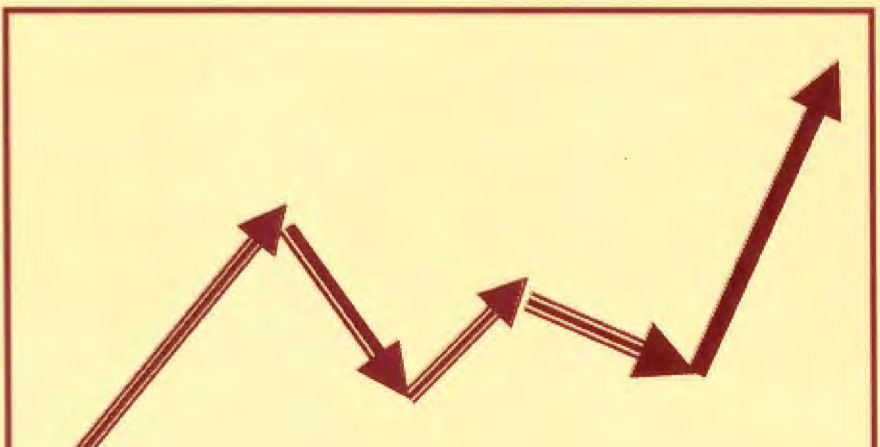
STANISŁAW PIASECKI

AN INTRODUCTION TO A THEORY OF MARKET COMPETITION

Volume II



Warsaw 2011

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INTRODUCTION

The purpose of the present book is to show the possibility of developing a quantitative description of the action of "invisible hand" on the market. This is why the text is full of mathematical expressions, even though they are kept purposefully at the possibly simple level.

At the same time, the book is a subsequent publication of results from the work on establishment of foundations for the theory of economic competition, limited, however, essentially to what is called price competition. Let us note at this point that the "competitors" here considered are the companies selling their produce on the common and limited market, and that price competition analysed takes place among the products of different companies, serving to satisfy the very same kind of demand from the side of the customers.

Price competition ought to be regarded as a dynamic market game, which takes place within the space of retail prices, i.e. in the "open", before the eyes of the consumers, or in the space of wholesale prices – behind the scenes. The strategies of the players consist in selection of prices, at which their products (services) are sold.

Under a close examination of the problem it turned out that most important for defining the market price game is determination of the "payoff function". The present book is devoted, therefore, mainly to this problem.

This volume constitutes a continuation of the considerations from the first volume of "*Introduction to a Theory of Market Competition*", in which territorial expansion strategy of companies has been analysed, this strategy allowing for expansion of sales and lowering of prices. Yet, sooner or later, the instant has to come when a company must enter an "alien" market, and, after a successful entry, face the problem of expanding its market share.

Hence, it is the last two issues that this book takes up and analyses.

The considerations forwarded therein are based on three fundamental assumptions:

- Increase of price of purchase entails a decrease of the number of products sold.
- Increase of the number of products turned out makes it possible to lower the cost of producing these products.
- The market secures the preservation of equilibrium between demand and supply. Customers are directed by reason when making purchasing decisions.

The first assumption results from the fact that each customer has limited financial capacities (of purchasing products and services).

The second assumption is justified by the commonly observed "production scale effect", which results from the continuous technological progress, taking place especially in the domain of production technologies. This fact finds its confirmation in the history of economic development – from handicraft through workshop production to the present-day mass (even if customised) production.

The third, double assumption is associated with the adoption of principles of free market.

In order to represent the "scale effect", the hyperbolic relation was used, resulting from the analysis of the constant and variable production costs.

To describe the dependence of demand upon the product price, stemming from the income structure of potential customers, the linear dependence was used, which is characteristic for the constant income density of customers.

Other adopted assumptions and simplifications are of technical character.

Many of the Readers shall certainly be disappointed, as they will not find in the book the statistical inquiries, based on what is called "real-life data", that would confirm the assumptions adopted and the results obtained. In order, though, for a theory to be subject to verification, it must first be formulated. It should be indicated that the precepts of this theory have been successfully implemented in economic reality, in the practice of quite a significant company in Poland.

Thus, the contents of this book ought to be regarded as an attempt of formulating a definite theory, by no means pretending to having exhausted the entire problem area. It should be added that the results contained in both volumes published so far result from the research done by the respective authors within the Systems Research Institute of the Polish Academy of Sciences. Separate thanks go to the NTT System S.A. company that supported financially the publication of both volumes.

The authors of both volumes hope that this modest contribution shall serve its purpose of providing to the Readers the very first insight into the possibility of representing and analysing in quantitative terms the processes we observe daily on the globalising markets. The authors would also like to announce the preparation of the subsequent volume, presenting the extension to the theory here expounded.

Warsaw, June 2011

Chapter V

OTHER SPECIAL PROBLEMS

1. On the nonlinear functions of demand and sharing

In this chapter we shall be analysing the strategies of market entry for a Company A, with its competitive product "A", into a market, on which a resident Company B has been a monopolist, in the situation, in which the dependence of demand upon price of the product sold in nonlinear and when the market sharing between the two competing companies also has a nonlinear character.

In this context we shall be distinguishing four definite special cases, namely the following ones:

I. Demand is defined by the linear dependence:

$$\lambda = \frac{\Lambda_{mx}}{C_{mx}}(C_{mx} - C)$$

and market division is also defined by the linear dependence:

$$\lambda_A = \lambda \cdot \frac{C}{C_A + C} \quad ; \quad \lambda_B = \lambda \cdot \frac{C_A}{C_A + C}.$$

II. Demand is defined by the linear dependence:

$$\lambda = \frac{\Lambda_{mx}}{C_{mx}}(C_{mx} - C),$$

while market split is defined by the nonlinear dependence:

$$\lambda_A = \lambda \cdot \frac{C^2}{C_A^2 + C^2} \quad ; \quad \lambda_B = \lambda \cdot \frac{C_A^2}{C_A^2 + C^2}.$$

III. Demand is defined by a nonlinear dependence:

$$\lambda = \frac{\Lambda_{mx}}{C_{mx}^2}(C_{mx} - C)^2$$

while market split is defined by a linear dependence:

$$\lambda_A = \lambda \cdot \frac{C}{C_A + C} \quad ; \quad \lambda_B = \lambda \cdot \frac{C_A}{C_A + C}$$

IV. Demand is defined by a nonlinear dependence:

$$\lambda = \frac{\Lambda_{mx}}{C_{mx}^2}(C_{mx} - C)^2$$

and market shares are also defined by nonlinear relations:

$$\lambda_A = \lambda \cdot \frac{C^2}{C_A^2 + C^2} \quad ; \quad \lambda_B = \lambda \cdot \frac{C_A^2}{C_A^2 + C^2}.$$

The first of the above cases was considered in detail in Chapter II. Before we start analysing the other three cases, we shall derive a somewhat different form of the expression for the difference of profits.

Namely, in Chapter II, the formula for the difference of profits had the form

$$\Delta Z = (C - b) \cdot \Delta\lambda_A - \lambda_A \cdot \Delta C$$

We shall modify this expression to a more convenient form, using the interdependences between the particular quantities, as shown in Fig. 5.1.

As we substitute these expressions and take into account the relations, shown in Fig. 5.1, we get

$$\Delta\lambda = \frac{\Lambda_{mx}}{C_{mx}} \cdot (C - C_A)$$

$$\Delta\lambda^0 = \frac{\Lambda_{mx}}{C_{mx}} \cdot (C_{mx} - C) \cdot \frac{C^2 - C_A^2}{C^2 + C_A^2}$$

$$\lambda_A = \lambda_A^0 + \Delta\lambda = \frac{\Lambda_{mx}}{C_{mx}} \cdot (C_{mx} - C) \cdot \frac{C^2}{C_A^2 + C^2} + \frac{\Lambda_{mx}}{C_{mx}} \cdot (C - C_A)$$

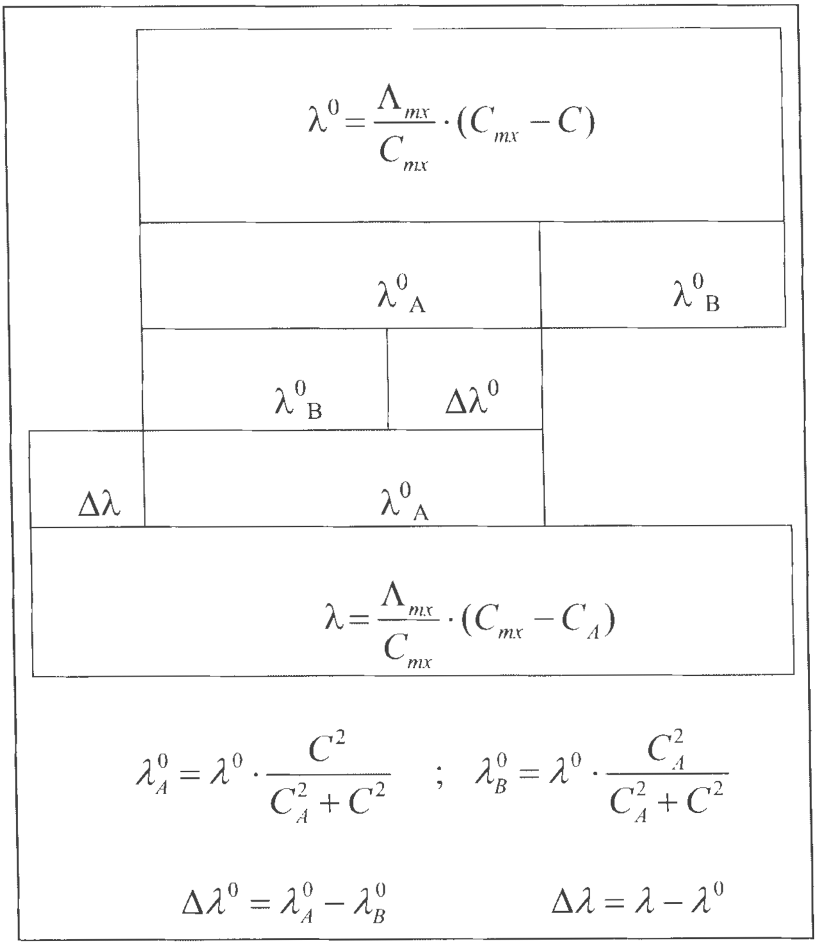

Figure 5.1.

If we next use the relations from Fig. 5.1, then we can apply the following transformation of the formula for the value of ΔZ:

$$\begin{aligned}\Delta Z &= (C-b)\cdot\Delta\lambda_A - \lambda_A\cdot(C-C_A) = \\ &= (C-b)\cdot(\lambda_A-\lambda_B) - (\lambda_A^0+\Delta\lambda)\cdot(C-C_A) = \\ &= (C-b)\cdot(\lambda_B^0+\Delta\lambda^0+\Delta\lambda-\lambda_B) - (\lambda_A^0+\Delta\lambda)\cdot(C-C_A) = \\ &= (C-b)\cdot(\Delta\lambda^0+\Delta\lambda) - (C-C_A)\cdot\Delta\lambda - (C-C_A)\cdot\lambda_A^0 = \\ &= (C_A-b)\cdot\Delta\lambda + (C-b)\cdot\Delta\lambda^0 - (C-C_A)\cdot\lambda_A^0\end{aligned}$$

As we substitute into the last expression the values of $\Delta\lambda$, $\Delta\lambda^0$ and $\lambda_A{}^0$, and perform simple transformations (for the Case II), we get

$$\begin{aligned}\Delta Z &= \frac{\Lambda_{mx}}{C_{mx}}\cdot\left\{(C-C_A)\cdot(C_A-b)-(C-C_A)\cdot(C_{mx}-C)\frac{(C-b)\cdot(C+C_A)-C^2}{C^2+C_A^2}\right\} = \\ &= \frac{\Lambda_{mx}}{C_{mx}}\cdot(C-C_A)\cdot(C_{mx}-C)\cdot\left\{\frac{C_A-b}{C_{mx}-C}-(C-b)\cdot\frac{B-C_A}{C^2+C_A^2}\right\} = \\ &= \Lambda_{mx}\cdot(C_{mx}-C)\cdot\frac{C-C_A}{C_{mx}}\cdot\left\{\frac{C_A-b}{C_{mx}-C}-(C-b)\cdot\frac{B-C_A}{C^2+C_A^2}\right\}\end{aligned}$$

After having performed a similar procedure for all the cases considered we obtain the following final formulae for the value of ΔZ:

I. $$\Delta Z = \Lambda_{mx}\cdot(C_{mx}-C)\cdot\frac{C-C_A}{C_{mx}}\cdot\left\{\frac{C_A-b}{C_{mx}-C}-\frac{b}{C+C_A}\right\}$$

II. $$\Delta Z = \Lambda_{mx}\cdot(C_{mx}-C)\cdot\frac{C-C_A}{C_{mx}}\cdot\left\{\frac{C_A-b}{C_{mx}-C}-(C-b)\cdot\frac{B-C_A}{C^2+C_A^2}\right\}$$

III. $$\Delta Z = \frac{\Lambda_{mx}}{C_{mx}}\cdot(C_{mx}-C)^2\cdot\frac{C-C_A}{C_{mx}}\cdot\left\{\frac{C_A-b}{C_{mx}-C}\cdot\frac{D-C_A}{C_{mx}-C}-\frac{b}{C+C_A}\right\}$$

$$\text{IV.}\quad \Delta Z = \frac{\Lambda_{mx}}{C_{mx}} \cdot (C_{mx} - C)^2 \cdot \frac{C - C_A}{C_{mx}} \cdot \left\{ \frac{C_A - b}{C_{mx} - C} \cdot \frac{D - C_A}{C_{mx} - C} + (C - b) \cdot \frac{B - C_A}{C^2 + C_A^2} \right\}$$

where $B = \dfrac{b \cdot C}{C - b}$; $D = 2C_{mx} - C$.

Now, if we introduce the following notations for the elementary functions of the variable C_A, whose values are dimensionless:

$$\varphi_0 = \frac{C - C_A}{C_{mx}}$$

$$\varphi_1 = \frac{C_A - b}{C_{mx} - C} \quad ; \quad \varphi_2 = \frac{C_A - b}{C_{mx} - C} \cdot \frac{D - C_A}{C_{mx} - C}$$

$$\varphi_3 = \frac{b}{C + C_A} \quad ; \quad \varphi_4 = (C - b) \cdot \frac{B - C_A}{C^2 + C_A^2}$$

then the formulae for ΔC take on the following forms:

$$I.\ \Delta Z = \Lambda_{mx} \cdot (C_{mx} - C) \cdot \varphi_0 \cdot \{\varphi_1 - \varphi_3\}$$

$$II.\ \Delta Z = \Lambda_{mx} \cdot (C_{mx} - C) \cdot \varphi_0 \cdot \{\varphi_1 - \varphi_4\}$$

$$III.\ \Delta Z = \frac{\Lambda_{mx}}{C_{mx}} \cdot (C_{mx} - C)^2 \cdot \varphi_0 \cdot \{\varphi_2 - \varphi_3\}$$

$$IV.\ \Delta Z = \frac{\Lambda_{mx}}{C_{mx}} \cdot (C_{mx} - C)^2 \cdot \varphi_0 \cdot \{\varphi_2 - \varphi_4\}$$

Due to such a notation of the formulae for the difference of profits, in order to determine the roots of the function ΔZ it suffices to verify the courses of the following types of elementary functions:

- $y = \dfrac{x - b}{S} \cdot \dfrac{D - x}{S}$, where $S = C_M - C$ is a constant;

- $y = \dfrac{b}{C + x}$;
- $y = S \cdot \dfrac{B - x}{C^2 + x^2}$, where $S = C \quad b$ is a constant.

The remaining functions are linear. The shapes of the above three functions are shown, respectively, in Figs. 5.2-4.

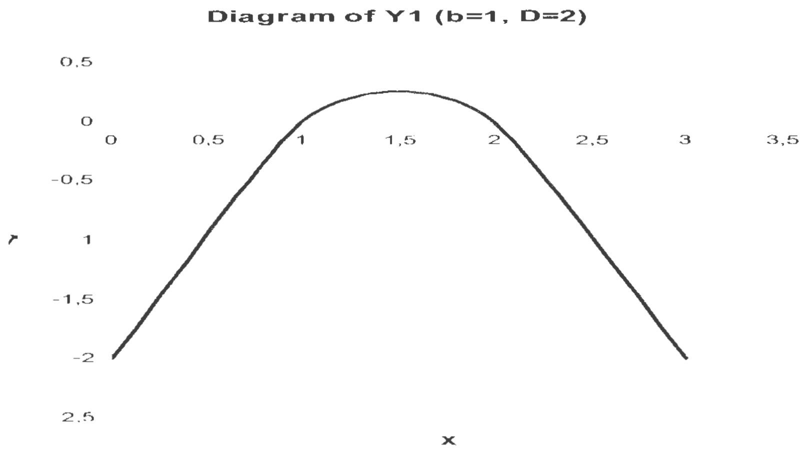

Figure 5.2.

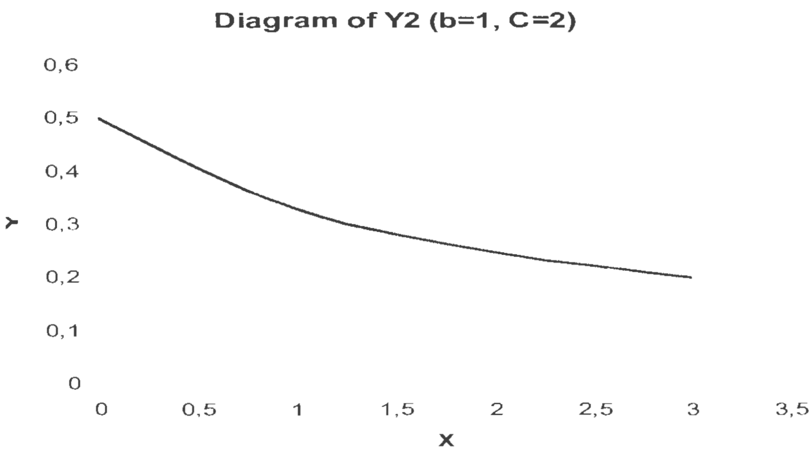

Figure 5.3.

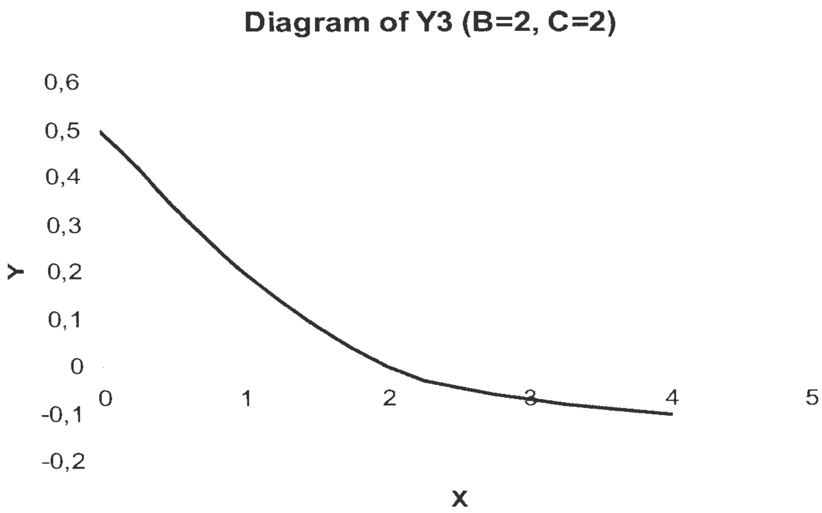

Figure 5.4.

Using the shapes of these elementary functions, we can easily realise the diagrams of the values of ΔZ for various cases, with the Case I having been considered in detail in Chapter I of this volume.

Case II.

Fig. 5.5 shows the diagrams of functions ϕ_0, ϕ_1, ϕ_4 and ΔZ. It can easily be seen that the range of values of C_A, in which the difference of profits, ΔZ, has a negative value is only the part (C_0,C) of the segment (b,C).

The figure shows an approximate course of the function ϕ_1-ϕ_4 intersecting the axis C_A at the point C^0. The difference in question crosses the straight line ϕ_0, forming a triangle over the segment mentioned. It defines the shape of the parabola, having two roots: C_0 and C. The positive part of the parabola corresponds to the range of values of C_A that guarantee a positive value of the difference of profits ΔZ. This parabola is, of course, an approximation of the entire diagram of the values of ΔZ, which is, naturally, a third order curve.

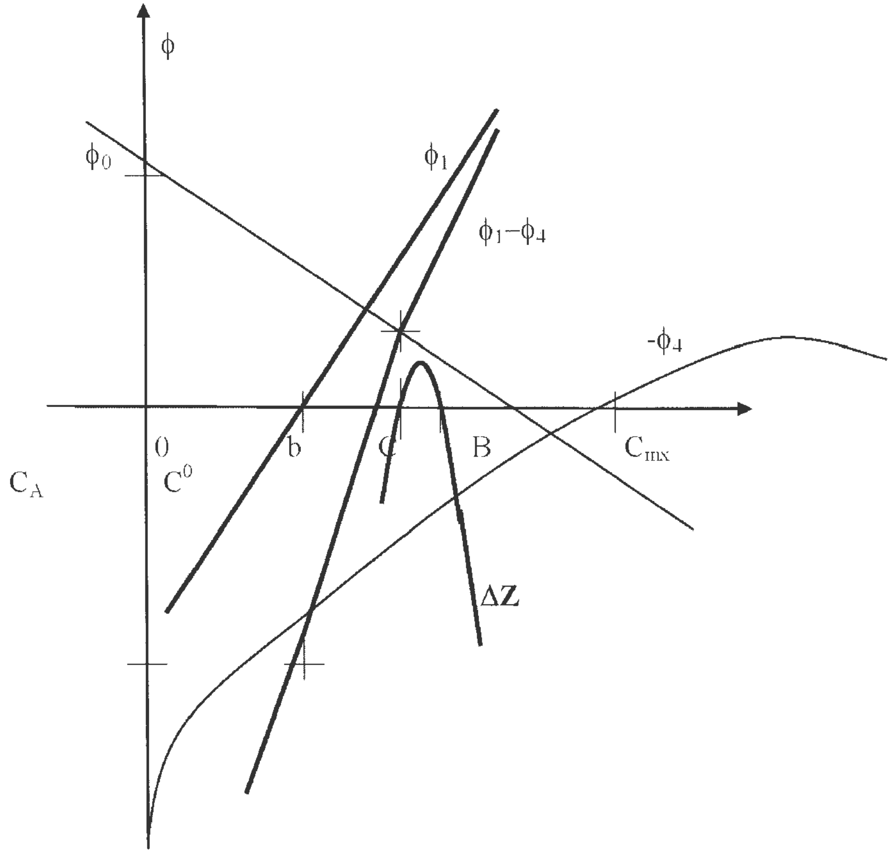

Figure 5.5.

Alas, determination of the value of C^0 requires solving of the third order equation. In practical terms it is much easier to tabularise the values of ΔZ, starting the calculations from the value of C downwards, until the values of ΔZ begin to decrease. For this last, biggest value of C_A, the value of ΔZ attains maximum.

Let us next analyse the way, in which the interval of variability of the value of C_A, in which the difference of profits has the positive value, shall change, if the sales price of the product to date has been optimal:

$$C^* = \frac{1}{2} \cdot (C_{mx} + b) \cdot$$

First of all let us define the shapes of functions ϕ_0, ϕ_1, ϕ_2, ϕ_3, ϕ_4 in the situation, when

$$C = C^* = \frac{1}{2} \cdot (C_{mx} + b)$$

and so the functions in question take on the forms

$$\phi_0 = \frac{C_M - 2C_A + b}{2C_M};$$

$$\phi_1 = 2\frac{C_A - b}{C_M - b};$$

$$\phi_2 = 4\frac{(C_A - b)(D - C_A)}{(C_M - b)(C_M - b)}, \text{ where } D = (3C_M - b)/2; \text{ or}$$

$$= 2\frac{(C_A - b)(3C_M - 2C_A - b)}{(C_M - b)^2};$$

$$\phi_3 = \frac{2b}{2C_A + C_M + b};$$

$$\phi_4 = 2\frac{(C_M - b)(B - C_A)}{4C_A^2 + (C_M - b)^2}, \text{ where } B = b\frac{C_M + b}{C_M - b}; \text{ or}$$

$$= 2\frac{b(C_M + b) - C_A(C_M - b)}{4C_A^2 + (C_M + b)^2};$$

as well as $C_M - C^* = (C_M \quad b)/2$.

Note that the quantities in the parentheses in the above expressions are positive, since $C_A > b$, $C_A < C = C^* < C_M$, and $b < C_M$. There may only be some problems with the quantities

$$C_{mx} - 2C_A + b;\ \ D - C_A;\ \ 3C_{mx} - 2C_A - b; \text{ and } B - C_A.$$

Namely, if we assume that the maximum value of C_A can be the number

$$C_A^{mx} = (1 - \delta)C^* = (1 - \delta)(C_{mx} + b)/2,$$

where δ is an arbitrarily small number, different from zero, then substituting this into the first and the third of the previous expressions, we conclude that they are positive.

If, then, we substitute the value of D into the second expression, we get

$$\frac{1}{2}\cdot(3C_{mx} - b) - C_A = \frac{1}{2}\cdot(3C_{mx} - b - 2C_A)$$

as we have shown before the positive value of the expression. Similarly, if we substitute the value of B into the last of the expressions, we obtain

$$b\cdot\frac{C_{mx}+b}{C_{mx}-b} - C_A = \frac{b\cdot(C_{mx}+b) - C_A(C_{mx}-b)}{C_{mx}-b}.$$

The sign of this expression depends upon the sign of the nominator. As we equate the nominator to zero, we conclude that the expression takes on positive values, when the inequality $C_A < D$ is satisfied.

We should yet determine the relative locations of the points corresponding to values of b, B, C C^* along the axis.

Analysis of values of b and D suggests that inequality $b < D$ is always fulfilled. Let us, then, check whether $D < C^*$. For this purpose we shall use explicit expressions for the two quantities:

$$b\cdot\frac{C_{mx}+b}{C_{mx}-b} < \frac{1}{2}\cdot(C_{mx}+b).$$

This inequality is true, when $3b < C_{mx}$. For this case we must narrow down the admissible interval of values of C_A, namely

$$b < C_A < \min \{ C^* ; D \}.$$

If $3b > C_{mx}$ and $D > C^*$, then we can retain the previous conditions on the values of C_A, i.e. $b < C_A < C^*$.

Since all the factors, appearing before the expression in the right-hand stylised brackets in the formulae for ΔZ in cases I through IV, are positive, including ϕ_0, so the difference of profits, ΔZ, will have a positive value, if the expression in these brackets is positive.

Besides, as it can be easily noted, functions ϕ_1, ϕ_2, ϕ_3 take positive values in the entire domain of values of C_A, that is, within the interval $b < C_A < C = C^*$. Only function ϕ_4 changes its sign within this interval. Namely:

$$\phi_4 > 0 \text{ when } b(C_M + b) - C_A(C_M - b) > 0 \Rightarrow C_A < C_A^0 = b\frac{C_M + b}{C_M - b},$$

or $\phi_4 < 0$ when $C_A > C_A^0$.

Consequently, the range of values of C_A, within which $\phi_4 > 0$, is limited to the interval $b < C_A < \min\{C^*, C_A^0\}$.

It is also easily noticed that the change in the right-hand side of the interval is insignificant, if $3b > C_M$, since then the inequality $C_A^0 > C = C^*$ holds. If, however, $3b < C_M$, then $C_A^0 < C^*$, and C_A^0 is situated in the range of admissible values of C_A that is of interest to us, and then $b < C_A < C_A^0$.

After having determined the signs of the functions ϕ_1, ϕ_2, ϕ_3 and ϕ_4, let us find the ranges of values of C_A, for which the differences of these functions take on the positive values. Thus, we are interested in the signs of the following expressions:

$$\text{I. } \{\phi_1 - \phi_3\} \text{ for } \Delta Z = \tfrac{1}{2} A_{mx}(C_{mx} - b)\frac{C_{mx} - 2C_A + b}{2C_{mx}} \{\phi_1 - \phi_3\};$$

$$\text{II. } \{\phi_1 - \phi_4\} \text{ for } \Delta Z = \tfrac{1}{2} A_{mx}(C_{mx} - b)\frac{C_{mx} - 2C_A + b}{C_{mx}} \{\phi_1 - \phi_4\};$$

III. $\{\phi_2 - \phi_3\}$ for $\Delta Z = ¼(A_{mx}/C_{mx})(C_{mx}-b)^2 \dfrac{C_{mx} - 2C_A + b}{C_{mx}}$ $\{\phi_1 - \phi_4\}$;

IV. $\{\phi_2 - \phi_3\}$ for $\Delta Z = ¼\,(A_{mx}/C_{mx})\,(C_{mx}-b)^2$ $\{\phi_1 - \phi_4\}$.

I. Let us consider the first case – for what values of C_A the difference $\phi_1 - \phi_3$ takes on the positive values. As we bring the respective expression to a common denominator, we obtain the nominator in the form

$$(C_A - b)\cdot(C_{mx} + C_A + b) > b\cdot(C_{mx} - b)\,.$$

After a transformation we get the inequality

$$2\cdot C_A^2 + (C_{mx} - b)\cdot C_A - 2\cdot b\cdot C_{mx} > 0\,.$$

The left-hand side of the above is the expression representing a parabola, which intersects the axis of C_A at the following two points

$$C_{A1} = -\frac{1}{4}\cdot(C_{mx} - b)\cdot\left\{\sqrt{1 + 8\frac{C_{mx}\cdot b}{(C_{mx} - b)^2}} + 1\right\} < 0$$

$$C_{A2} = \frac{1}{4}\cdot(C_{mx} - b)\cdot\left\{\sqrt{1 + 8\frac{C_{mx}\cdot b}{(C_{mx} - b)^2}} - 1\right\} > 0$$

and attains minimum value at the point

$$C_A^* = -\frac{1}{4}\cdot(C_{mx} - b)\,.$$

We should yet check, whether the value C_{A2} is contained in the interval of admissible values of C_A, i.e. $b < C_A < C^*$. Let us verify, whether the inequality C^* C_{A2} holds. As we introduce the expressions for the particular quantities, we get

$$\frac{1}{2}\cdot(C_{mx} + b) - \frac{1}{4}\cdot(C_{mx} - b)\cdot\left\{\sqrt{1 + 8\frac{C_{mx}\cdot b}{C_{mx} - b)^2}} - 1\right\} > 0\,.$$

After a series of transformations we obtain the inequality

$$(3C_{mx} + b)^2 - (C_{mx} - b)^2 > 8 \cdot b \cdot C_{mx},$$

which implies that, in fact, $C_{A2} < C^*$. This result limits the admissible range of values of C_A, since the difference ϕ_1-ϕ_3 takes positive values for $C_A > C_{A2}$. Ultimately, then, we obtain the following range of values of C_A, guaranteeing fulfilment of the condition $\Delta Z > 0$:

$$\max\left\{ b,\ C_{A2} \right\} < C_A < C^*.$$

II. We shall now consider the second case and determine the range of values of C_A, for which the difference ϕ_1-ϕ_4 takes on positive values:

$$\varphi_1 - \varphi_4 = \frac{C_A - b}{C_{mx} - b} - \frac{b \cdot (C_{mx} + b) - C_A(C_{mx} - b)}{4 \cdot C_A^2 + (C_{mx} + b)^2} > 0 \cdot$$

As we bring the above expression to the common denominator, we obtain the following form of the inequality:

$$(C_A - b) \cdot \left[(C_{mx} + b)^2 + 4C_A^2) \right] > (C_{mx} - b) \cdot \left[b \cdot (C_{mx} + b) - C_A \cdot (C_{mx} - b) \right] \cdot$$

Then, as we transform the expression obtained and reduce the similar elements, we ultimately get

$$2C_A^3 - 2bC_A^2 - 2(C_{mx}^2 + b^2)C_A - 2bC_{mx}(C_{mx} + b) > 0.$$

This is an expression for a third-order curve, which has two extremes, whose coordinates are defined by the equation

$$3C_A^2 - 2bC_A - (C_{mx}^2 + b^2) = 0$$

i.e.

$$C_{A1,2} = \frac{b}{3} \cdot \left\{ 1 \pm \sqrt{1 + 3\frac{C_{mx}^2 + b^2}{b^2}} \right\}.$$

The function in question takes at the point C_{A2} negative minimum value; this is true also for $C_A = 0$ and for $C_A = b$. For $C_A < 2b$ the values of the function are also negative, if

$$\frac{C_{mx}}{b} < \frac{2}{3},$$

which is not possible.

It is only for $C_A = 3b$ that the function may take positive values, if

$$\frac{C_{mx}}{b} <\approx 2{,}9\cdot$$

Consequently, it can be concluded that the root of the third-order function that is of interest to us, is usually located beneath the value $3b$. Thus, we can assume that – in practice – there do not exist values of C_A, for which the difference ϕ_1-ϕ_4 and ΔZ may take positive values. We shall become virtually certain as to this conclusion, when the following inequality holds:

$$C = C^* = \frac{1}{2}\cdot(C_{mx} + b) < 3b\cdot$$

III. Next, let us analyse the third case, in which we consider the sign of the difference ϕ_2-ϕ_3. As we introduce the respective expressions, we obtain

$$\varphi_2 - \varphi_3 = \frac{(C_A - b)\cdot(3C_{mx} - 2C_A - b)}{(C_{mx} - b)^2} - \frac{b}{C_{mx} + 2C_A + b}\cdot$$

Like before, we bring the expressions obtained to the common denominator, we get the following form of the nominator, whose value is decisive for the sign of the difference analysed:

$$(C_A - b)\cdot(3C_{mx} - 2C_A - b)\cdot(C_{mx} + 2C_A + b) - b\cdot(C_{mx} - b)^2.$$

After having performed the respective operations and reduced similar elements, we obtain an equation of the third order

$$-4\cdot C_A^3+4\cdot(C_{mx}+b)\cdot C_A^2+\left[(2C_{mx}+b)^2-(2b^2+C_{mx}^2)\right]\cdot C_A-2\cdot C_{mx}\cdot b\cdot(2C_{mx}-b)$$

For this equation, roots cannot be defined through simple formulae. If we differentiate this function then we can determine the roots of the derivative, i.e. of

$$-12C_A^2+8(C_{mx}+b)C_A+(2C_{mx}+b)^2-(C_{mx}^2+2b^2)$$

by equating this expression to zero. Thereby, we get

$$C_{A1,2}=\frac{1}{3}\cdot(C_{mx}+b)\cdot\left\{1\pm\sqrt{1+\frac{3}{4}\cdot\frac{(2C_{mx}+b)^2-(2b^2+C_{mx}^2)}{(C_{mx}+b)^2}}\right\}.$$

One of these roots is negative, while the other one – positive. We are, of course, interested in the positive one. The value of this root does exceed $0.8(C_{mx}+b)$.

At this point the function attains a local maximum with a positive value if the following inequality holds approximately:

$$3C_{mx}^3-5bC_{mx}^2-2b^2C_{mx}-b^3>0.$$

Yet, since $C = C^* = 0.5(C_{mx}+b)$, the function stays positive over the entire range of the admissible values of C_A, i.e. $b < C_A < C^*$.

IV. Let us now pass over to the analysis of the fourth case, for which the difference ϕ_2-ϕ_4 takes on the form

$$2\cdot\frac{(C_A-b)\cdot(3C_{mx}-2C_A-b)}{(C_{mx}-b)^2}-2\cdot\frac{b\cdot(C_{mx}+b)-C_A\cdot(C_{mx}-b)}{4C_A^2+(C_{mx}-b)^2}.$$

After we bring the formula to the common denominator and reduce the similar elements, the nominator we obtain is the fourth-order polynomial.

$$W_4 = -8C_A^4 + 4(3C_{mx} + b)C_A^3 - 2(C_{mx}^2 - 4bC_{mx} + 3b^2)C_A^2 +$$
$$+ (3C_{mx}^2 + 7bC_{mx}^2 + 5b^2C_{mx} + b^3)C_A - 4bC_{mx}^2(C_{mx} + b)$$

Since the denominator of the thus obtained fractional expression is positive (within the range of values that is of interest to us), the quantity ΔZ shall be positive, when the nominator is positive, that is – when $W_4 > 0$.

The course of the function W_4 can be determined only in an approximate manner, using for this purpose the derivatives, having the courses as shown in Fig. 5.6.

In particular, the course of the square polynomial W_4'' can be easily drawn, since it attains the maximum value at the point

$$C_A^{mx} = \frac{1}{8} \cdot (3 \cdot C_{mx} + b)$$

while the roots are the numbers

$$C_{A1,2} = \frac{1}{8} \cdot (3C_{mx} + b) \cdot \left\{ 1 \pm \sqrt{1 - \frac{8}{3} \cdot \frac{C_{mx}^2 - 4bC_{mx} + 3b^2}{(3C_{mx} + b)^2}} \right\}.$$

As this can easily be seen the roots (zeroes) of the function are located symmetrically and close to the value C_A^{mx}. Besides, we know that $W_4'' < 0$ and $C_A^{mx} \approx C^*$.

Consequently, we can determine only in a qualitative manner the courses of the functions $W_4''(C_A)$, $W_4'(C_A)$, $W_4(C_A)$, consecutively, starting with the function W_4''. Fig. 5.6, announced already before, depicts the respective function courses.

As can be seen from this figure, the chance that there exists such a value $C_A < C^*$ that the nominator of the expression has a positive value, is marginal.

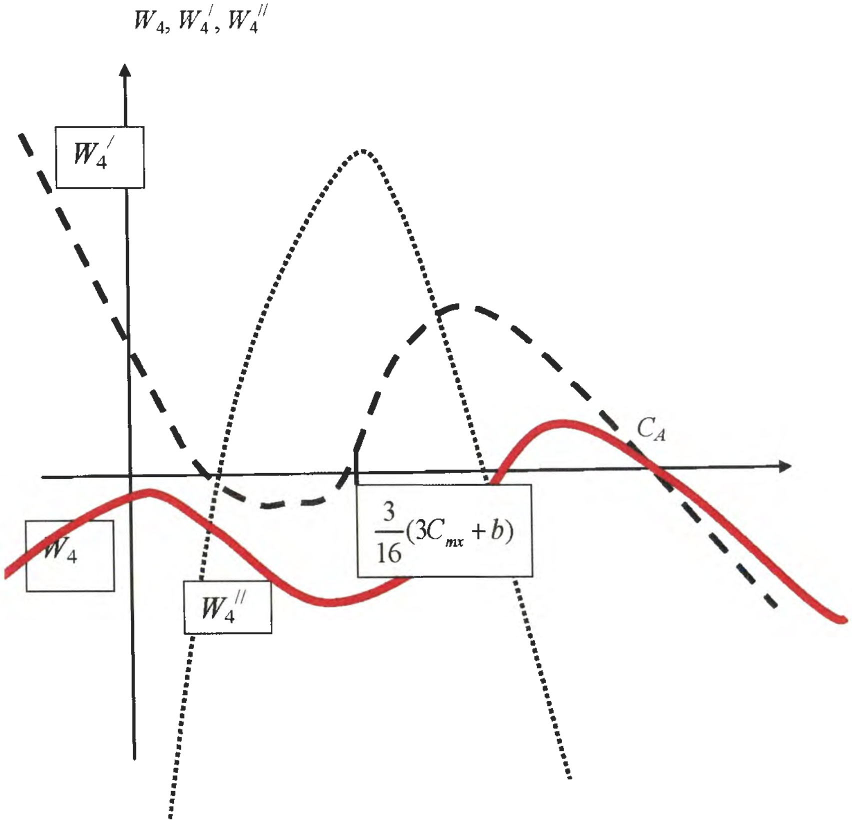

Figure 5. 6.

We shall assume that the dependence of demand upon product price is linear, i.e.

$$\lambda = \frac{\Lambda_{mx}}{C_{mx}} \cdot (C_{mx} - C)$$

while the functions of the shares of companies A and B in market demand are nonlinear, namely

$$\lambda_A = \lambda \cdot \frac{C_B^2}{C_A^2 + C_B^2} \qquad \lambda_B = \lambda \cdot \frac{C_A^2}{C_A^2 + C_B^2}.$$

Naturally, in the case the two companies sell their competitive products for the same price, the shares are equal (50% each).

Now, presume that before the time instant t_0, when Company A started the action, aiming at conquering the market, the prices of the competitive products were identical: $C_A = C_B = C$, while total sales equalled

$$\lambda_0 = \frac{\Lambda_{mx}}{C_{mx}} \cdot (C_{mx} - C).$$

At the time instant t_0 Company A lowered the sales price of its competitive product down to $C_A = C - \Delta C$, where $\Delta C > 0$. After the standard unit time interval elapses, the set of customers gets divided in the way different from before, namely those, who decided to still purchase the more expensive products of Company B (who can, therefore, be referred to as conservationists), and the ones, who either have been already purchasing the products of Company A, or changed their habits and started to buy the cheaper products of Company A.

In this manner an uneven division of the market, and of the set of customers, develops, defined in the following way:

$$\lambda_A = \lambda_0 \cdot \frac{C^2}{C_A^2 + C^2} \qquad \lambda_B = \lambda_0 \cdot \frac{C_A^2}{C_A^2 + C^2}$$

Yet, this is not the only thing that happens. Namely, new customers shall appear on the market considered, the ones who have not been, until t_0, purchasing the products in question, since they have been (at the price C) too expensive for them. Now, when the products of Company A, sold for the lower price C_A, appeared on the market, they shall start buying them.

So, with the lower prices, total demand shall increase to the value

$$\lambda = \frac{\Lambda_{mx}}{C_{mx}} \cdot (C_{mx} - C_A),$$

meaning that total demand shall increase by the value

$$\Delta\lambda = \lambda - \lambda_0 = \frac{\Lambda_{mx}}{C_{mx}}\left[(C_{mx} - C_A) - (C_{mx} - C)\right] = \frac{\Lambda_{mx}}{C_{mx}}(C - C_A).$$

The entire increase, expressed above, shall occur within the market share of Company A. Consequently, demand for the products of Company A shall become equal

$$\lambda_A = \lambda_A^0 + \Delta\lambda = \frac{\Lambda_{mx}}{C_{mx}^2}(C_{mx} \quad C)^2 \cdot \frac{C^2}{C_A^2 + C^2} + \frac{\Lambda_{mx}}{C_{mx}^2} \cdot (C - C_A) \cdot (C_{mx} - C - C_A).$$

while the demand for products of Company B shall decrease to the value

$$\lambda_B = \lambda_B^0 = \frac{\Lambda_{mx}}{C_{mx}^2}(C_{mx} - C)^2 \cdot \frac{C_A^2}{C_A^2 + C^2}.$$

Having established the above, let us pass over to the determination of the value of ΔZ by applying the respective formula:

$$\Delta Z = (C_A - b) \cdot \Delta\lambda + (C - b)\ \Delta\lambda^0 - (C - C_A)\ \lambda^0_A$$

$$= \frac{\Lambda_{mx}}{C_{mx}^2}(C - C_A) \cdot$$

$$\cdot \{(C_a - b) \cdot (2C_{mx} - C - C_A) + (C - b) \cdot (C_{mx} - C)^2 \cdot \frac{C + C_A}{C^2 + C_A^2} - (C_{mx} - C)^2 \cdot \frac{C^2}{C^2 + C_A^2}\}.$$

Now, after simple transformations, we get the following expression:

$$\Delta Z = \frac{\Lambda_{mx}}{C_{mx}^2}(C - C_A)\left\{(C_A - b) \cdot (2C_{mx} - C - C_A) - \frac{(C_{mx} - C)^2}{C^2 + C_A^2} \cdot \left[bC - (C - b) \cdot C_A\right]\right\}.$$

2. On identification of the demand function

We measure demand through the magnitude of **free sales**, expressed as the number of similar products sold within a definite sector or segment of the market. The segment or the sector may concern a definite geographical area (like a country, a group of countries or a continent).

The condition of "free sales" refers to the fact that the sale is not carried out under pressure, and that it is not limited by the lack of adequate supply of the product on the market, etc.

Similarly, we must make the reservation that we consider only the case, when the customers consciously purchase products, being aware of both the benefits accruing from the use of the product and the own financial capacities related to the purchase and exploitation of the product. In other words, we assume that the customers are not under the (decisive) influence of mass hysteria or a fad.

In an obvious way, the magnitude of demand depends upon the price C of the product unit and its relation to the income level of the potential buyer. Then, demand depends also upon the cost of using the product, which in many cases can be expressed through the ratio of the purchasing price (C) to the period of use (T) or validity of the product.

The empirical study of dependence of demand upon the purchasing price consists in the temporary or persistent lowering of the product price and comparing the demand, expressed in quantitative terms, before and after the price change. More precisely, the comparison ought to be made between two time periods of equal length, having similar seasonality characteristics, the price being equal C over the entire the first period and then C-ΔC over the entire second period.

The length of the time periods can be extended, in the course of the analysis of the rate of change of demand, so as to become sufficiently certain that demand within a given time period stabilised, especially after the price change. On the other hand, the period of observation can be shortened, to the instant, when the positive acceleration of demand increase diminishes to zero and changes its sign. The thus measured time interval is equal to the inverted value of α (i.e. its duration τ equals $1/\alpha$).

The study of demand can also be carried out with the historical method, by comparing the sales volumes in particular time periods, especially when the prices of competitive products undergo relatively fast changes. If, however, the time periods compared are separated by longer time periods (e.g. longer than one year) then we must take into account the potential necessity of introducing a correction of calculations, allowing for consideration of the possible change in the **purchasing power of the customers**, estimated according to the appropriate "**consumer basket**".

An example

Assume that during our measurement the increase of sales took place, so that we obtained two numbers:

- A_0 as the cumulated demand value for the first time interval, when product price was equal C, and
- $A_1 = A_0 + \Delta A$ for demand in the second time interval, when product price was equal $C_1 = C\text{-}\Delta C$.

The proper values of demand, A_0 and A_1 are obtained by dividing the numbers A_0 and A_1 of the products sold in respective time intervals by the lengths of these intervals.

By comparing the magnitudes of the increments ΔA and ΔC we can determine the approximation for the derivative $\partial A/\partial C$, allowing for the determination of the linear approximation to the values of the demand function, in the neighbourhood of the point C, in dependence upon the value of price, in particular – a price decrease. Of course, determination of the demand function over a broader area and with greater precision would require multiple repetitions of the measurements here outlined for different starting values C.

Usually, knowledge of the course of the demand function around the current value of price, C, suffices, since in the normal business activity we rarely can afford bigger changes of product prices.

Let us note that for the determination of the linear dependence of demand upon product price we can use also the seasonal or periodical price decreases, performed on the occasion of some holidays, anniversaries of the company, etc.

Determination of the appropriate time period that should elapse since the introduction of price decrease is in reality much more complicated.

The temporal course of the transition process, from one stable sales level to another stable sales level, requires definite time period, called duration of the transition process. This duration depends upon numerous factors, first of all upon:

- accessibility of the cheaper product for each potential customer,
- encouragement to purchasing of the cheaper product from the side of mass media,
- strength of the conservatism in the habits of the customers.

This duration time characterises the inertia of the process of changes in the demand function.

In connection with the above it can happen that at the end of the pre-defined period of the measurement experiment, meant to establish the dependence of demand upon the product price, demand may still not be stabilised, and then the period should yet be extended.

As we analyse deeper the demand process, we can represent the quantitative measure of the volume of demand, generated by a statistical customer, in the form of the expression

$$\nu N/T$$

where N is the number of buyers (in a pre-defined time period), ν is the expected (mean) magnitude of a single purchase event that the statistical buyer makes, and T is the mean time interval between successive purchases.

As this has been already mentioned, we shall be assuming further on that $\nu = 1$, and then T is also the period of use of the product.

Hence, the frequency of making purchases equals the inverted time period of use of the product ($1/T$), and so we get

$$\lambda = 1/T \text{ and } \Lambda = \lambda N = N/T.$$

In this manner we can (also for $n = 1$) identify the form of dependence of demand upon the period T of exploitation (wear) of product unit.

Namely, if at a definite time instant we introduce onto the market the very same product, having the same price, but characterised by a significant improvement in durability, then, after a certain period of time, when the sales stabilise at a higher level, we may be able to establish the magnitude of the increase of sales, caused by the increase of durability. The ratio of these quantities determines, approximately, the value of the first partial derivative of the function of demand in dependence upon product durability, similarly as we made use of the results of measurement of dependence of demand upon price.

As we denote with $\Lambda(C,T)$ the dependence of demand upon the price C and the durability T, we can write down the expression for the linear approximation of this function around the point (C_0,T_0) in the form

$$\Lambda(C,T) \approx \Lambda(C_0,T_0) + \frac{\partial\Lambda}{\partial C}(C\text{-}C_0) + \frac{\partial\Lambda}{\partial T}(T\text{-}T_0).$$

We can, of course, write down the approximate dependence of demand upon many other variables (factors), under, naturally, definite assumptions. This question is addressed in some other publications of the first author.

Given that dependence of demand upon price is usually stronger than upon product durability, the latter shall be neglected in further considerations, especially when we investigate only the question of price competition.

There exists one more method for estimating the dependence of demand Λ upon the product price, through indirect estimation of parameters of the demand function. This, however, is possible only under the assumption that the respective dependence is linear. The course of the function is then fully defined by two parameters: L_{mx} and C_{mx}. The entire difficulty in determining their values consists in the possibly precise estimation of the number of customers, N©, whose needs are fulfilled by the product considered.

In order to arrive at such an estimation, we must in a possibly precise manner the group (set) of persons, natural and / or legal, who are the potential customers and may become actual buyers.

So, in particular, we should possibly precisely define the characteristic features of these customers, such as:

-- age, gender, occupation, education, area of residence – with due account of the access to the sales facilities of the given product (for natural persons);

-- sector of economy, nature of products manufactured, turnover value, location of activity (for legal persons), etc.

Of special importance is to establish, whether the potential customers are within the reach (R) of influence (accessibility) of the sales facilities of the product (see Volume I). For the thus defined group of potential customers we should determine the upper and lower boundaries of their incomes.

Having the data as specified above, we can perform estimations of:

- N_{mx}, the maximum number of potential customers, for the price of the product tending to zero,
- C_{mx}, the maximum price of the product, for which the number of buyers would tend to zero.

When we have the value of N_{mx}, then we can determine the value of the maximum demand (for $\nu = 1$):

$$\Lambda = \Lambda_{mx}(1 - C/C_{mx}) \text{ or } \Lambda = \boldsymbol{a}(C_{mx} - C),$$

where $\boldsymbol{a} = \Lambda_{mx}/C_{mx}$, and $\Lambda_{mx} = N/T$.

Having the thus established values of these quantities, we can determine the dependence of demand upon price.

Yet, this is not the end of the respective procedure. We ought to check the correctness of the function obtained. Namely, we must introduce into the thus determined function the current value of the product sales price, C_0, and the current magnitude A_0 of the sales of the given products on the market considered, irrespective of the companies that produce them. We then check the consistency of the two numbers: $A(C_0)$ and A_0. In the case the difference between the two is too big, in our opinion, we should return to the attempts of further, more precise determination of the values of A_{mx} and C_{mx}.

In some cases the quantities N_{mx} and C_{mx} can be estimated in a much simpler manner. An explanation of the method of estimation will be shown on the example below.

An example

Assume a bakery produces some kind of bakery, supplied to the bakery shops in the neighbourhood. The commonly assumed price of a unit of supplied products (say, a box) by the local bakeries is C_B. We wish to determine the linear function of demand, hence to determine the values of parameters of the demand function: A_{mx} and C_{mx}. We do also assume, of course, that the price, reigning on the market, C_B, is higher than own cost $\boldsymbol{b}$ of producing the unit mentioned.

If we increase the price, then it shall turn out that even the closest purchasers shall give up buying bakery products from us, when the price exceeds the value of C'.

In this manner we have determined the value of $C_{mx} = c'$. We can now establish the linear demand function, since we dispose of the coordinates of two points on this function:

$$(C_B, A_B) \text{ and } (C', 0).$$

By solving the resulting problem with two unknown values, we obtain the formula for A_{mx}:

$$\Lambda_{mx} = \frac{\Lambda_B}{1 - \frac{C_B}{C'}}.$$

In this context, let us note that Λ_B is the current magnitude of sales, for the sales price C_B. Besides, we should remember that we are interested in the course of the demand function for the values of C from b to $C_{mx} = C'$.

It is somewhat less complicated to **estimate the value of the coefficient** α, defining the speed of reaction of the market to the appearance of the new, competitive product, sold at a lower price.

Thus, it is recommended to observe the demand of at least one of the two competitive products, preferably of the one, whose price C remained unchanged. The measurement to make concerns the time that has elapsed since the instant t_0, when the competitive product started to be sold for the price decreased by ΔC until the time instant when demand for the product, whose price remained unchanged shall decrease to the level defined by

$$a = \frac{C - \Delta C}{C}.$$

The length of this time period is the inverse of the coefficient α, and at the same time this value is the measure of the inertia of the process, τ_0.

* * *

It is obvious that the issues related to the measurement of parameters of the demand and cost functions form a much broader domain than the remarks here forwarded. We simply wished to illustrate the possibilities of determining these parameters.

CONCLUDING REMARKS

This book, like the previous one, constituting Volume I of the introduction to a theory of market competition, contains considerations that involve a number of approximations and simplifications. We, that is – the authors of both these volumes – would like to draw the attention of the Reader to them.

In general, we do not explicitly consider the vague, uncertain or "fuzzy" character of some of the quantities we refer to. This concerns, in particular, such quantities as the limit value of d in the determination of the demand function, $\lambda(C)$.

Likewise, the uncertain, or specific character of some relations has not been treated in an explicit manner. Thus, for instance, we state in the book that the situation when the market shares of two competitors are equal, 50% each, constitutes indeed a kind of equilibrium, but this is an unstable equilibrium point, for any disturbance to this situation shall drive it far away from the equilibrium (assuming, of course, that this disturbance, due to behaviour of one of the competitors, does not find any "appropriate" reaction from the side of the other competitor).

In reality, though, this equilibrium point is not that unstable, i.e. it is not that sensitive to the very small disturbances. Actually, an interval of insensitivity always exists, due to various reasons, such as delays, information shortage, lack of reaction of customers to very small price changes etc. It is even possible that the "hysteresis" effect may appear. In terms of the notions introduced in this book, the magnitude of the zone of insensitivity depends upon the slope of the production characteristics (the value of the derivative $dk/d\mu$).

Independently of the above remarks the considerations here presented neglect the effect of the change in the number of potential customers due to the change in product price. Namely, along with the change in product price, there is also change in the value of the difference C-b, exerting the decisive influence on the magnitude of the optimum radius R^* of the area, over which the company effectively caters to its potential customers. This radius R^* defines, in turn, the number of such potential customers, i.e. the ones, to whom the products are effectively supplied. This number, in turn, together with the income structure of the customers, defines the value of Λ_{mx} (see also Volume I). Yet, in the book, for both companies selling their products for different prices, the very same value of Λ_{mx} was adopted.

Neglecting this particular aspect is justified by the following circumstances:

- a company that just enters the market (as well as the one, which tries to expand its market share) can hardly afford the advertising saying that its product is not worse than the one of the competitor, even though it is cheaper – and this not for all the potential customers, exception being constituted by the farthest ones;
- on the other hand, the company defending its market share and for this purpose decreasing the sales price of its products, ought not get rid of its more distant customers, since this would make a very disadvantageous effect on the remaining customers and would actually accelerate elimination of such a company from the market.

Of course, the fact that we neglected the influence, exerted by the changes in the reach, R, was also largely due to the wish of simplifying the complicated interrelations, constituting the description of the market process, the mechanism of functioning of the "invisible hand of the market".

Considerations, contained in the book, do not account, either, for the influence of advertising, although certain remarks on this subject are forwarded in Chapter I.

Likewise, we did not forward the estimates for the cost of entry onto an alien market, which could be formulated with the use of the formulae for the sales magnitudes (Λ_A and Λ_B).

When considering the (initial) shares of two competing companies, we analysed the case, when they start from equal market shares. For modelling and analysing other possible situations, we could use the coefficient u.

In the case when more than two companies have (non-negligible) shares in a market, the struggle for the market share ought to be started with the weakest company, avoiding the appearance of a hostile coalition of the remaining companies on the market. Otherwise, it would become necessary to establish an own coalition that would be able to withstand the competition of the other coalition. In such a case the struggle for the market shares would reduce to the case of two competitors, that is – to the situation described in the book.

In view of these and, indeed, many other aspects that remain to be accounted for, it is obvious that the description of the mechanism behind the functioning of the "invisible hand" is far from complete.

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This book is the second part of an exposition of a coherent and far-reaching theory of market competition. The theory is based on simple precepts, does not require deep knowledge of either economics or mathematics, and is therefore aimed primarily at undergraduate students and all those trying to put in order their vision of how the essential market mechanisms might work. The present Volume II constitutes a complement to the considerations, contained in Volume I.

The logic of the presentation is straightforward; it associates the easily grasped microeconomic elements of quantitative character in order to arrive at both more general conclusions and at concrete formulae defining the way the market mechanisms work under definite assumed conditions.

Some may consider this exposition too simplistic. In fact, it is deliberately kept very simple, for heuristic purposes, as well as in order to make the conclusions more clear. Adding a lot of details that make theory more realistic – these details, indeed, changing from country to country, and from sector to sector – is mainly left to the Reader, who is supposed to be able to design the more accurate image on the basis of the foundations, provided in the book.

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