Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

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On intuitionistic fuzzy trees

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1 Introduction

The concept of Intuitionistic Fuzzy Graph (IFG) was introduced in 1994 in [11]. It was an object of some subsequent extensions (see [4, 6, 12]), representations (see [2, 3, 5]) and applications (see [5]).

In [8] we discussed an intuitionistic fuzzy version of the special particular case of a graph – the tree, called an *Intuitionistic Fuzzy Tree (IFTree).* In [9] we gave the index matrix (IM, see [1, 7]) interpretation and gave an example for an application of the IFTrees.

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Here we shall introduce operation "substitution" of a leaf of an IFTrees with whole IFTrees.

2 Definition and properties of intuitionistic fuzzy trees

Let a set E be fixed. An IFS A in E is an object of the following form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ determine the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Let the oriented graph $G = (V, A)$ be given, where V is a set of vertices and A is a set of arcs. Every graph arc connects one or two graph vertices (see, e.g., [10]).

Following [11], we shall note that the set

$$A^* = \{\langle\langle v, w\rangle, \mu_A(v,w), \nu_A(v,w)\rangle \mid \langle v, w\rangle \in V \times V\}$$

is called an IFG if the functions $\mu_A : V \times V \to [0,1]$ and $\nu_A : V \times V \to [0,1]$ define the respective degrees of membership and non-membership of the element $\langle v, w\rangle \in V \times V$ and for all $\langle v, w\rangle \in V \times V$:

$$0 \leq \mu_A(v,w) + \nu_A(v,w) \leq 1.$$

The above definition can be transformed directly to the case of IFTree, but we will extend the new object.

Let us have a (fixed) set of vertices $\mathcal{V}$. An IFTree T (over $\mathcal{V}$) will be the ordered pair $T = (V^*, A^*)$, where

$$V \subset \mathcal{V},$$
$$V^* = \{\langle v, \mu_V(v), \nu_V(v)\rangle | v \in V\},$$
$$A \subset V \times V,$$
$$A^* = \{\langle g, \mu_A(g), \nu_A(g)\rangle | (\exists v, w \in V)(g = \langle v, w\rangle \in A)\},$$

where
$\mu_V(v)$ and $\nu_V(v)$ are degrees of membership and non-membership of the element $v \in \mathcal{V}$ to V and

$$0 \leq \mu_V(v) + \nu(v) \leq 1.$$

$\mu_A(g)$ and $\nu_A(g)$ are degrees of membership and non-membership of the element $g = \langle v, w\rangle \in V \times V$ to A and

$$0 \leq \mu_A(g) + \nu_A(g) \leq 1.$$

The IFTree $T = (V^*, A^*)$ is:
a) *weak well constructed (WWC-IFTree)* if

$$(\forall v, w \in V)((\exists g \in A)(g = \langle v, w\rangle) \rightarrow (\mu_V(v) \geq \mu_V(w) \& \nu_V(v) \leq \nu_V(w));$$

b) *strong well constructed (SWC-IFTree)* if

$$(\forall v, w \in V)((\exists g \in A)(g = \langle v, w\rangle)$$
$$\rightarrow (\mu_V(v) \geq \max(\mu_V(w), \mu_A(g)) \& \nu_V(v) \leq \min(\nu_V(w), \nu_A(g)));$$

c) *average well constructed (AWC-IFTree)* if

$$(\forall v, w \in V)((\exists g \in A)$$
$$(g = \langle v, w\rangle) \rightarrow (\mu_V(v) \geq \frac{\mu_V(w) + \mu_A(g)}{2} \& \nu_V(v) \leq \frac{\nu_V(w) + \nu_A(g)}{2}).$$

Let two IFTrees $T_1 = (V_1^*, G_1^*)$ and $T_2 = (V_2^*, G_2^*)$ be given. We define:

$$T_1 \cup T_2 = (V_1^*, A_1^*) \cup (V_2^*, A_2^*) = (V_1^* \cup V_2^*, A_1^* \cup A_2^*),$$
$$T_1 \cap T_2 = (V_1^*, A_1^*) \cap (V_2^*, A_2^*) = (V_1^* \cap V_2^*, A_1^* \cap A_2^*).$$

Let

$$\mathcal{P}(X) = \{Y | Y \subset X\},$$

and let for $T = (V^*, A^*)$

$$T_{full} = (E(V), E(A)),$$
$$T_{empty} = (O(V), O(A)),$$

where

$$E(V) = \{\langle v, 1, 0\rangle | v \in \mathcal{V}\},$$
$$O(V) = \{\langle v, 0, 1\rangle | v \in \mathcal{V}\},$$
$$E(A) = \{\langle g, 1, 0\rangle | (\exists v, w \in V)(g = \langle v, w\rangle) \in \mathcal{V} \times \mathcal{V}\},$$
$$O(A) = \{\langle g, 0, 1\rangle | (\exists v, w \in V)(g = \langle v, w\rangle) \in \mathcal{V} \times \mathcal{V}\}.$$

Theorem: $(\mathcal{P}(\mathcal{V}), \cup, T_{empty})$ and $(\mathcal{P}(\mathcal{V}), \cap, T_{full})$ are commutative monoids.

3 Index matrix interpretation of the intuitionistic fuzzy trees

Following [1] the basic definitions and properties related to IMs will be given.

Let I be a fixed set of indices and $\mathcal{R}$ be the set of the real numbers. By an IM with index sets K and L $(K, L \subset I)$ we will consider the object:

$$[K, L, \{a_{k_i,l_j}\}] \equiv \begin{array}{c|cccc} & l_1 & l_2 & \dots & l_n \\ \hline k_1 & a_{k_1,l_1} & a_{k_1,l_2} & \dots & a_{k_1,l_n} \\ k_2 & a_{k_2,l_1} & a_{k_2,l_2} & \dots & a_{k_2,l_n} \\ \vdots & & & & \\ k_m & a_{k_m,l_1} & a_{k_m,l_2} & \dots & a_{k_m,l_n} \end{array}$$

(or briefly: $[K, L, \{a_{k_i,l_j}\}]$), where $K = \{k_1, k_2, ..., k_m\}$, $L = \{l_1, l_2, ..., l_n\}$, for $1 \le i \le m$, and for $1 \le j \le n : a_{k_i,l_j} \in \mathcal{R}$ – the set of the real numbers.

For any two IMs different operations and relations are defined in [1, 7]. Here we will give only one of them.

Let $G = (V, A)$ be a given IFTree. We can construct its standard incidence matrix. After this, we can change the elements of the matrix with their degrees of membership and non-membership. Finally, numbering the rows and columns of the matrix with the identifiers of the IFTree vertices, we will obtain an IM.

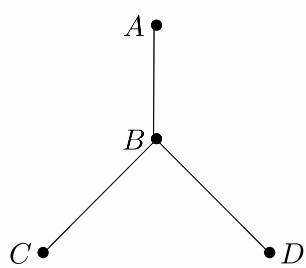

Figure 1.

For example, if we have the IFTree from Fig. 1, we can construct the IM that corresponds to its incidence matrix:

$$[\{A,B,C,D\},\{A,B,C,D\},$$

$$\begin{array}{c|cc} & A & B \\ \hline A & \langle\mu(A,A),\nu(A,A)\rangle & \langle\mu(A,B),\nu(A,B)\rangle \\ B & \langle\mu(B,A),\nu(B,A)\rangle & \langle\mu(B,B),\nu(B,B)\rangle \\ C & \langle\mu(C,A),\nu(C,A)\rangle & \langle\mu(C,B),\nu(C,B)\rangle \\ D & \langle\mu(D,A),\nu(D,A)\rangle & \langle\mu(D,B),\nu(D,B)\rangle \end{array} \ ...$$

$$... \begin{array}{c|cc} & C & D \\ \hline A & \langle\mu(A,C),\nu(A,C)\rangle & \langle\mu(A,D),\nu(A,D)\rangle \\ B & \langle\mu(B,C),\nu(B,C)\rangle & \langle\mu(B,D),\nu(B,D)\rangle \\ C & \langle\mu(C,C),\nu(C,C)\rangle & \langle\mu(C,D),\nu(C,D)\rangle \\ D & \langle\mu(D,C),\nu(D,C)\rangle & \langle\mu(D,D),\nu(D,D)\rangle \end{array}].$$

Having in mind that arcs AA, AC, AD, BB, CC, CD and DD do not exist, we can modify the above IM to the form:

$$[\{A,B,C,D\},\{A,B,C,D\},$$

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & \langle 0,1\rangle & \langle\mu(A,B),\nu(A,B)\rangle & \langle 0,1\rangle & \langle 0,1\rangle \\ B & \langle 0,1\rangle & \langle 0,1\rangle & \langle\mu(B,C),\nu(B,C)\rangle & \langle\mu(B,D),\nu(B,D)\rangle \\ C & \langle 0,1\rangle & \langle 0,1\rangle & \langle 0,1\rangle & \langle 0,1\rangle \\ D & \langle 0,1\rangle & \langle 0,1\rangle & \langle 0,1\rangle & \langle 0,1\rangle \end{array}].$$

Now, we see, that all elements of the column indexed with A and all elements of the rows indexed with C and D are $\langle 0,1\rangle$. Therefore, we can omit these two rows and the column and we will obtain the essentially simpler IM:

$$[\{A,B,C,D\},\{A,B,C,D\},$$

$$\begin{array}{c|ccc} & B & C & D \\ \hline A & \langle\mu(A,B),\nu(A,B)\rangle & \langle 0,1\rangle & \langle 0,1\rangle \\ B & \langle 0,1\rangle & \langle\mu(B,C),\nu(B,C)\rangle & \langle\mu(B,D),\nu(B,D)\rangle \end{array}].$$

Finally, having in mind that there is no more a column indexed with A and rows indexed with C and D, we obtain as a final form of the IM:

$$[\{A,B\},\{B,C,D\},$$

$$\begin{array}{c|ccc} & B & C & D \\ \hline A & \langle\mu(A,B),\nu(A,B)\rangle & \langle 0,1\rangle & \langle 0,1\rangle \\ B & \langle 0,1\rangle & \langle\mu(B,C),\nu(B,C)\rangle & \langle\mu(B,D),\nu(B,D)\rangle \end{array}].$$

4 Operation “substitution of an IFTree’s leaf with an IFTree”

Let us have an IFTree $G = (V, A)$ and let L be one of its leaves. Let $F = (W, B)$ be another IFTree so that

$$V \cap W = \{L\},$$

$$A \cup B = \emptyset.$$

Then, following definitions from [7], we can describe the result of operation “substitution of an IFTree’s leaf L with the IFTree F. The result will have the form of the IFTree $(V \cup W, A \cup B)$.

For example, if G is the IFTree from Fig. 1 and if we like to substitute its leaf D with the IFTree F from Fig. 2 that has the shorter IM-representation

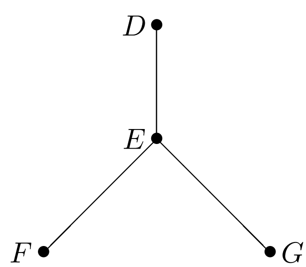

Figure 2.

$$[\{D, E\}, \{E, F, G\},$$

	E	F	G
D	$\langle \mu(D,E), \nu(D,E) \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$
E	$\langle 0, 1 \rangle$	$\langle \mu(E,F), \nu(E,F) \rangle$	$\langle \mu(E,G), \nu(E,G) \rangle$

then, the result will be the IFTree from Fig. 3 and it will have the IM-representation

$$[\{A, B, D, E\}, \{B, C, D, E, F, G\},$$

	B	C	D	
A	$\langle \mu(A,B), \nu(A,B) \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	
B	$\langle 0, 1 \rangle$	$\langle \mu(B,C), \nu(B,C) \rangle$	$\langle \mu(B,D), \nu(B,D) \rangle$	$\ldots$
D	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	
E	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	

…

	E	F	G
A	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$
B	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$
D	$\langle \mu(D,E),\nu(D,E)\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$
E	$\langle 0,1\rangle$	$\langle \mu(E,F),\nu(E,F)\rangle$	$\langle \mu(E,G),\nu(E,G)\rangle$

$$= [\{A,B,D,E\},\{B,C,D,E,F,G\},$$

	B	C	D
A	$\langle \mu(A,B),\nu(A,B)\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$
B	$\langle 0,1\rangle$	$\langle \mu(B,C),\nu(B,C)\rangle$	$\langle \mu(B,D),\nu(B,D)\rangle$

$\oplus$

	E	F	G
D	$\langle \mu(D,E),\nu(D,E)\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$
E	$\langle 0,1\rangle$	$\langle \mu(E,F),\nu(E,F)\rangle$	$\langle \mu(E,G),\nu(E,G)\rangle$

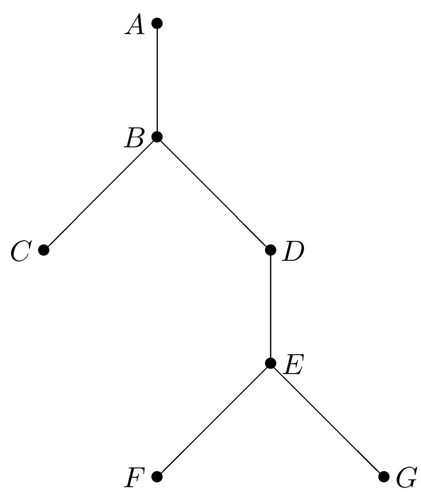

Figure 3.

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) organized in Warsaw on October 16, 2009 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Centre for Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT – Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bistrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) has been meant to commence a new series of scientific events primarily focused on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Moreover, other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems are discussed.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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