an open problem to characterize the above families.

The definition of a fuzzy implication on $\mathcal{L}$ is based on the notation from fuzzy set theory introduced by Fodor, Roubens [15].

Definition 8. Let $\mathcal{L} = (L, \leq_L)$ be a complete lattice. A function $\mathcal{I}\colon L^2 \to L$ is called a fuzzy implication on $\mathcal{L}$ if it is decreasing with respect to the first variable, increasing with respect to the second variable and fulfills the border conditions $\mathcal{I}(0_L, 0_L) = \mathcal{I}(1_L, 1_L) = 1_L, \mathcal{I}(1_L, 0_L) = 0_L$.

Implications in Atanassov's intuitionistic and interval-valued fuzzy set theory have been investigated by many authors (see e.g. [8], [7], [11], [12], [2], [18]).

4 (S,N)-implications

It is well-known in the classical logic that the unary negation operation $\neg$ can combine with any other binary operation to generate rest of the binary operations. This distinction of the unary $\neg$ is also shared by the Boolean implication $\to$, if defined in the following usual way:

$$p \to q \equiv \neg p \vee q.$$

The definition as given above was the first to catch the attention of the researchers leading to the following class of fuzzy implications.

Definition 9. Let $\mathcal{L} = (L, \leq_L)$ be a complete lattice. A function $\mathcal{I}\colon L^2 \to L$ is called an (S,N)-implication on $\mathcal{L}$ if there exists a t-conorm $\mathcal{S}$ on $\mathcal{L}$ and a fuzzy negation $\mathcal{N}$ on $\mathcal{L}$ such that

$$\mathcal{I}(x, y) = \mathcal{S}(\mathcal{N}(x), y), \qquad x, y \in L.$$

If $\mathcal{N}$ is a strong negation, then $\mathcal{I}$ is called an S-implication (or a strong implication) on $\mathcal{L}$. Moreover, if $\mathcal{I}$ is an (S,N)-implication generated from $\mathcal{S}$ and $\mathcal{N}$, then we will often denote it by $\mathcal{I}_{\mathcal{S},\mathcal{N}}$.

Remark 1. For a fixed fuzzy negation $\mathcal{N}$ on $\mathcal{L}$, if $\mathcal{S}_1$, $\mathcal{S}_2$ are two comparable t-conorms on $\mathcal{L}$ such that $\mathcal{S}_1 \leq \mathcal{S}_2$, then $\mathcal{I}_{\mathcal{S}_1,\mathcal{N}} \leq \mathcal{I}_{\mathcal{S}_2,\mathcal{N}}$. On the other side, if $\mathcal{S}$ is a fixed t-conorm on $\mathcal{L}$, then for two comparable fuzzy negations $\mathcal{N}_1$, $\mathcal{N}_2$ on $\mathcal{L}$ such that $\mathcal{N}_1 \leq \mathcal{N}_2$ we get $\mathcal{I}_{\mathcal{S},\mathcal{N}_2} \leq \mathcal{I}_{\mathcal{S},\mathcal{N}_1}$.

Example 3. (i) If $\mathcal{S}$ is any t-conorm on $\mathcal{L}$ and $\mathcal{N} = N_{\mathbf{G1}}$, then we obtain the least (S,N)-implication on $\mathcal{L}$:

$$I_{\mathbf{G1}}(x, y) = \begin{cases} 1_L, & \text{if } x = 0_L \\ y, & \text{if } x > 0_L \end{cases}, \qquad x, y \in L.$$

(ii) If $\mathcal{S}$ is any t-conorm on $\mathcal{L}$ and $\mathcal{N} = N_{\mathbf{G2}}$, then we obtain the greatest (S,N)-implication on $\mathcal{L}$:

$$I_{\mathbf{G2}}(x, y) = \begin{cases} 1_L, & \text{if } x < 1_L \\ y, & \text{if } x = 1_L \end{cases}, \qquad x, y \in L.$$

One can easily check the following algebraic properties of (S,N)-implications defined on any complete lattice L.

Theorem 3 (cf. [8])**.** *If $\mathcal{I}_{\mathcal{S},\mathcal{N}}$ is an (S,N)-implication on a complete lattice $\mathcal{L}$, then*

(i) $\mathcal{I}_{\mathcal{S},\mathcal{N}}$ is always a fuzzy implication on $\mathcal{L}$ and it satisfies the left neutrality property, i.e., $\mathcal{I}_{\mathcal{S},\mathcal{N}}(1, y) = y$, for $y \in L$, and the exchange principle, i.e.,

$$\mathcal{I}_{\mathcal{S},\mathcal{N}}(x, \mathcal{I}_{\mathcal{S},\mathcal{N}}(y, z)) = \mathcal{I}_{\mathcal{S},\mathcal{N}}(y, \mathcal{I}_{\mathcal{S},\mathcal{N}}(x, z)), \qquad x, y, z \in L. \tag{EP}$$

(ii) $\mathcal{I}_{\mathcal{S},\mathcal{N}}$ satisfies the law of contraposition with respect to $\mathcal{N}$, i.e.,

$$\mathcal{I}_{\mathcal{S},\mathcal{N}}(x, y) = \mathcal{I}_{\mathcal{S},\mathcal{N}}(\mathcal{N}(y), \mathcal{N}(x)), \qquad x, y \in L, \tag{CP}$$

if and only if $\mathcal{N}$ is a strong negation.

Remark 2 (see [8], Theorem 5)**.** The ordering property is given by the following formula

$$x \le y \Longleftrightarrow \mathcal{I}(x, y) = 1, \qquad x, y \in L. \tag{OP}$$

Let us consider the Atanassov's intuitionistic or interval-valued lattice. It is important to note that in these cases if $\mathcal{S}$ is a t-representable t-conorm and $\mathcal{N}$ is a strong negation, then $\mathcal{I}_{\mathcal{S},\mathcal{N}}$ does not satisfy the (OP).

We have the following general characterization of S-implications defined on a complete lattice $\mathcal{L}$. The proof of this result is analogous to the proof of Theorem 2.6 in [4].

Theorem 4 (cf. Baczyński, Jayaram 2007)**.** *Let $\mathcal{L} = (L, \le_L)$ be a complete lattice. For a function $\mathcal{I}\colon L^2 \to L$ the following statements are equivalent:*

(i) $\mathcal{I}$ is an S-implication on $\mathcal{L}$.

(ii) $\mathcal{I}$ is decreasing with respect to the first variable, it satisfies the exchange principle (EP) *and the function $\mathcal{N}_{\mathcal{I}}\colon L \to L$ defined as $\mathcal{N}_I(x) = \mathcal{I}(x, 0)$, for $x \in L$ is a strong negation on $\mathcal{L}$.*

Note that properties in the above theorem are mutually independent.

4.1 S-implications on lattice $\mathcal{L}^*$

In this subsection we consider the situation when a t-conrom on $\mathcal{L}^*$ is t-representable.

Theorem 5 (cf. [3], [6])**.** *Let $\mathcal{N}$ be a strong negation on $\mathcal{L}^*$ generated from the strong negation N on $([0, 1], \le)$ and let $\mathcal{S}$ be a t-representable t-conorm on L^* generated from a t-norm T and a t-conorm S such that $T(x, y) \le 1 - S(1-x, 1-y)$, for $x, y \in [0, 1]$. Then for a function $\mathcal{I}\colon (L^*)^2 \to L^*$ the following statements are equivalent:*

(i) $\mathcal{I}$ is an S-implication on $\mathcal{L}^*$ based on the t-conorm $\mathcal{S}$ and the strong negation $\mathcal{N}$.

(ii) $\mathcal{I}$ is an implication on $\mathcal{L}^*$ generated from the S-implication based on S and N, and the co-implication J based on S and the standard negation $N_{\mathbf{S}}$, respectively, and the strong negation N, i.e., it has the following form

$$\mathcal{I}((x_1, x_2), (y_1, y_2)) = (I_{S,N}(1 - x_2, y_1), J_{T,N_{\mathbf{S}}}(N(x_1), y_2))$$

for all $(x_1, x_2), (y_1, y_2) \in L^*$.

Proof. $(i) \implies (ii)$ Let us assume that $\mathcal{I}$ is an S-implication on $\mathcal{L}^*$ based on the t-representable t-conorm $\mathcal{S}$ and the strong negation $\mathcal{N}$. Then

$$\begin{aligned}\mathcal{I}((x_1,x_2),(y_1,y_2)) &= \mathcal{S}(\mathcal{N}(x_1,x_2),(y_1,y_2)) \\ &= \mathcal{S}((N(1-x_2),1-N(x_1)),(y_1,y_2)) \\ &= (S(N(1-x_2),y_1),T(1-N(x_1),y_2)) \\ &= (I_{S,N}(1-x_2,y_1),J_{T,N_\mathbf{S}}(N(x_1),y_2)),\end{aligned}$$

for any $(x_1,x_2),(y_1,y_2) \in L^*$.

$(ii) \implies (i)$ This fact can be obtained by reversing the above steps. □

For the notion of co-implications see [15], p. 31.

4.2 S-implications on lattice $\mathcal{L}^I$

In this subsection we consider the situation when a t-conrom on $\mathcal{L}^I$ is t-representable.

Theorem 6 (cf. [2], Theorem 6)**.** *Let $\mathcal{N}$ be a strong negation on L^I generated from the strong negation N on $([0,1],\leq)$ and let $\mathcal{S}$ be a t-representable t-conorm on L^I generated from two t-conorms S_1, S_2 such that $S_1 \leq S_2$. Then for a function $\mathcal{I}\colon (L^I)^2 \to L^I$ the following statements are equivalent:*

(i) *$\mathcal{I}$ is an S-implication on $\mathcal{L}^I$ based on the t-conorm $\mathcal{S}$ and the strong negation $\mathcal{N}$.*

(ii) *$\mathcal{I}$ is an implication on $\mathcal{L}^I$ generated from two S-implications based on S_1 and S_2, respectively, and the strong negation N, i.e., it has the following form*

$$\mathcal{I}([x_1,x_2],[y_1,y_2]) = [I_{S_1,N}(x_2,y_1), I_{S_2,N}(x_1,y_2)],$$

for all $[x_1,x_2],[y_1,y_2] \in L^I$.

Proof. $(i) \implies (ii)$ Let us assume that $\mathcal{I}$ is an S-implication on $\mathcal{L}^I$ based on the t-conorm $\mathcal{S}$ and the strong negation $\mathcal{N}$. Then

$$\begin{aligned}\mathcal{I}([x_1,x_2],[y_1,y_2]) &= \mathcal{S}(\mathcal{N}([x_1,x_2]),[y_1,y_2]) \\ &= \mathcal{S}([N(x_2),N(x_1)],[y_1,y_2]) \\ &= [S_1(N(x_2),y_1),S_2(N(x_1),y_2)] \\ &= [I_{S_1,N}(x_2,y_1),I_{S_2,N}(x_1,y_2)],\end{aligned}$$

for any $[x_1,x_2],[y_1,y_2] \in L^I$.

$(ii) \implies (i)$ This fact can be easily obtained by reversing the above steps. □

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) organized in Warsaw on October 16, 2009 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Centre for Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT – Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bistrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) has been meant to commence a new series of scientific events primarily focused on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Moreover, other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems are discussed.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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