Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

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Systems Research Institute
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Product of intuitionistic fuzzy groups

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Abstract

Cartesian product of intuitionistic fuzzy sets have been defined by Atanassov using the function minimum and the function maximum. In this paper we introduce and analyse the properties of a product of intuitionistic fuzzy groups. In particular we investigate conditions under which an intuitionistic fuzzy group can be expressed as the product of the simple ones.

Keywords: intuitionistic fuzzy sets, intuitionistic fuzzy groups, intuitionistic fuzzy normal subgroups, product of intuitionistic fuzzy sets.

1 Introduction

Using the notion of a fuzzy set introduced by Zadeh [12], Rosenfeld [8] defined a fuzzy subgroup of a group. It is worth to be pointed that Foster [6] gave the concept of products of fuzzy subgroups. This concept was modified by Sherwood [9] by the use of the notion of a triangular norm. Since then, many scholars have studied the theories of products of fuzzy subgroups [1, 4, 7]. After the introduction of the concept of fuzzy sets, several researches were conducted on the generalizations of the notion of fuzzy sets. The idea of 'intuitionistic fuzzy set' was introduced by Atanassov in 1983 in [3]. Xiaoping and Guijun in [10] put forward the notion of an intuitionistic fuzzy group. In the present paper we introduce the concept of the product of intuitionistic fuzzy groups and obtain some related

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results. In particular we prove that an intuitionistic fuzzy group on the Cartesian product of groups can be expressed as the product of the respective intuitionistic fuzzy groups.

2 Basic definitions

First, we recall some basic definitions for the sake of completeness. In the whole paper G (or G_i) denotes a given group.

Definition 1 (cf. [2]). *An intuitionistic fuzzy set in a universe X is an object of the form*

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\} \tag{1}$$

where $\mu_A : X \to [0,1]$ *and* $\nu_A : X \to [0,1]$ *satisfy the following condition:*

$$\mu_A(x) + \nu_A(x) \leq 1 \text{ for all } x \in X \tag{2}$$

μ_A *is called the "degree of membership of x in A",* ν_A *is called the "degree of non-membership of x in A". The class of intuitionistic fuzzy sets in a universe X will be denoted IF(X).*

Let $A, B \in IF(X)$ and
$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}, \;\; B = \{< x, \mu_B(x), \nu_B(x) > | x \in X\}$.
Then the order is defined as follows:

$$A \subseteq B \text{ iff } \mu_A(x) \leq \mu_B(x) \text{ and } \nu_A(x) \geq \nu_B(x) \text{ for all } x \in X. \tag{3}$$

Definition 2 (cf. [10]). *Let G be a classical group, the intuitionistic fuzzy set* $A = \{< x, \mu_A(x), \nu_A(x) > | x \in G\}$ *is called an intuitionistic fuzzy group on G, if the following conditions are satisfied*

$$\mu_A(xy) \geq \min\left(\mu_A(x), \mu_A(y)\right), \; \nu_A(xy) \leq \max\left(\nu_A(x), \; \nu_A(y)\right) \tag{4}$$

for all x,y∈ G,

$$\mu_A(x^{-1}) \geq \mu_A(x), \; \nu_A(x^{-1}) \leq \; \nu_A(x) \text{ for all } x \in G \tag{5}$$

The class of intuitionistic fuzzy groups on G will be denoted IFG(G).

Definition 3 (cf. [11]). *Let* $A = \{< x, \mu_A(x), \nu_A(x) > | x \in G\}$. *A∈IFG(G) is called an intuitionistic fuzzy normal subgroup on G, if*

$$\mu_A(xyx^{-1}) \geq \mu_A(y), \; \nu_A(xyx^{-1}) \leq \nu_A(y), \text{ for all } x,y \in G. \tag{6}$$

The class of intuitionistic fuzzy normal subgroups on G will be denoted IFNS(G).

Definition 4 (cf. [11]). *Let A∈IF(X),* $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$. *Then for arbitrary* $t \in [0,1]$ *the set*

$$A^t = \{x \in X | \nu_A(x) \leq t \leq \mu_A(x)\} \tag{7}$$

is called a t-level set of A.

Theorem 1 (cf. [11]). *For every* $A = \{< x, \mu_A(x), \nu_A(x) > | x \in G\}$, $A \in$ *IFNS*(G) *the condition (6) is equivalent to*

$$\mu_A(xy) = \mu_A(yx),\ \nu_A(xy) = \nu_A(yx),\ \textit{for all x,y} \in G. \tag{8}$$

Theorem 2 (cf. [11]). *Let G be a classical group. Then the intuitionistic fuzzy group A is an intuitionistic fuzzy normal subgroup on G iff* A^t *is a classical normal subgroup on G for all* $t \in [0,1]$.

3 Main result

In [2] a number of Cartesian products is defined, amongst which the following. Let $A \in IF(X_1)$, $B \in IF(X_2)$, $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X_1\}$, $B = \{< x, \mu_B(x), \nu_B(x) > | x \in X_2\}$. Then the following Cartesian product is defined:

$$A \times_4 B =$$
$$= \{< (x,y), \min(\mu_A(x), \mu_B(y), \max(\nu_A(x), \nu_B(y)) > | x \in X_1, y \in X_2\}.$$

We can adopt the above Cartesian products for intuitionistic fuzzy groups as follows.

Definition 5. *Suppose that* $A_i = \{< x_i,\ \mu_{A_i}(x_i), \nu_{A_i}(x_i) > | x_i \in G_i\}$, $A_i \in$ *IFG*(G_i) *for each i=1,2,...,n. Then the product* $A_1 \times A_2 \times ... \times A_n$ *is defined as follows*

$$A_1 \times A_2 \times ... \times A_n = \{< (x_1, x_2, ..., x_n), \min(\mu_{A_1}(x_1), ..., \mu_{A_n}(x_n)),$$
$$\max(\nu_{A_1}(x_1), ..., \nu_{A_n}(x_n)) > | x_1 \in G_1, ..., x_n \in G_n\}. \tag{9}$$

Remark 1. *The product* $A_1 \times A_2 \times ... \times A_n$ *is an intuitionistic fuzzy set in* $G_1 \times G_2 \times ... \times G_n$ *(see [2]). We see that the minimum function is used for the degree of membership and the maximum for the degree of non-membership, i.e.* $\mu_{A_1 \times A_2 \times ... \times A_n}(x_1, x_2, ..., x_n) = \min(\mu_{A_1}(x_1), ..., \mu_{A_n}(x_n))$, $\nu_{A_1 \times A_2 \times ... \times A_n}(x_1, x_2, ..., x_n) = \max(\nu_{A_1}(x_1), ..., \nu_{A_n}(x_n))$.

Theorem 3. *Let A_i ∈IFG(G_i) for each i=1,2,...,n. Then $A_1 \times A_2 \times ... \times A_n$ ∈IFG($G_1 \times G_2 \times ... \times G_n$).*

Proof. For all elements $(x_1, x_2, ..., x_n), (y_1, y_2, ..., y_n) \in G_1 \times G_2 \times ... \times G_n$ we get

$$\mu_{A_1\times A_2\times...\times A_n}((x_1,x_2,...,x_n)(y_1,y_2,...,y_n)) =$$
$$\mu_{A_1\times A_2\times...\times A_n}(x_1y_1,x_2y_2,...,x_ny_n) =$$
$$\min(\mu_{A_1}(x_1y_1),....,\mu_{A_n}(x_ny_n)) \geq$$
$$\min(\min(\mu_{A_1}(x_1),\mu_{A_1}(y_1)),...,\min(\mu_{A_n}(x_n),\mu_{A_n}(y_n))) =$$
$$\min(\min(\mu_{A_1}(x_1),...,\mu_{A_n}(x_n)),\min(\mu_{A_1}(y_1),....,\mu_{A_n}(y_n))) =$$
$$\min(\mu_{A_1\times A_2\times...\times A_n}(x_1,x_2,...,x_n),\nu_{A_1\times A_2\times...\times A_n}(y_1,y_2,...,y_n)).$$

Also,

$$\nu_{A_1\times A_2\times...\times A_n}((x_1,x_2,...,x_n)(y_1,y_2,...,y_n)) =$$
$$\nu_{A_1\times A_2\times...\times A_n}(x_1y_1,x_2y_2,...,x_ny_n) =$$
$$\max(\nu_{A_1}(x_1y_1),....,\nu_{A_n}(x_ny_n)) \leq$$
$$\max(\max(\nu_{A_1}(x_1),\nu_{A_1}(y_1)),...,\max(\nu_{A_n}(x_n),\nu_{A_n}(y_n))) =$$
$$\max(\max(\nu_{A_1}(x_1),...,\nu_{A_n}(x_n)),\max(\nu_{A_1}(y_1),...,\nu_{A_n}(y_n))) =$$
$$\max(\nu_{A_1\times A_2\times...\times A_n}(x_1,x_2,...,x_n),\nu_{A_1\times A_2\times...\times A_n}(y_1,y_2,...,y_n)).$$

The condition (4) is fulfilled. We also have

$$\mu_{A_1\times A_2\times...\times A_n}((x_1,x_2,...,x_n)^{-1}) = \mu_{A_1\times A_2\times...\times A_n}(x_1^{-1},x_2^{-1},...,x_n^{-1}) =$$
$$\min(\mu_{A_1}(x_1^{-1}),...,\mu_{A_n}(x_n^{-1})) \geq \min(\mu_{A_1}(x_1),...,\mu_{A_n}(x_n)) =$$
$$\mu_{A_1\times A_2\times...\times A_n}(x_1,x_2,...,x_n).$$

And

$$\nu_{A_1\times A_2\times...\times A_n}((x_1,x_2,...,x_n)^{-1}) = \nu_{A_1\times A_2\times...\times A_n}(x_1^{-1},x_2^{-1},...,x_n^{-1}) =$$
$$\max(\nu_{A_1}(x_1^{-1}),...,\nu_{A_n}(x_n^{-1}) \leq \max(\nu_{A_1}(x_1),...,\nu_{A_n}(x_n)) =$$
$$\nu_{A_1\times A_2\times...\times A_n}(x_1,x_2,...,x_n).$$

Thus $A_1 \times A_2 \times ... \times A_n$ is an intuitionistic fuzzy group on $G_1 \times G_2 \times ... \times G_n$. □

Theorem 4. *Let $A_i \in$ IFNS(G_i) for each $i = 1, 2, ..., n$. Then $A_1 \times A_2 \times ... \times A_n$ ∈IFNS($G_1 \times G_2 \times ... \times G_n$).*

Proof. From previous theorem we obtain that $A_1 \times A_2 \times ... \times A_n \in$ IFG($G_1 \times G_2 \times ... \times G_n$). Let us show that $A_1 \times A_2 \times ... \times A_n$ is normal.

For $(x_1, ..., x_n), (y_1, ..., y_n) \in G_1 \times G_2 \times ... \times G_n$ we have

$$\mu_{A_1 \times A_2 \times ... \times A_n}((x_1, ..., x_n)(y_1, ..., y_n)) =$$
$$\min(\mu_{A_1}(x_1 y_1), ..., \mu_{A_n}(x_n y_n)) =$$
$$\min(\mu_{A_1}(y_1 x_1), ..., \mu_{A_n}(y_n x_n)) =$$
$$\mu_{A_1 \times A_2 \times ... \times A_n}((y_1, ..., y_n)(x_1, ..., x_n)).$$

Also,

$$\nu_{A_1 \times A_2 \times ... \times A_n}((x_1, ..., x_n)(y_1, ..., y_n)) =$$
$$\max(\nu_{A_1}(x_1 y_1), ..., \nu_{A_n}(x_n y_n)) =$$
$$\max(\nu_{A_1}(y_1 x_1), ..., \nu_{A_n}(y_n x_n)) =$$
$$\nu_{A_1 \times A_2 \times ... \times A_n}((y_1, ..., y_n)(x_1, ..., x_n)).$$

Thus $A_1 \times A_2 \times ... \times A_n$ is an intuitionistic fuzzy normal subgroup on $G_1 \times G_2 \times ... \times G_n$. □

Theorem 5. *Suppose, for each i=1,2,...,n, A_i ∈IFNS(G_i) and $t \in [0, 1]$. Then $(A_1 \times A_2 \times ... \times A_n)^t$ is a classical normal subgroup on $G_1 \times G_2 \times ... \times G_n$ and $(A_1 \times A_2 \times ... \times A_n)^t = A_1^t \times A_2^t \times ... \times A_n^t$.*

Proof. By previous Theorem $A_1 \times A_2 \times ... \times A_n$ ∈IFNS($G_1 \times G_2 \times ... \times G_n$). Hence and Theorem 2 we get that for arbitrary $t \in [0, 1]$, $(A_1 \times A_2 \times ... \times A_n)^t$ is a classical normal subgroup on $G_1 \times G_2 \times ... \times G_n$.
Let $A_i = \{< x_i, \mu_{A_i}(x_i), \nu_{A_i}(x_i) > | x_i \in G_i\}$.
For the element $(a_1, a_2, ..., a_n) \in (A_1 \times A_2 \times ... \times A_n)^t$, using Definition 4 we can write

$$\nu_{A_1 \times A_2 \times ... \times A_n}(a_1, a_2, ..., a_n) \leq t \leq \mu_{A_1 \times A_2 \times ... \times A_n}(a_1, a_2, ..., a_n).$$

From here

$$\max(\nu_{A_1}(a_1), ..., \nu_{A_n}(a_n)) \leq t \leq \min(\mu_{A_1}(a_1), ..., \mu_{A_n}(a_n)).$$

Then we have

$$\mu_{A_1}(a_1) \geq t, ..., \mu_{A_n}(a_n) \geq t \ \ and \ \ \nu_{A_1}(a_1) \leq t, ..., \nu_{A_n}(a_n) \leq t.$$

Hence we get

$$\nu_{A_1}(a_1) \leq t \leq \mu_{A_1}(a_1), ..., \nu_{A_n}(a_n) \leq t \leq \mu_{A_n}(a_n).$$

Consequently, we obtain $a_1 \in A_1^t, ..., a_n \in A_n^t$. And so

$$(a_1, a_2, ..., a_n) \in A_1^t \times A_2^t \times ... \times A_n^t.$$

Conversely, let $(a_1, a_2, ..., a_n) \in A_1^t \times A_2^t \times ... \times A_n^t$. Then $a_i \in A_i^t$ for $i = 1, 2, ..., n$. Using Definition 4 we have

$$\nu_{A_1}(a_1) \leq t \leq \mu_{A_1}(a_1), ..., \nu_{A_n}(a_n) \leq t \leq \mu_{A_n}(a_n).$$

This gives us

$$\max(\nu_{A_1}(a_1), ..., \nu_{A_n}(a_n)) \leq t \leq \min(\mu_{A_1}(a_1), ..., \mu_{A_n}(a_n)).$$

Then we have

$$\nu_{A_1 \times A_2 \times ... \times A_n}(a_1, a_2, ..., a_n) \leq t \leq \mu_{A_1 \times A_2 \times ... \times A_n}(a_1, a_2, ..., a_n).$$

Finally we get

$$(a_1, a_2, ..., a_n) \in (A_1 \times A_2 \times ... \times A_n)^t.$$

□

When the intuitionistic fuzzy group can be expressed as the product of the simple ones? The following theorem deal with such a question.

Theorem 6. *Let $A_1, A_2, ..., A_n$ be intuitionistic fuzzy subsets in the groups $G_1, G_2, ..., G_n$, respectively, such that*
$A_i = \{< x_i, \mu_{A_i}(x_i), \nu_{A_i}(x_i) > | x_i \in G_i\}$ and

$$\mu_{A_i}(x_i) \leq \mu_{A_1 \times ... \times A_{i-1} \times A_{i+1} \times ... \times A_n}(e_1, ..., e_{i-1}, e_{i+1}, ..., e_n) \quad (10)$$
$$\nu_{A_i}(x_i) \geq \nu_{A_1 \times ... \times A_{i-1} \times A_{i+1} \times ... \times A_n}(e_1, ..., e_{i-1}, e_{i+1}, ..., e_n) \quad (11)$$

for all x_i in G_i, e_i being the identity element of G_i. If $A_1 \times A_2 \times ... \times A_n \in$IFG($G_1 \times G_2 \times ... \times G_n$), then $A_i \in$IFG(G_i).

Proof. Let $A_1 \times A_2 \times ... \times A_n$ be an intuitionistic fuzzy group on $G_1 \times G_2 \times ... \times G_n$ and $x_i, y_i \in G_i$.

Then

$$(e_1, ..., e_{i-1}, x_i, e_{i+1}, ..., e_n), (e_1, ..., e_{i-1}, y_i, e_{i+1}, ..., e_n) \in G_1 \times G_2 \times ... \times G_n.$$

Now, using (10) and (11), we get

$$\mu_{A_i}(x_i y_i) =$$
$$\min(\mu_{A_i}(x_i y_i), \mu_{A_1 \times ... \times A_{i-1} \times A_{i+1} \times ... \times A_n}((e_1, ..., e_{i-1},$$
$$e_{i+1}, ..., e_n)(e_1, ..., e_{i-1}, e_{i+1}, ..., e_n))) =$$

$$
\begin{aligned}
&\min(\mu_{A_i}(x_iy_i), \min(\mu_{A_1}(e_1e_1), ..., \mu_{A_{i-1}}(e_{i-1}e_{i-1}),\\
&\mu_{A_{i+1}}(e_{i+1}e_{i+1}), ..., \mu_{A_n}(e_ne_n))) =\\
&\min(\mu_{A_1}(e_1e_1), ..., \mu_{A_{i-1}}(e_{i-1}e_{i-1}), \mu_{A_i}(x_iy_i),\\
&\mu_{A_{i+1}}(e_{i+1}e_{i+1}), ..., \mu_{A_n}(e_ne_n)) =\\
&\mu_{A_1\times...\times A_i\times...\times A_n}(e_1e_1, ..., e_{i-1}e_{i-1}, x_iy_i, e_{i+1}e_{i+1}, ..., e_ne_n) =\\
&\mu_{A_1\times...\times A_i\times...\times A_n}((e_1, ..., e_{i-1},\\
&x_i, e_{i+1}, ..., e_n)(e_1, ..., e_{i-1}, y_i, e_{i+1}, ..., e_n)) \geq\\
&\min(\mu_{A_1\times...\times A_i\times...\times A_n}(e_1, ..., e_{i-1}, x_i, e_{i+1}, ..., e_n),\\
&\mu_{A_1\times...\times A_i\times...\times A_n}(e_1, ..., e_{i-1}, y_i, e_{i+1}, ..., e_n)) =\\
&\min(\min(\mu_{A_i}(x_i), \mu_{A_1\times...\times A_{i-1}\times A_{i+1}\times...\times A_n}(e_1, ..., e_{i-1}, e_{i+1}, ..., e_n)),\\
&\min(\mu_{A_i}(y_i), \mu_{A_1\times...\times A_{i-1}\times A_{i+1}\times...\times A_n}(e_1, ..., e_{i-1}, e_{i+1}, ..., e_n))) =\\
&= \min(\mu_{A_i}(x_i), \mu_{A_i}(y_i)).
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
&\nu_{A_i}(x_iy_i) = \max(\nu_{A_i}(x_iy_i), \mu_{A_1\times...\times A_{i-1}\times A_{i+1}\times...\times A_n}((e_1, ..., e_{i-1},\\
&e_{i+1}, ..., e_n)(e_1, ..., e_{i-1}, e_{i+1}, ..., e_n))) = \nu_{A_1\times...\times A_i\times...\times A_n}((e_1, ..., e_{i-1},\\
&x_i, e_{i+1}, ..., e_n)(e_1, ..., e_{i-1}, y_i, e_{i+1}, ..., e_n)) \leq \max(\nu_{A_1\times...\times A_i\times...\times A_n}(e_1,\\
&, ..., e_{i-1}, x_i, e_{i+1}, ..., e_n), \nu_{A_1\times...\times A_i\times...\times A_n}(e_1, ..., e_{i-1}, y_i, e_{i+1}, ..., e_n))\\
&= \max(\max(\nu_{A_i}(x_i), \nu_{A_1\times...\times A_{i-1}\times A_{i+1}\times...\times A_n}(e_1, ..., e_{i-1}, e_{i+1}, ..., e_n)),\\
&\max(\nu_{A_i}(y_i), \mu_{A_1\times...\times A_{i-1}\times A_{i+1}\times...\times A_n}(e_1, ..., e_{i-1}, e_{i+1}, ..., e_n))) =\\
&= \max(\nu_{A_i}(x_i), \nu_{A_i}(y_i)).
\end{aligned}
$$

Also,

$$
\begin{aligned}
&\mu_{A_i}(x_i^{-1}) = \min(\mu_{A_i}(x_i^{-1}), \mu_{A_1\times...\times A_{i-1}\times A_{i+1}\times...\times A_n}(e_1^{-1}, ..., e_{i-1}^{-1}, e_{i+1}^{-1},\\
&, ..., e_n^{-1})) = \mu_{A_1\times...\times A_i\times...\times A_n}(e_1^{-1}, ..., e_{i-1}^{-1}, x_i^{-1}, e_{i+1}^{-1}, ..., e_n^{-1}) =\\
&\mu_{A_1\times...\times A_i\times...\times A_n}((e_1, ..., e_{i-1}, x_i, e_{i+1}, ..., e_n)^{-1}) \geq\\
&\geq \mu_{A_1\times...\times A_i\times...\times A_n}(e_1, ..., e_{i-1}, x_i, e_{i+1}, ..., e_n) =\\
&\min(\mu_{A_i}(x_i), \mu_{A_1\times...\times A_{i-1}\times A_{i+1}\times...\times A_n}(e_1, ..., e_{i-1}, e_{i+1}, ..., e_n))\\
&= \mu_{A_i}(x_i).
\end{aligned}
$$

And

$$
\begin{aligned}
&\nu_{A_i}(x_i^{-1}) = \max(\nu_{A_i}(x_i^{-1}), \nu_{A_1\times...\times A_{i-1}\times A_{i+1}\times...\times A_n}(e_1^{-1}, ..., e_{i-1}^{-1}, e_{i+1}^{-1},\\
&, ..., e_n^{-1})) = \nu_{A_1\times...\times A_i\times...\times A_n}(e_1^{-1}, ..., e_{i-1}^{-1}, x_i^{-1}, e_{i+1}^{-1}, ..., e_n^{-1}) =\\
&\nu_{A_1\times...\times A_i\times...\times A_n}((e_1, ..., e_{i-1}, x_i, e_{i+1}, ..., e_n)^{-1}) \leq\\
&\leq \nu_{A_1\times...\times A_i\times...\times A_n}(e_1, ..., e_{i-1}, x_i, e_{i+1}, ..., e_n) =\\
&\max(\nu_{A_i}(x_i), \nu_{A_1\times...\times A_{i-1}\times A_{i+1}\times...\times A_n}(e_1, ..., e_{i-1}, e_{i+1}, ..., e_n))\\
&= \nu_{A_i}(x_i).
\end{aligned}
$$

Hence A_i is an intuitionistic fuzzy group on G_i. This completes the proof. □

4 Conclusions

In this contribution the definition of the product of intuitionistic fuzzy groups is considered. Some properties of this product are proved. These results can be useful in further examinations in the field of generalized Cartesian product (see [5]).

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) organized in Warsaw on October 16, 2009 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Centre for Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT – Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bistrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) has been meant to commence a new series of scientific events primarily focused on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Moreover, other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems are discussed.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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