Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

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On the Hamming-metric-based Hausdorff distance for intuitionistic fuzzy sets and interval-valued fuzzy sets

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Abstract

The problem of constructing the Hausdorff distance between Atanassov's intuitionistic fuzzy sets (A-IFSs for short) based on the Hamming metric is considered. Particular attention is paid to the consistency of the metric used and the essence of the Hausdorff distances. We also consider the same problem for interval-valued fuzzy sets. It is shown that essence of solutions obtained is different for the case of intuitionistic and interval-valued fuzzy sets.

Keywords: Intuitionistic fuzzy sets, distances, Hausdorff distance, Hamming metric.

1 Introduction

Distances are one of the most important measures used both in theoretical considerations and for practical purposes in many areas. It is not possible to overestimate their importance also in the context of fuzzy sets (Zadeh [45]) or their

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generalizations. Distances are necessary while concluding about entropy, similarity, when making group decisions, assessing soft consensus, in classifications, pattern recognition, etc.

The Hausdorff distances (cf. Grünbaum [9]) play an important role in practical applications, especially in many visual tasks, such as image matching, image analysis, motion tracking, visual navigation of robots, computer-assisted surgery and so on (cf. e.g., Huttenlocher et al. [10], Huttenlocher and Rucklidge [11], Olson [14], Peitgen et al. [15], Rucklidge [17]-[21]).

The definition of the Hausdorff distances is simple but the calculations needed to solve the real problems are complex. In result the efficiency of the algorithms for computing the Hausdorff distances is decisive and computing approximations are of most interest (e.g, Aichholzer [1], Atallah [2], Huttenlocher et al. [10], Preparata and Shamos [16], Rucklidge [21], Veltkamp [44]).

The formulas proposed for calculating the distances should be reliable. It is the motivation of this paper. Namely, we consider the results of using the Hamming distances between A-IFSs calculated in two possible ways - taking into account two term representation (membership and non-membership values) of A-IFSs, and next - taking into account three term representation (membership, non-membership values, and the hesitation margins) of A-IFSs. We will verify if the resulting distances fulfill the properties of the Hausdorff distances.

We also consider the problem of calculating the Hausdorff distance (based on the Hamming metric) for the interval-valued fuzzy sets. We show that the formulas that are effective and efficient for interval-valued fuzzy sets do not work well in the case of A-IFSs.

2 Brief introduction to A-IFSs

One of the generalizations of a fuzzy set in X (Zadeh [45]) , given by

$$A^{'} = \{< x, \mu_{A'}(x) > | x \in X\} \tag{1}$$

where $\mu_{A'}(x) \in [0,1]$ is the membership function of the fuzzy set $A^{'}$, is an A-IFS (Atanassov [3], [4]) A given by

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\} \tag{2}$$

where: $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ such that

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \tag{3}$$

and $\mu_A(x)$, $\nu_A(x) \in [0, 1]$ denote a degree of membership and a degree of non-membership of $x \in A$, respectively. (Two approaches to the assigning memberships and non-memberships for A-IFSs are proposed by Szmidt and Baldwin [24]).

Obviously, each fuzzy set may be represented by the following A-IFS

$$A = \{< x, \mu_{A'}(x), 1 - \mu_{A'}(x) > | x \in X\} \tag{4}$$

An additional concept for each A-IFS in X, that is not only an obvious result of (2) and (3) but which is also relevant for applications, we will call (Atanassov [4])

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \tag{5}$$

a *hesitation margin (an intuitionistic fuzzy index)* of $x \in A$ which expresses a lack of knowledge of whether x belongs to A or not (cf. Atanassov [4]). It is obvious that $0 \leq \pi_A(x) \leq 1$, for each $x \in X$.

The hesitation margin turns out to be important while considering the distances (Szmidt and Kacprzyk [26], [28], [35], entropy (Szmidt and Kacprzyk [31], [38]), similarity (Szmidt and Kacprzyk [39]) for the A-IFSs, etc. i.e., the measures that play a crucial role in virtually all information processing tasks.

Also from the point of view of the applications, the hesitation margin is crucial - in image processing (cf. Bustince et al. [6], [7]), classification of imbalanced and overlapping classes (cf. Szmidt and Kukier [37], [40], [41]), group decision making, negotiations, voting and other situations (cf. Szmidt and Kacprzyk [25], [27], [29], [30], [32], [33], [34], [36]).

2.1 Distances between the A-IFSs

Distances between A-IFSs are calculated in the literature in two ways, using two terms only (e.g., Atanassov [4]) or all three terms (Szmidt and Kacprzyk [28], [35], Tasseva et al. [43], Atanassov et al. [5], Szmidt and Baldwin [22], [23], Deng-Feng [8], Tan and Zhang [42], Narukawa and Torra [13])) describing elements belonging to the sets. Both ways are proper from the point of view of pure mathematical conditions concerning distances (all properties are fulfilled in both cases). Unfortunately one cannot say that both ways are equal when assessing the results obtained by the two approaches.

In Szmidt and Kacprzyk [28], [35], Szmidt and Baldwin [22], [23], it is shown why in the calculation of distances between A-IFSs one should use all three terms describing A-IFSs. Examples of the distances between any two A-IFSs A and B in $X = \{x_{1,}\ x_2, \ldots, x_n\}$ while using three term representation (Szmidt and Kacprzyk [28], Szmidt and Baldwin [22], [23]):

- the normalized Hamming distance:

$$l_{IFS}(A,B) = \frac{1}{2n}\sum_{i=1}^{n}(|\mu_A(x_i)-\mu_B(x_i)| + |\nu_A(x_i)-\nu_B(x_i)| + |\pi_A(x_i)-\pi_B(x_i)|) \quad (6)$$

- the normalized Euclidean distance:

$$e_{IFS}(A,B) = (\frac{1}{2n}\sum_{i=1}^{n}(\mu_A(x_i)-\mu_B(x_i))^2 + (\nu_A(x_i)-\nu_B(x_i))^2 + (\pi_A(x_i)-\pi_B(x_i))^2)^{\frac{1}{2}} \quad (7)$$

Both distances are from the interval [0,1].

The counterparts of the above distances while using the two term representation of A-IFSs are:

- the normalized Hamming distance:

$$l'(A,B) = \frac{1}{2n}\sum_{i=1}^{n}(|\mu_A(x_i)-\mu_B(x_i)| + |\nu_A(x_i)-\nu_B(x_i)|) \quad (8)$$

- the normalized Euclidean distance:

$$q'(A,B) = (\frac{1}{2n}\sum_{i=1}^{n}(\mu_A(x_i)-\mu_B(x_i))^2 + (\nu_A(x_i)-\nu_B(x_i))^2)^{\frac{1}{2}} \quad (9)$$

3 The Hausdorff distance

The Hausdorff distance is *the maximum distance of a set to the nearest point in the other set.* More formal description is given by the following

Definition 1 Given two finite sets $A = \{a_1, ..., a_p\}$ and $B = \{b_1, ..., b_q\}$, the Hausdorff distance $H(A,B)$ is defined as:

$$H(A,B) = \max\{h(A,B), h(B,A)\} \quad (10)$$

where

$$h(A,B) = \max_{a\in A}\min_{b\in B} d(a,b) \quad (11)$$

– a and b are elements of sets A and B respectively,
– $d(a,b)$ is any metric between these elements,
– the two distances $h(A,B)$ and $h(B,A)$ (11) are called directed Hausdorff distances.

The function $h(A, B)$ (the directed Hausdorff distance from A to B) ranks each element of A based on its distance to the nearest element of B, and then the largest ranked such element (the most mismatched element of A) specifies the value of the distance. Intuitively, if $h(A, B) = c$, then each element of A must be within distance c of some element of B, and there also is some element of A that is exactly distance c from the nearest element of B (the most mismatched element). In general $h(A, B)$ and $h(B, A)$ can attain very different values (the directed distances are not symmetric).

The algorithm of calculating the directed Hausdorff distances (11):

1. $h(A, B) = 0$
2. for every element a_i of A,
   - 2.1 $shortest\ \ distance = \infty$;
   - 2.2 for every point b_j of B
     - $d_{ij} = d(a_i, b_j)$
     - if $d_{ij} < shortest\ \ distance$ then
       - $shortest\ \ distance = d_{ij}$
   - 2.3 if $shortest\ \ distance > h(A, B)$ then
     - $h(A, B) = shortest\ \ distance$

The above algorithm is illustrated in Fig. 1

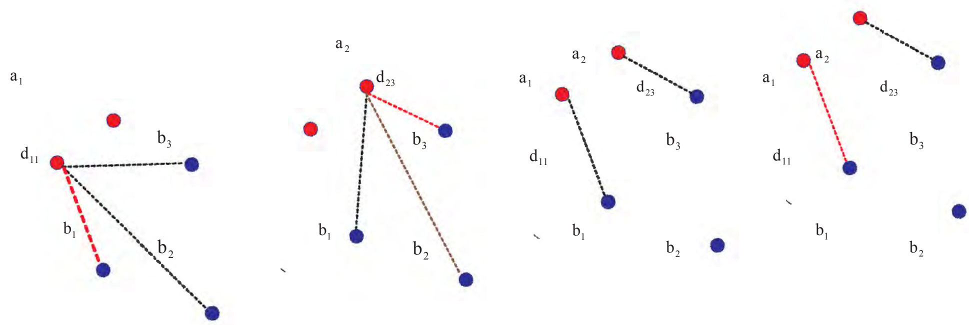

Figure 1: Steps of calculations (Hausdorff distance is one of the calculated $d(a, b)$)

It follows from Definition 1, the above algorithm, and Fig. 1 that if A and B contain one element each (a_1 and b_1, respectively), the Hausdorff distance is just equal to $d(a_1, b_1)$. In other words, if a formula which is expected to express the Hausdorff distance gives for the separate elements the results not consistent with the used metric d (e.g., the Hamming distance, the Euclidean distance etc.), the considered formula is not a proper definition of the Hausdorff distance.

3.1 Hausdorff distance between interval-valued fuzzy sets

The Hausdorff distance between two intervals: $U = [u_1, u_2]$ and $W = [w_1, w_2]$ is (Moore [12]):

$$h(U, W) = \max\{|u_1 - w_1|,\ |u_2 - w_2|\} \tag{12}$$

If we assume the two-term representation for A-IFSs: $A = \{x, \mu_A(x), \nu_A(x)\}$ and $B = \{x, \mu_B(x), \nu_B(x)\}$, we may consider A-IFSs as two intervals, namely:

$$[\mu_A(x), 1 - \nu_A(x)] \quad \text{and} \quad [\mu_B(x), 1 - \nu_B(x)] \tag{13}$$

then

$$h(A, B) = \max\{|\mu_A(x) - \mu_B(x)|,\ |\nu_A(x) - \nu_B(x)|\} \tag{14}$$

Later we will verify if (14) is a properly calculated Hausdorff distance between the A-IFSs while using the Hamming metric.

3.2 Hausdorff distance between A-IFSs while using the two term Hamming distance

Due to the algorithm of calculating the directed Hausdorff distances, when applying the two term distance (8) for A-IFSs, we obtain:

$$d_h(A, B) \quad = \quad \frac{1}{n}\sum_{i=1}^{n} max\{|\mu_A(x_i) - \mu_B(x_i)|\,, |\nu_A(x_i) - \nu_B(x_i)|\} \tag{15}$$

If the above distance is properly calculated Hausdorff distance, in the case of degenerated, i.e., one-element sets $A = \{< x, \mu_A(x), \nu_A(x) >\}$ and $B = \{< x,\ \mu_B(x), \nu_B(x) >\}$, it should give the same results as the two term Hamming distance. It means that in the case of the two parameter Hamming distance, for one element A-IFSs, the following equations should give just the same results:

$$l^{'}(A, B) = \frac{1}{2}(|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)|) \tag{16}$$

$$d_h(A, B) = \max\{|\mu_A(x) - \mu_B(x)|\,, |\nu_A(x) - \nu_B(x)|\} \tag{17}$$

where (16) is the normalized two term Hamming distance, and (17) should be its counterpart Hausdorff distance.

We will verify on the simple examples if (16) and (17) gives the same results as it should be following the essence of of the Hausdorff measures.

Example 1
Let consider the following one-element A-IFSs: A, B, D, G, $E \in X = \{x\}$

$$\begin{aligned} A &= \{< x, 1, 0 >\}, \quad B = \{< x, 0, 1 >\}, \quad D = \{< x, 0, 0 >\}, \\ G &= \{< x, \frac{1}{2}, \frac{1}{2} >\}, \quad E = \{< x, \frac{1}{4}, \frac{1}{4} >\} \end{aligned} \tag{18}$$

The results obtained from (17) are:

$$\begin{aligned} d_h(A, B) &= \max\{|1-0|, |0-1|\} = 1 \\ d_h(A, D) &= \max\{|1-0|, |0-0|\} = 1 \\ d_h(B, D) &= \max\{|0-0|, |1-0|\} = 1 \\ d_h(A, G) &= \max\{|1-1/2|, |0-1/2|\} = 0.5 \\ d_h(A, E) &= \max\{|1-1/4|, |0-1/4|\} = 0.75 \\ d_h(B, G) &= \max\{|0-1/2|, |1-1/2|\} = 0.5 \\ d_h(B, E) &= \max\{|0-1/4|, |1-1/4|\} = 0.75 \\ d_h(D, G) &= \max\{|0-1/2|, |0-1/2|\} = 0.5 \\ d_h(D, E) &= \max\{|0-1/4|, |1-1/4|\} = 0.25 \\ d_h(G, E) &= \max\{|1/2-1/4|, |1/2-1/4|\} = 0.25 \end{aligned}$$

Their counterpart Hamming distances calculated from (16) are:

$$\begin{aligned} l^{'}(A, B) &= 0.5(|1-0| + |0-1|) = 1 \\ l^{'}(A, D) &= 0.5(|1-0| + |0-0||) = 0.5 \\ l^{'}(B, D) &= 0.5(|0-0| + |1-0||) = 0.5 \\ l^{'}(A, G) &= 0.5(|0-1/2| + |0-1/2|) = 0.5 \\ l^{'}(A, E) &= 0.5(|1-1/4| + |0-1/4||) = 0.5 \\ l^{'}(B, G) &= 0.5(|1-1/4| + |0-1/4|) = 0.5 \\ l^{'}(B, E) &= 0.5(|1-1/4| + |0-1/4|) = 0.5 \\ l^{'}(D, G) &= 0.5(|0-1/2| + |0-1/2|) = 0.5 \\ l^{'}(D, E) &= 0.5(|0-1/4| + |0-1/4|) = 0.25 \\ l^{'}(G, E) &= 0.5(|1/2-1/4| + |1/2-1/4|) = 0.25 \end{aligned}$$

i.e. the values of the Hamming distances (16) used to propose the Hausdorff measures (17), and the values of the resulting Hausdorff distances (17) calculated for the separate elements are not consistent (as they should be). The differences:

$$d_h(A,D) \neq l^{'}(A,D) \tag{19}$$
$$d_h(B,D) \neq l^{'}(B,D) \tag{20}$$
$$d_h(A,E) \neq l^{'}(A,E) \tag{21}$$
$$d_h(B,E) \neq l^{'}(B,E) \tag{22}$$

Now we will show that the inconsistencies as showed above occur not only for (19) – (22) but for infinite number of other cases.

Let us verify the conditions under which the equation (16) and (17) give the consistent results, i.e., when for the separate elements we have

$$\begin{aligned} & \frac{1}{2}(|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)|) = \\ = \; & \max\{|\mu_A(x) - \mu_B(x)|, |\nu_A(x) - \nu_B(x)|\} \end{aligned} \tag{23}$$

Having in mind that

$$\mu_A(x) + \nu_A(x) + \pi_A(x) = 1 \tag{24}$$

$$\mu_B(x) + \nu_B(x) + \pi_B(x) = 1 \tag{25}$$

from (24) and (25) we obtain

$$(\mu_A(x) - \mu_B(x)) + (\nu_A(x) - \nu_B(x)) + (\pi_A(x) - \pi_B(x)) = 0 \tag{26}$$

It is easy to verify that (26) is not fulfilled for all elements belonging to an A-IFSs but for some elements only. The following conditions guarantee that (23) is fulfilled

- for $\pi_A(x) - \pi_B(x) = 0$, from (26) we have

$$|\mu_A(x) - \mu_B(x)| = |\nu_A(x) - \nu_B(x)| \tag{27}$$

  and taking into account (27), we can express (23) in the following way:

$$\begin{aligned} & 0.5(|\mu_A(x) - \mu_B(x)| + |\mu_A(x) - \mu_B(x)|) = \\ = \; & \max\{|\mu_A(x) - \mu_B(x)|, |\mu_A(x) - \mu_B(x)|\} \end{aligned} \tag{28}$$

- if $\pi_A(x) - \pi_B(x) \neq 0$ but the same time

$$\mu_A(x) - \mu_B(x) = \nu_A(x) - \nu_B(x) = -\frac{1}{2}(\pi_A(x) - \pi_B(x)) \tag{29}$$

  guarantee that (23) boils down again to (28).

In other words, (23) is fulfilled (what means that the Hausdorff measure given by (17) is a natural counterpart of (16)) only for such elements belonging to an A-IFS, for which some additional conditions are given like: $\pi_A(x) - \pi_B(x) = 0$ or (29). But in general, for infinite numbers of elements, (23) is not valid.

In the above context it seems unfortunate trying to construct the Hausdorff distance using the two term Hamming distance between A-IFSs.

Returning to the results from Section 3.1, the Hausdorff distance for A-IFSs can not be constructed the same way as for the interval-valued fuzzy sets.

3.3 A straightforward generalizations of the Hamming distance based on the Hausdorff metric

Now we will show that applying the three term Hamming distance for A-IFSs, we obtain its correct (in the sense of Definition 1) counterpart in terms of $\max$ function, i.e. obtain a generalization of the Hamming distance based on the Hausdorff metric.

Namely, if we calculate the three term Hamming distance between two degenerated, i.e. one-element IFSs, A and B in the spirit of Szmidt and Kacprzyk [28], [35], Szmidt and Baldwin [22], [23], i.e., in the following way:

$$\begin{aligned} l_{IFS}(A,B) &= \frac{1}{2}(|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)| + \\ &+ |\pi_A(x) - \pi_B(x)|) \end{aligned} \tag{30}$$

we can give a counterpart of the above distance in terms of $\max$ function:

$$\begin{aligned} H_3(A,B) &= \max\{|\mu_A(x) - \mu_B(x)|, |\nu_A(x) - \nu_B(x)|, \\ &, |\pi_A(x) - \pi_B(x)|\} \end{aligned} \tag{31}$$

If $H_3(A,B)$ (31) is properly calculated Hausdorff distance, the following condition should be fulfilled:

$$\begin{aligned} &\frac{1}{2}(|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)|) + |\pi_A(x) - \pi_B(x)|) = \\ = \; &\max\{|\mu_A(x) - \mu_B(x)|, |\nu_A(x) - \nu_B(x)|, |\pi_A(x) - \pi_B(x)|\} \end{aligned} \tag{32}$$

Let us verify if (32) is valid. Without loss of generality we can assume

$$\begin{aligned} \max \; &\{|\mu_A(x) - \mu_B(x)|, |\nu_A(x) - \nu_B(x)|, |\pi_A(x) - \pi_B(x)|\} = \\ = \; &|\mu_A(x) - \mu_B(x)| \end{aligned} \tag{33}$$

For $|\mu_A(x) - \mu_B(x)|$ fulfilling (33), and because of (24) and (25), we conclude that both $\nu_A(x) - \nu_B(x)$, and $\pi_A(x) - \pi_B(x)$ are of the same sign (both values are either positive or negative). Therefore

$$|\mu_A(x) - \mu_B(x)| = |\nu_A(x) - \nu_B(x)| + |\pi_A(x) - \pi_B(x)| \tag{34}$$

Applying (34) we can verify that (32) always is valid as

$$\begin{aligned} & 0.5\{|\mu_A(x) - \mu_B(x)| + |\mu_A(x) - \mu_B(x)|\} = \\ = \; & \max\{|\mu_A(x) - \mu_B(x)|, |\nu_A(x) - \nu_B(x)|, |\pi_A(x) - \pi_B(x)|\} = \\ = \; & |\mu_A(x) - \mu_B(x)| \end{aligned} \tag{35}$$

Now we will use the above formulas (30) and (31) for the data used in Example 1. But now, as we also take into account the hesitation margins $\pi(x)$ (5), instead of (18) we use the "full description" of the data $\{< x, \mu(x), \nu(x), \pi(x) >\}$, i.e. employing all three functions (membership, non-membership and hesitation margin) describing the considered A-IFSs:

$$\begin{aligned} A &= \{< x, 1, 0, 0 >\}, \quad B = \{< x, 0, 1, 0 >\}, \quad D = \{< x, 0, 0, 1 >\}, \\ G &= \{< x, \frac{1}{2}, \frac{1}{2}, 0 >\}, \quad E = \{< x, \frac{1}{4}, \frac{1}{4}, \frac{1}{2} >\} \end{aligned} \tag{36}$$

and obtain from (31):

$$\begin{aligned} H_3(A,B) &= \max(|1-0|, |0-1|, |0-0|) = 1 \\ H_3(A,D) &= \max(|1-0|, |0-0|, |0-1|) = 1 \\ H_3(B,D) &= \max(|0-0|, |1-0|, |0-1|) = 1 \\ H_3(A,G) &= \max(|0-1/2|, |0-1/2|, |0-0|) = 0.5 \\ H_3(A,E) &= \max(|1-1/4|, |0-1/4|, |0-1/2|) = 0.75 \\ H_3(B,G) &= \max(|1-1/4|, |0-1/4|, |0-1/2|) = 0.75 \\ H_3(B,E) &= \max(|1-1/4|, |0-1/4|, |0-1/2|) = 0.75 \\ H_3(D,G) &= \max(|0-1/2|, |0-1/2|, |1-0|) = 1 \\ H_3(D,E) &= \max(|0-1/4|, |0-1/4|, |1-1/2|) = 0.5 \\ H_3(G,E) &= \max(|1/2-1/4|, |1/2-1/4|, |0-1/2|) = 0.5 \end{aligned}$$

Now we calculate the counterpart Hamming distances using (30) (with all three functions). The results are

$$
\begin{aligned}
l_{IFS}(A,B) &= 0.5(|1-0|+|0-1|+|0-0|)=1\\
l_{IFS}(A,D) &= 0.5(|1-0|+|0-0|+|0-1|)=1\\
l_{IFS}(B,D) &= 0.5(|0-0|+|1-0|+|0-1|)=1\\
l_{IFS}(A,G) &= 0.5(|0-1/2|+|0-1/2|+|0-0|)=0.5\\
l_{IFS}(A,E) &= 0.5(|1-1/4|+|0-1/4|+|0-1/2|)=0.75\\
l_{IFS}(B,G) &= 0.5(|1-1/4|+|0-1/4|+|0-1/2|)=0.75\\
l_{IFS}(B,E) &= 0.5(|1-1/4|+|0-1/4|+|0-1/2|)=0.75\\
l_{IFS}(D,G) &= 0.5(|0-1/2|+|0-1/2|+|1-0|)=1\\
l_{IFS}(D,E) &= 0.5(|0-1/4|+|0-1/4|+|1-1/2|)=0.5\\
l_{IFS}(G,E) &= 0.5(|1/2-1/4|+|1/2-1/4|+|0-1/2|)=0.5
\end{aligned}
$$

As we can see, the Hausdorff distance (31) proposed in this paper (using memberships, non-memberships and hesitation margins) and the Hamming distance (30) give for one-element IFS sets fully consistent results.

In other words, for the normalized Hamming distance expressed in the spirit of (Szmidt and Kacprzyk [28], [35]) given by (6) we can give the following equivalent representation in terms of $\max$ function:

$$
\begin{aligned}
H_3(A,B) &= \frac{1}{n}\sum_{i=1}^{n}\max\{|\mu_A(x_i)-\mu_B(x_i)|,|\nu_A(x_i)-\nu_B(x_i)|,\\
&\quad |\pi_A(x_i)-\pi_B(x_i)|\} \qquad (37)
\end{aligned}
$$

Unfortunately, it can be easily verified that it is impossible to give the counterpart pairs of the formulas as (6)–(37) for $r > 1$ in the Minkowski r–metrics ($r = 1$ is the Hamming distances, $r = 2$ is the Euclidean distances, etc.)

For the details on other distances between A-IFSs we refer the interested reader to Szmidt and Kacprzyk [28] and especially [35]. More details are given in [5] and [43]. The counterpart results, but in respect to mass assignment theory, are given by Szmidt and Baldwin [22], [23].

4 Conclusions

A correct method of the calculating distances between A-IFSs based on the Hausdorff metric (being a counterpart of the Hamming distance) was proposed. The method employs all three functions describing A-IFSs. The proposed method is both mathematically valid and intuitively appealing (cf. [35]). It is worth mentioning that the method is not the same as in the case of interval-valued fuzzy sets.

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.

It may be viewed as a result of fruitful discussions held during the Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) organized in Warsaw on October 16, 2009 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Centre for Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT – Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bistrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) has been meant to commence a new series of scientific events primarily focused on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Moreover, other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems are discussed.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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