

**Developments in Fuzzy Sets,
Intuitionistic Fuzzy Sets,
Generalized Nets and Related Topics.
Volume I: Foundations**

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Michał Baczyński
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**Systems Research Institute
Polish Academy of Sciences**

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Systems Research Institute
Polish Academy of Sciences
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Dedicated to Professor Beloslav Riečan on his 75th anniversary

On a new approach towards defining intuitionistic fuzzy subtractions

Krassimir Atanassov

Dept. of Bioinformatics and Mathematical Modelling,
Institute of Biophysics and Biomedical Engineering,
Bulgarian Academy of Sciences
105 Acad. G. Bonchev Str., 1113 Sofia, Bulgaria
krat@bas.bg

Abstract

A new set of operations subtraction over intuitionistic fuzzy sets are defined and some of their basic properties are studied.

Keywords: intuitionistic fuzzy set, operation, subtraction.

To my friend Prof. Beloslav Riečan

1 Introduction

In a series of papers, part of which written together with Prof. Beloslav Riečan, the concept of “subtraction” operation over an Intuitionistic Fuzzy Set (IFS, see [1]), was introduced for the first time (see, [2, 3, 4, 6, 7, 8, 9]).

In the first two papers [6, 7], we offered direct definitions of subtractions. Later, an approach providing a series of definitions was introduced and 67 different instances of the “subtraction” operation were constructed and their properties were studied. B. Riečan participated actively in this research [8, 9].

Now, a new approach to defining different “subtraction” operations is constructed and some of the basic properties of the derived new instances will be studied.

New Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations (K.T. Atanassov, M. Baczyński, J. Drewniak, J. Kacprzyk, M. Krawczak, E. Szmidt, M. Wygralak, S. Zadrozny, Eds.), IBS PAN - SRI PAS, Warsaw, 2012.

2 Some preliminary results

Up to now, different operations have been defined over IFS. Let

$$A^* = \{\langle x, \mu_A(x), \nu_A(x) \rangle | x \in E\},$$

where the functions $\mu_A : E \rightarrow [0, 1]$ and $\nu_A : E \rightarrow [0, 1]$ stand for the degrees of membership and non-membership of the element x from a fixed universe E to the set $A \subset E$, respectively, and every x satisfies that: $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Let for every $x \in E$:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x).$$

Therefore, function π determines the degree of uncertainty.

Below, for brevity, we write A instead of A^* . When the IFSs A and B are given, we can construct the IFS $A - B$. The currently existing forms of this operation are given below. The first two forms are taken, respectively, from [6] and [7] and we will denote them as $BR1$ and $BR2$:

$$A -_{BR1} B = \{\langle x, \mu_{A-B}(x), \nu_{A-B}(x) \rangle | x \in E\},$$

where

$$\mu_{A-B}(x) = \begin{cases} \frac{\mu_A(x) - \mu_B(x)}{1 - \mu_B(x)}, & \text{if } \mu_A(x) \geq \mu_B(x) \text{ and } \nu_A(x) \leq \nu_B(x) \\ & \text{and } \nu_B(x) > 0 \\ & \text{and } \nu_A(x)\pi_B(x) \leq \pi_A(x)\nu_B(x) \\ 0, & \text{otherwise} \end{cases}$$

and

$$\nu_{A-B}(x) = \begin{cases} \frac{\nu_A(x)}{\nu_B(x)}, & \text{if } \mu_A(x) \geq \mu_B(x) \text{ and } \nu_A(x) \leq \nu_B(x) \\ & \text{and } \nu_B(x) > 0 \\ & \text{and } \nu_A(x)\pi_B(x) \leq \pi_A(x)\nu_B(x) \\ 1, & \text{otherwise} \end{cases}$$

and

$$A -_{BR2} B = \{\langle \min(\mu_A(x), \nu_B(x)), \max(\mu_B(x), \nu_A(x)) \rangle | x \in E\}.$$

In some definitions below, we use functions sg and $\overline{\text{sg}}$, defined by

$$\text{sg}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases},$$

$$\overline{\text{sg}}(x) = \begin{cases} 0 & \text{if } x > 0 \\ 1 & \text{if } x \leq 0 \end{cases}$$

The next definitions of instances of the “subtraction” operation are based on the well-known formula from set theory:

$$A - B = A \cap \neg B$$

where A and B are given sets. In the IFS case, if the IFSs A and B are given, we define the following versions of “subtraction” operation:

$$A -'_i B = A \cap \neg_i B, \text{ and } A -''_i B = \neg_i \neg_i A \cap \neg_i B,$$

where $i = 1, 2, \dots, 34$.

Of course, for every two IFSs A and B , it is valid that

$$A -'_1 B = A -''_1 B,$$

because the first negation will satisfy the Law of Excluded Middle, but in the other cases this equality is not valid.

All new subtractions are given in Table 1.

Table 1: List of intuitionistic fuzzy subtractions

$-'_1$	$\{\langle x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x)) \rangle x \in E\}$
$-'_2$	$\{\langle x, \min(\mu_A(x), \overline{\text{sg}}(\mu_B(x))), \max(\nu_A(x), \text{sg}(\mu_B(x))) \rangle x \in E\}$
$-'_3$	$\{\langle x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x) \cdot \nu_B(x) + \mu_B(x)^2) \rangle x \in E\}$
$-'_4$	$\{\langle x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), 1 - \nu_B(x)) \rangle x \in E\}$
$-'_5$	$\{\langle x, \min(\mu_A(x), \overline{\text{sg}}(1 - \nu_B(x))), \max(\nu_A(x), \text{sg}(1 - \nu_B(x))) \rangle x \in E\}$
$-'_6$	$\{\langle x, \min(\mu_A(x), \overline{\text{sg}}(1 - \nu_B(x))), \max(\nu_A(x), \text{sg}(\mu_B(x))) \rangle x \in E\}$
$-'_7$	$\{\langle x, \min(\mu_A(x), \overline{\text{sg}}(1 - \nu_B(x))), \max(\nu_A(x), \mu_B(x)) \rangle x \in E\}$
$-'_8$	$\{\langle x, \min(\mu_A(x), 1 - \mu_B(x)), \max(\nu_A(x), \mu_B(x)) \rangle x \in E\}$
$-'_9$	$\{\langle x, \min(\mu_A(x), \overline{\text{sg}}(\mu_B(x))), \max(\nu_A(x), \mu_B(x)) \rangle x \in E\}$

$-'_{10}$	$\{\langle x, \min(\mu_A(x), \overline{\text{sg}}(1 - \nu_B(x))), \max(\nu_A(x), 1 - \nu_B(x)) \rangle x \in E\}$
$-'_{11}$	$\{\langle x, \min(\mu_A(x), \text{sg}(\nu_B(x))), \max(\nu_A(x), \overline{\text{sg}}(\nu_B(x))) \rangle x \in E\}$
$-'_{12}$	$\{\langle x, \min(\mu_A(x), \nu_B(x) \cdot (\mu_B(x) + \nu_B(x))),$ $\max(\nu_A(x), \mu_B(x) \cdot (\nu_B(x)^2 + \mu_B(x) + \mu_B(x) \cdot \nu_B(x))) \rangle x \in E\}$
$-'_{13}$	$\{\langle x, \min(\mu_A(x), \text{sg}(1 - \mu_B(x))),$ $\max(\nu_A(x), \overline{\text{sg}}(1 - \mu_B(x))) \rangle x \in E\}$
$-'_{14}$	$\{\langle x, \min(\mu_A(x), \text{sg}(\nu_B(x))), \max(\nu_A(x), \overline{\text{sg}}(1 - \mu_B(x))) \rangle x \in E\}$
$-'_{15}$	$\{\langle x, \min(\mu_A(x), \overline{\text{sg}}(1 - \nu_B(x))),$ $\max(\nu_A(x), \overline{\text{sg}}(1 - \mu_B(x))) \rangle x \in E\}$
$-'_{16}$	$\{\langle x, \min(\mu_A(x), \overline{\text{sg}}(\mu_B(x))), \max(\nu_A(x), \overline{\text{sg}}(1 - \mu_B(x))) \rangle x \in E\}$
$-'_{17}$	$\{\langle x, \min(\mu_A(x), \overline{\text{sg}}(1 - \nu_B(x))), \max(\nu_A(x), \overline{\text{sg}}(\nu_B(x))) \rangle x \in E\}$
$-'_{18}$	$\{\langle x, \min(\mu_A(x), \nu_B(x), \text{sg}(\mu_B(x))),$ $\max(\nu_A(x), \min(\mu_B(x), \text{sg}(\nu_B(x)))) \rangle x \in E\}$
$-'_{19}$	$\{\langle x, \min(\mu_A(x), \nu_B(x), \text{sg}(\mu_B(x))), \nu_A(x) \rangle x \in E\}$
$-'_{20}$	$\{\langle x, \min(\mu_A(x), \nu_B(x)), \nu_A(x) \rangle x \in E\}$
$-'_{21}$	$\{\langle x, \min(\mu_A(x), 1 - \mu_B(x), \text{sg}(\mu_B(x))),$ $\max(\nu_A(x), \min(\mu_B(x), \text{sg}(1 - \mu_B(x)))) \rangle x \in E\}$
$-'_{22}$	$\{\langle x, \min(\mu_A(x), 1 - \mu_B(x), \text{sg}(\mu_B(x))), \nu_A(x) \rangle x \in E\}$
$-'_{23}$	$\{\langle x, \min(\mu_A(x), 1 - \mu_B(x)), \nu_A(x) \rangle x \in E\}$
$-'_{24}$	$\{\langle x, \min(\mu_A(x), \nu_B(x), \text{sg}(1 - \nu_B(x))),$ $\max(\nu_A(x), \min(1 - \nu_B(x), \text{sg}(\nu_B(x)))) \rangle x \in E\}$
$-'_{25}$	$\{\langle x, \min(\mu_A(x), \nu_B(x), \text{sg}(1 - \nu_B(x))), \nu_A(x) \rangle x \in E\}$
$-'_{26}$	$\{\langle x, \min(\mu_A(x), \nu_B(x)),$ $\max(\nu_A(x), \mu_B(x) \cdot \nu_B(x) + \overline{\text{sg}}(1 - \mu_B(x))) \rangle x \in E\}$
$-'_{27}$	$\{\langle x, \min(\mu_A(x), 1 - \mu_B(x)),$ $\max(\nu_A(x), \mu_B(x) \cdot (1 - \mu_B(x)) + \overline{\text{sg}}(1 - \mu_B(x))) \rangle x \in E\}$
$-'_{28}$	$\{\langle x, \min(\mu_A(x), \nu_B(x)),$ $\max(\nu_A(x), (1 - \nu_B(x)) \cdot \nu_B(x) + \overline{\text{sg}}(\nu_B(x))) \rangle x \in E\}$
$-'_{29}$	$\{\langle x, \min(\mu_A(x), \max(0, \mu_B(x) \cdot \nu_B(x) + \overline{\text{sg}}(1 - \nu_B(x))),$ $\max(\nu_A(x), \mu_B(x) \cdot (\mu_B(x) \cdot \nu_B(x)$ $+ \overline{\text{sg}}(1 - \nu_B(x)) + \overline{\text{sg}}(1 - \mu_B(x)))) \rangle x \in E\}$
$-'_{30}$	$\{\langle x, \min(\mu_A(x), \mu_B(x) \cdot \nu_B(x)),$ $\max(\nu_A(x), \mu_B(x) \cdot (\mu_B(x) \cdot \nu_B(x)$ $+ \overline{\text{sg}}(1 - \nu_B(x)) + \overline{\text{sg}}(1 - \mu_B(x)))) \rangle x \in E\}$
$-'_{31}$	$\{\langle x, \min(\mu_A(x), (1 - \mu_B(x)) \cdot \mu_B(x) + \overline{\text{sg}}(\mu_B(x))),$ $\max(\nu_A(x), \mu_B(x) \cdot ((1 - \mu_B(x)) \cdot \mu_B(x)$ $+ \overline{\text{sg}}(\mu_B(x)) + \overline{\text{sg}}(1 - \mu_B(x)))) \rangle x \in E\}$

$-'_{32}$	$\{\langle x, \min(\mu_A(x), (1 - \mu_B(x)) \cdot \mu_B(x)),$ $\max(\nu_A(x), \mu_B(x) \cdot ((1 - \mu_B(x)) \cdot \mu_B(x)$ $+ \overline{\text{sg}}(\mu_B(x)) + \overline{\text{sg}}(1 - \mu_B(x)))) \mid x \in E \}$
$-'_{33}$	$\{\langle x, \min(\mu_A(x), \nu_B(x) \cdot (1 - \nu_B(x)) + \overline{\text{sg}}(1 - \nu_B(x))),$ $\max(\nu_A(x), (1 - \nu_B(x)) \cdot (\nu_B(x) \cdot (1 - \nu_B(x))$ $+ \overline{\text{sg}}(1 - \nu_B(x)) + \overline{\text{sg}}(\nu_B(x)))) \mid x \in E \}$
$-'_{34}$	$\{\langle x, \min(\mu_A(x), \nu_B(x) \cdot (1 - \nu_B(x))),$ $\max(\nu_A(x), (1 - \nu_B(x)) \cdot (\nu_B(x) \cdot (1 - \nu_B(x))$ $+ \overline{\text{sg}}(1 - \nu_B(x)) + \overline{\text{sg}}(\nu_B(x)))) \mid x \in E \}$
$-''_1$	$\{\langle x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x)) \mid x \in E \}$
$-''_2$	$\{\langle x, \min(\text{sg}(\mu_A(x)), \overline{\text{sg}}(\mu_B(x))),$ $\max(\overline{\text{sg}}(\mu_A(x)), \text{sg}(\mu_B(x))) \mid x \in E \}$
$-''_3$	$\{\langle x, \min(\mu_A(x) \cdot \nu_A(x) + \mu_A(x)^2, \nu_B(x)),$ $\max(\nu_A(x) \cdot (\mu_A(x) \cdot \nu_A(x) + \mu_A(x)^2) + \nu_A(x)^2,$ $\mu_B(x) \cdot \nu_B(x) + \mu_B(x)^2) \mid x \in E \}$
$-''_4$	$\{\langle x, \min(1 - \nu_A(x), \nu_B(x)), \max(\nu_A(x), 1 - \nu_B(x)) \mid x \in E \}$
$-''_5$	$\{\langle x, \min(\text{sg}(1 - \nu_A(x)), \overline{\text{sg}}(1 - \nu_B(x))),$ $\max(\overline{\text{sg}}(1 - \nu_A(x)), \text{sg}(1 - \nu_B(x))) \mid x \in E \}$
$-''_6$	$\{\langle x, \min(\text{sg}(\mu_A(x)), \overline{\text{sg}}(1 - \nu_B(x))),$ $\max(\overline{\text{sg}}(1 - \nu_A(x)), \text{sg}(\mu_B(x))) \mid x \in E \}$
$-''_7$	$\{\langle x, \min(\overline{\text{sg}}(1 - \mu_A(x)), \overline{\text{sg}}(1 - \nu_B(x))),$ $\max(\overline{\text{sg}}(1 - \nu_A(x)), \mu_B(x)) \mid x \in E \}$
$-''_8$	$\{\langle x, \min(\mu_A(x), 1 - \mu_B(x)), \max(1 - \mu_A(x), \mu_B(x)) \mid x \in E \}$
$-''_9$	$\{\langle x, \min(\text{sg}(\mu_A(x)), \overline{\text{sg}}(\mu_B(x))), \max(\overline{\text{sg}}(\mu_A(x)), \mu_B(x)) \mid x \in E \}$
$-''_{10}$	$\{\langle x, \min(\overline{\text{sg}}(\nu_A(x)), \overline{\text{sg}}(1 - \nu_B(x))),$ $\max(\nu_A(x), 1 - \nu_B(x)) \mid x \in E \}$
$-''_{11}$	$\{\langle x, \min(\overline{\text{sg}}(\nu_A(x)), \text{sg}(\nu_B(x))),$ $\max(\text{sg}(\nu_A(x)), \overline{\text{sg}}(\nu_B(x))) \mid x \in E \}$
$-''_{12}$	$\{\langle x, \min(\mu_A(x) \cdot (\nu_A(x)^2 + \mu_A(x) + \mu_A(x) \cdot \nu_A(x)) \cdot (\mu_A(x) \cdot (\nu_A(x)^2$ $+ \mu_A(x) + \mu_A(x) \cdot \nu_A(x)) + (\nu_A(x) \cdot (\mu_A(x) + \nu_A(x))))),$ $\nu_B(x) \cdot (\mu_B(x) + \nu_B(x))),$ $\max(\nu_A(x) \cdot (\mu_A(x) + \nu_A(x)) \cdot (\mu_A(x)^2 \cdot (\nu_A(x)^2 + \mu_A(x)$ $+ \mu_A(x) \cdot \nu_A(x))^2 + \nu_A(x) \cdot (\mu_A(x) + \nu_A(x))) + \mu_A(x) \cdot \nu_A(x)$ $\cdot (\nu_A(x)^2 + \mu_A(x) + \mu_A(x) \cdot \nu_A(x)) \cdot (\mu_A(x) + \nu_A(x)),$ $\mu_B(x) \cdot (\nu_B(x)^2 + \mu_B(x) + \mu_B(x) \cdot \nu_B(x))) \mid x \in E \}$
$-''_{13}$	$\{\langle x, \min(\overline{\text{sg}}(1 - \mu_A(x)), \text{sg}(1 - \mu_B(x))),$ $\max(\text{sg}(1 - \mu_A(x)), \overline{\text{sg}}(1 - \mu_B(x))) \mid x \in E \}$

$-\frac{''}{14}$	$\{\langle x, \min(\overline{\text{sg}}(1 - \mu_A(x)), \text{sg}(\nu_B(x))), \max(\text{sg}(\nu_A(x)), \overline{\text{sg}}(1 - \mu_B(x))) \rangle x \in E\}$
$-\frac{''}{15}$	$\{\langle x, \min(\overline{\text{sg}}(1 - \mu_A(x)), \overline{\text{sg}}(1 - \nu_B(x))), \max(\overline{\text{sg}}(1 - \nu_A(x)), \overline{\text{sg}}(1 - \mu_B(x))) \rangle x \in E\}$
$-\frac{''}{16}$	$\{\langle x, \min(\text{sg}(\mu_A(x)), \overline{\text{sg}}(\mu_B(x))), \max(\overline{\text{sg}}(\mu_A(x)), \overline{\text{sg}}(1 - \mu_B(x))) \rangle x \in E\}$
$-\frac{''}{17}$	$\{\langle x, \min(\overline{\text{sg}}(\nu_A(x)), \overline{\text{sg}}(1 - \nu_B(x))), \max(\text{sg}(\nu_A(x)), \overline{\text{sg}}(\nu_B(x))) \rangle x \in E\}$
$-\frac{''}{18}$	$\{\langle x, \min(\mu_A(x), \text{sg}(\nu_A(x)), \nu_B(x), \text{sg}(\mu_B(x))), \max(\min(\nu_A(x), \text{sg}(\mu_A(x))), \min(\mu_B(x), \text{sg}(\nu_B(x)))) \rangle x \in E\}$
$-\frac{''}{19}$	$\{\langle x, 0, 0 \rangle x \in E\}$
$-\frac{''}{20}$	$\{\langle x, 0, 0 \rangle x \in E\}$
$-\frac{''}{21}$	$\{\langle x, \mu_A(x) \cdot \text{sg}(1 - \mu_A(x)), \max((1 - \mu_A(x)) \cdot \text{sg}(\mu_A(x)), \min(\mu_B(x), \text{sg}(1 - \mu_B(x)))) \rangle x \in E\}$
$-\frac{''}{22}$	$\{\langle x, \min(\mu_A(x) \cdot \text{sg}(\mu_A(x)), 1 - \mu_B(x), \text{sg}(\mu_B(x))), 0 \rangle x \in E\}$
$-\frac{''}{23}$	$\{\langle x, \min(\mu_A(x), 1 - \mu_B(x)), 0 \rangle x \in E\}$
$-\frac{''}{24}$	$\{\langle x, \min(1 - \nu_A(x), \text{sg}(\nu_A(x)), \nu_B(x), \text{sg}(1 - \nu_B(x))), \max(\nu_A(x) \cdot \text{sg}(1 - \nu_A(x)), \min(1 - \nu_B(x), \text{sg}(\nu_B(x)))) \rangle x \in E\}$
$-\frac{''}{25}$	$\{\langle x, 0, 0 \rangle x \in E\}$
$-\frac{''}{26}$	$\{\langle x, \min(\mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \mu_A(x)), \nu_B(x)), \max(\nu_A(x) \cdot (\mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \mu_A(x))) + \overline{\text{sg}}(1 - \nu_A(x)), \mu_B(x) \cdot \nu_B(x) + \overline{\text{sg}}(1 - \mu_B(x))) \rangle x \in E\}$
$-\frac{''}{27}$	$\{\langle x, \min(\mu_A(x), 1 - \mu_B(x)), \max(((1 - \mu_A(x)) \cdot \mu_A(x)) + \overline{\text{sg}}(\mu_A(x)), \mu_B(x) \cdot (1 - \mu_B(x)) + \overline{\text{sg}}(1 - \mu_B(x))) \rangle x \in E\}$
$-\frac{''}{28}$	$\{\langle x, \min((1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}(\nu_A(x)), \nu_B(x)), \max((1 - (1 - \nu_A(x)) \cdot \nu_A(x)) - \overline{\text{sg}}(\nu_A(x))) \cdot ((1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}((1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}(\nu_A(x))), (1 - \nu_B(x)) \cdot \nu_B(x) + \overline{\text{sg}}(\nu_B(x))) \rangle x \in E\}$
$-\frac{''}{29}$	$\{\langle x, \min((\mu_A(x) \cdot (\mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(1 - \mu_A(x))) \cdot (\mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(1 - \mu_A(x)) \cdot (\mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))) - \overline{\text{sg}}(1 - \mu_A(x))), \mu_B(x) \cdot \nu_B(x) + \overline{\text{sg}}(1 - \nu_B(x))), \max((\mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))) \cdot ((\mu_A(x) \cdot (\mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(1 - \mu_A(x))) \cdot (\mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(1 - \mu_A(x)) \cdot (\mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))) - \overline{\text{sg}}(1 - \mu_A(x)))) + \overline{\text{sg}}(1 - \mu_A(x)) \cdot \nu_A(x) - \overline{\text{sg}}(1 - \nu_A(x))), \mu_B(x) \cdot (\mu_B(x) \cdot \nu_B(x) + \overline{\text{sg}}(1 - \nu_B(x))) + \overline{\text{sg}}(1 - \mu_B(x))) \rangle x \in E\}$

$-\frac{''}{30}$	$\{ \langle x, \min(((\mu_A(x) \cdot (\mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(1 - \mu_A(x))) \cdot \mu_A(x) \cdot \nu_A(x)), \mu_B(x) \cdot \nu_B(x)), \max(\mu_A(x) \cdot \nu_A(x) \cdot ((\mu_A(x) \cdot (\mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(1 - \mu_A(x))) \cdot \mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))) - \overline{\text{sg}}(1 - \mu_A(x)))) + \overline{\text{sg}}(1 - (\mu_A(x) \cdot \nu_A(x))), \mu_B(x) \cdot (\mu_B(x) \cdot \nu_B(x) + \overline{\text{sg}}(1 - \nu_B(x))) + \overline{\text{sg}}(1 - \mu_B(x))) \rangle x \in E \}$
$-\frac{''}{31}$	$\{ \langle x, \min((1 - (1 - \mu_A(x)) \cdot \mu_A(x) - \overline{\text{sg}}(\mu_A(x))) \cdot ((1 - \mu_A(x)) \cdot \mu_A(x) + \overline{\text{sg}}(\mu_A(x))) + \overline{\text{sg}}((1 - \mu_A(x)) \cdot \mu_A(x) + \overline{\text{sg}}(\mu_A(x))))), (1 - \mu_B(x)) \cdot \mu_B(x) + \overline{\text{sg}}(\mu_B(x))), \max(((1 - \mu_A(x)) \cdot \mu_A(x) + \overline{\text{sg}}(\mu_A(x))) \cdot ((1 - (1 - \mu_A(x)) \cdot \mu_A(x)) - \overline{\text{sg}}(\mu_A(x))) \cdot ((1 - \mu_A(x)) \cdot \mu_A(x) + \overline{\text{sg}}(\mu_A(x))) + \overline{\text{sg}}((1 - \mu_A(x)) \cdot \mu_A(x) + \overline{\text{sg}}(\mu_A(x)))) + \overline{\text{sg}}(1 - (1 - \mu_A(x)) \cdot \mu_A(x) - \overline{\text{sg}}(\mu_A(x))), \mu_B(x) \cdot ((1 - \mu_B(x)) \cdot \mu_B(x) + \overline{\text{sg}}(\mu_B(x))) + \overline{\text{sg}}(1 - \mu_B(x))) \rangle x \in E \}$
$-\frac{''}{32}$	$\{ \langle x, \min((1 - (1 - \mu_A(x)) \cdot \mu_A(x)) \cdot (1 - \mu_A(x)) \cdot \mu_A(x), (1 - \mu_B(x)) \cdot \mu_B(x)), \max(((1 - \mu_A(x)) \cdot \mu_A(x) \cdot ((1 - (1 - \mu_A(x)) \cdot \mu_A(x)) \cdot (1 - \mu_A(x)) \cdot \mu_A(x) + \overline{\text{sg}}((1 - \mu_A(x)) \cdot \mu_A(x))) + \overline{\text{sg}}(1 - (1 - \mu_A(x)) \cdot \mu_A(x))), \mu_B(x) \cdot ((1 - \mu_B(x)) \cdot \mu_B(x) + \overline{\text{sg}}(\mu_B(x))) + \overline{\text{sg}}(1 - \mu_B(x))) \rangle x \in E \}$
$-\frac{''}{33}$	$\{ \langle x, \min(((1 - \nu_A(x)) \cdot (\nu_A(x) \cdot (1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(\nu_A(x))) \cdot (1 - (1 - \nu_A(x)) \cdot (\nu_A(x) \cdot (1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x))) - \overline{\text{sg}}(\nu_A(x))) + \overline{\text{sg}}(1 - (1 - \nu_A(x)) \cdot (\nu_A(x) \cdot (1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x))) - \overline{\text{sg}}(\nu_A(x))), \nu_B(x) \cdot (1 - \nu_B(x)) + \overline{\text{sg}}(1 - \nu_B(x))), \max((1 - (1 - \nu_A(x)) \cdot (\nu_A(x) \cdot (1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x))) - \overline{\text{sg}}(\nu_A(x))) \cdot ((1 - \nu_A(x)) \cdot (\nu_A(x) \cdot (1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(\nu_A(x))) \cdot (1 - (1 - \nu_A(x)) \cdot (\nu_A(x) \cdot (1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x))) - \overline{\text{sg}}(\nu_A(x))) + \overline{\text{sg}}(1 - (1 - \nu_A(x)) \cdot (\nu_A(x) \cdot (1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x))) - \overline{\text{sg}}(\nu_A(x)))) + \overline{\text{sg}}((1 - \nu_A(x)) \cdot (\nu_A(x) \cdot (1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x))) - \overline{\text{sg}}(\nu_A(x))), (1 - \nu_B(x)) \cdot (\nu_B(x) \cdot (1 - \nu_B(x)) + \overline{\text{sg}}(1 - \nu_B(x))) + \overline{\text{sg}}(\nu_B(x))) \rangle x \in E \}$

$-''_{34}$	$\{ \langle x, \min(((1-\nu_A(x)).(\nu_A(x)).(1-\nu_A(x)) + \overline{\text{sg}}(1-\nu_A(x))) + \overline{\text{sg}}(\nu_A(x))).(1 - (1 - \nu_A(x)).(\nu_A(x)).(1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x))) - \overline{\text{sg}}(\nu_A(x))),$ $\nu_B(x).(1 - \nu_B(x)), \max(((1 - (1 - \nu_A(x)).(\nu_A(x)).(1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x))) - \overline{\text{sg}}(\nu_A(x))).((1 - \nu_A(x)).(\nu_A(x))$ $.(1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(\nu_A(x))).(1 - (1 - \nu_A(x))$ $.(\nu_A(x).(1-\nu_A(x))+\overline{\text{sg}}(1-\nu_A(x)))-\overline{\text{sg}}(\nu_A(x)))+\overline{\text{sg}}(1-(1-\nu_A(x))$ $.(\nu_A(x).(1-\nu_A(x)) + \overline{\text{sg}}(1-\nu_A(x)))-\overline{\text{sg}}(\nu_A(x))))$ $+\overline{\text{sg}}((1-\nu_A(x)).(\nu_A(x).(1-\nu_A(x)) + \overline{\text{sg}}(1-\nu_A(x))) + \overline{\text{sg}}(\nu_A(x))),$ $(1-\nu_B(x)).(\nu_B(x).(1 - \nu_B(x)) + \overline{\text{sg}}(1 - \nu_B(x)))$ $+\overline{\text{sg}}(\nu_B(x))) \mid x \in E \}$
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We immediately see that operation $-_{BR1}$ does not occur in Table 1, while operations $-_{BR2}$, $-'_1$ and $-''_2$ coincide.

3 Main results

Initially, we give the list of all intuitionistic fuzzy implications (see Table 2). They generate 34 different negations, given in Table 3. The relations between the implications and negations are shown in Table 4.

Table 2: List of the intuitionistic fuzzy implications

\rightarrow_1	$\{ \langle x, \max(\nu_A(x), \min(\mu_A(x), \mu_B(x))), \min(\mu_A(x), \nu_B(x)) \rangle \mid x \in E \}$
\rightarrow_2	$\{ \langle x, \overline{\text{sg}}(\mu_A(x) - \mu_B(x)), \nu_B(x). \text{sg}(\mu_A(x) - \mu_B(x)) \rangle \mid x \in E \}$
\rightarrow_3	$\{ \langle x, 1 - (1 - \mu_B(x)). \text{sg}(\mu_A(x) - \mu_B(x)) \rangle, \nu_B(x). \text{sg}(\mu_A(x) - \mu_B(x)) \rangle \mid x \in E \}$
\rightarrow_4	$\{ \langle x, \max(\nu_A(x), \mu_B(x)), \min(\mu_A(x), \nu_B(x)) \rangle \mid x \in E \}$
\rightarrow_5	$\{ \langle x, \min(1, \nu_A(x) + \mu_B(x)), \max(0, \mu_A(x) + \nu_B(x) - 1) \rangle \mid x \in E \}$
\rightarrow_6	$\{ \langle x, \nu_A(x) + \mu_A(x)\mu_B(x), \mu_A(x)\nu_B(x) \rangle \mid x \in E \}$
\rightarrow_7	$\{ \langle x, \min(\max(\nu_A(x), \mu_B(x)), \max(\mu_A(x), \nu_A(x)), \max(\mu_B(x), \nu_B(x))), \max(\min(\mu_A(x), \nu_B(x)), \min(\mu_A(x), \nu_A(x)), \min(\mu_B(x), \nu_B(x)))) \rangle \mid x \in E \}$
\rightarrow_8	$\{ \langle x, 1 - (1 - \min(\nu_A(x), \mu_B(x))). \text{sg}(\mu_A(x) - \mu_B(x)), \max(\mu_A(x), \nu_B(x)). \text{sg}(\mu_A(x) - \mu_B(x)), \text{sg}(\nu_B(x) - \nu_A(x)) \rangle \mid x \in E \}$
\rightarrow_9	$\{ \langle x, \nu_A(x) + \mu_A(x)^2 \mu_B(x), \mu_A(x)\nu_A(x) + \mu_A(x)^2 \nu_B(x) \rangle \mid x \in E \}$

\rightarrow_{10}	$\{\langle x, \mu_B(x). \overline{\text{sg}}(1 - \mu_A(x)) + \text{sg}(1 - \mu_A(x)).(\overline{\text{sg}}(1 - \mu_B(x)) + \nu_A(x). \text{sg}(1 - \mu_B(x))), \nu_B(x). \overline{\text{sg}}(1 - \mu_A(x)) + \mu_A(x). \text{sg}(1 - \mu_A(x)). \text{sg}(1 - \mu_B(x)) \rangle x \in E\}$
\rightarrow_{11}	$\{\langle x, 1 - (1 - \mu_B(x)). \text{sg}(\mu_A(x) - \mu_B(x)), \nu_B(x). \text{sg}(\mu_A(x) - \mu_B(x)). \text{sg}(\nu_B(x) - \nu_A(x)) \rangle x \in E\}$
\rightarrow_{12}	$\{\langle x, \max(\nu_A(x), \mu_B(x)), 1 - \max(\nu_A(x), \mu_B(x)) \rangle x \in E\}$
\rightarrow_{13}	$\{\langle x, \nu_A(x) + \mu_B(x) - \nu_A(x). \mu_B(x), \mu_A(x). \nu_B(x) \rangle x \in E\}$
\rightarrow_{14}	$\{\langle x, 1 - (1 - \mu_B(x)). \text{sg}(\mu_A(x) - \mu_B(x)) - \nu_B(x). \overline{\text{sg}}(\mu_A(x) - \mu_B(x)). \text{sg}(\nu_B(x) - \nu_A(x)), \nu_B(x). \text{sg}(\nu_B(x) - \nu_A(x)) \rangle x \in E\}$
\rightarrow_{15}	$\{\langle x, 1 - \min(\nu_A(x), \mu_B(x)) . \text{sg}(\text{sg}(\mu_A(x)) - \mu_B(x)) + \text{sg}(\nu_B(x) - \nu_A(x)) \min(\nu_A(x), \mu_B(x)). \text{sg}(\mu_A(x) - \mu_B(x)). \text{sg}(\nu_B(x) - \nu_A(x)), 1 - (1 - \min(\mu_A(x), \nu_B(x))). \text{sg}(\overline{\text{sg}}(\mu_A(x) - \mu_B(x)) + \overline{\text{sg}}(\nu_B(x) - \nu_A(x))) - \max(\mu_A(x), \nu_B(x)). \overline{\text{sg}}(\mu_A(x) - \mu_B(x)) . \overline{\text{sg}}(\nu_B(x) - \nu_A(x)) \rangle x \in E\}$
\rightarrow_{16}	$\{\langle x, \max(\overline{\text{sg}}(\mu_A(x)), \mu_B(x)), \min(\text{sg}(\mu_A(x)), \nu_B(x)) \rangle x \in E\}$
\rightarrow_{17}	$\{\langle x, \max(\nu_A(x), \mu_B(x)), \min(\mu_A(x). \nu_A(x) + \mu_A(x)^2, \nu_B(x)) \rangle x \in E\}$
\rightarrow_{18}	$\{\langle x, \max(\nu_A(x), \mu_B(x)), \min(1 - \nu_A(x), \nu_B(x)) \rangle x \in E\}$
\rightarrow_{19}	$\{\langle x, \max(1 - \text{sg}(\text{sg}(\mu_A(x)) + \text{sg}(1 - \nu_A(x))), \mu_B(x)), \min(\text{sg}(1 - \nu_A(x)), \nu_B(x)) \rangle x \in E\}$
\rightarrow_{20}	$\{\langle x, \max(\overline{\text{sg}}(\mu_A(x)), \text{sg}(\mu_B(x))), \min(\text{sg}(\mu_A(x)), \overline{\text{sg}}(\mu_B(x))) \rangle x \in E\}$
\rightarrow_{21}	$\{\langle x, \max(\nu_A(x), \mu_B(x).(\mu_B(x) + \nu_B(x))), \min(\mu_A(x).(\mu_A(x) + \nu_A(x)), \nu_B(x).(\mu_B(x)^2 + \nu_B(x) + \mu_B(x). \nu_B(x))) \rangle x \in E\}$
\rightarrow_{22}	$\{\langle x, \max(\nu_A(x), 1 - \nu_B(x)), \min(1 - \nu_A(x), \nu_B(x)) \rangle x \in E\}$
\rightarrow_{23}	$\{\langle x, 1 - \min(\text{sg}(1 - \nu_A(x)), \overline{\text{sg}}(1 - \nu_B(x))), \min(\text{sg}(1 - \nu_A(x)), \overline{\text{sg}}(1 - \nu_B(x))) \rangle x \in E\}$
\rightarrow_{24}	$\{\langle x, \overline{\text{sg}}(\mu_A(x) - \mu_B(x)). \overline{\text{sg}}(\nu_B(x) - \nu_A(x)), \text{sg}(\mu_A(x) - \mu_B(x)). \text{sg}(\nu_B(x) - \nu_A(x)) \rangle x \in E\}$
\rightarrow_{25}	$\{\langle x, \max(\nu_A(x), \overline{\text{sg}}(\mu_A(x)). \overline{\text{sg}}(1 - \nu_A(x)), \mu_B(x). \overline{\text{sg}}(\nu_B(x)). \overline{\text{sg}}(1 - \mu_B(x))), \min(\mu_A(x), \nu_B(x)) \rangle x \in E\}$
\rightarrow_{26}	$\{\langle x, \max(\overline{\text{sg}}(1 - \nu_A(x)), \mu_B(x)), \min(\text{sg}(\mu_A(x)), \nu_B(x)) \rangle x \in E\}$
\rightarrow_{27}	$\{\langle x, \max(\overline{\text{sg}}(1 - \nu_A(x)), \text{sg}(\mu_B(x))), \min(\text{sg}(\mu_A(x)), \overline{\text{sg}}(1 - \nu_B(x))) \rangle x \in E\}$

→28	$\{\langle x, \max(\overline{\text{sg}}(1 - \nu_A(x)), \mu_B(x)), \min(\mu_A(x), \nu_B(x)) \rangle x \in E\}$
→29	$\{\langle x, \max(\overline{\text{sg}}(1 - \nu_A(x)), \overline{\text{sg}}(1 - \mu_B(x))), \min(\mu_A(x), \overline{\text{sg}}(1 - \nu_B(x))) \rangle x \in E\}$
→30	$\{\langle x, \max(1 - \mu_A(x), \min(\mu_A(x), 1 - \nu_B(x))), \min(\mu_A(x), \nu_B(x)) \rangle x \in E\}$
→31	$\{\langle x, \overline{\text{sg}}(\mu_A(x) + \nu_B(x) - 1), \nu_B(x) \cdot \text{sg}(\mu_A(x) + \nu_B(x) - 1) \rangle x \in E\}$
→32	$\{\langle x, 1 - \nu_B(x) \cdot \text{sg}(\mu_A(x) + \nu_B(x) - 1), \nu_B(x) \cdot \text{sg}(\mu_A(x) + \nu_B(x) - 1) \rangle x \in E\}$
→33	$\{\langle x, 1 - \min(\mu_A(x), \nu_B(x)), \min(\mu_A(x), \nu_B(x)) \rangle x \in E\}$
→34	$\{\langle x, \min(1, 2 - \mu_A(x) - \nu_B(x)), \max(0, \mu_A(x) + \nu_B(x) - 1) \rangle x \in E\}$
→35	$\{\langle x, 1 - \mu_A(x) \cdot \nu_B(x), \mu_A(x) \cdot \nu_B(x) \rangle x \in E\}$
→36	$\{\langle x, \min(1 - \min(\mu_A(x), \nu_B(x)), \max(\mu_A(x), (1 - \mu_A(x)), \max(1 - \nu_B(x), \nu_B(x))), \max(\min(\mu_A(x), \nu_B(x)), \min(\mu_A(x), 1 - \mu_A(x))), \min(1 - \nu_B(x), \nu_B(x))) \rangle x \in E\}$
→37	$\{\langle x, 1 - \max(\mu_A(x), \nu_B(x)) \cdot \text{sg}(\mu_A(x) + \nu_B(x) - 1), \max(\mu_A(x), \nu_B(x)) \cdot \text{sg}(\mu_A(x) + \nu_B(x) - 1) \rangle x \in E\}$
→38	$\{\langle x, 1 - \mu_A(x) + (\mu_A(x))^2 \cdot (1 - \nu_B(x)), \mu_A(x) \cdot (1 - \mu_A(x)) + \mu_A(x)^2 \cdot \nu_B(x) \rangle x \in E\}$
→39	$\{\langle x, (1 - \nu_B(x)) \cdot \overline{\text{sg}}(1 - \mu_A(x)) + \text{sg}(1 - \mu_A(x)) \cdot (\overline{\text{sg}}(\nu_B(x)) + (1 - \mu_A(x)) \cdot \text{sg}(\nu_B(x))), \nu_B(x) \cdot \overline{\text{sg}}(1 - \mu_A(x)) + \mu_A(x) \cdot \text{sg}(1 - \mu_A(x)) \cdot \text{sg}(\nu_B(x)) \rangle x \in E\}$
→40	$\{\langle x, 1 - \text{sg}(\mu_A(x) + \nu_B(x) - 1), 1 - \overline{\text{sg}}(\mu_A(x) + \nu_B(x) - 1) \rangle x \in E\}$
→41	$\{\langle x, \max(\overline{\text{sg}}(\mu_A(x)), (1 - \nu_B(x))), \min(\text{sg}(\mu_A(x)), \nu_B(x)) \rangle x \in E\}$
→42	$\{\langle x, \max(\overline{\text{sg}}(\mu_A(x)), \text{sg}(1 - \nu_B(x))), \min(\text{sg}(\mu_A(x)), \overline{\text{sg}}(1 - \nu_B(x))) \rangle x \in E\}$
→43	$\{\langle x, \max(\overline{\text{sg}}(\mu_A(x)), 1 - \nu_B(x)), \min(\text{sg}(\mu_A(x)), \nu_B(x)) \rangle x \in E\}$
→44	$\{\langle x, \max(\overline{\text{sg}}(\mu_A(x)), 1 - \nu_B(x)), \min(\mu_A(x), \nu_B(x)) \rangle x \in E\}$
→45	$\{\langle x, \max(\overline{\text{sg}}(\mu_A(x)), \overline{\text{sg}}(\nu_B(x))), \min(\mu_A(x), \overline{\text{sg}}(1 - \nu_B(x))) \rangle x \in E\}$
→46	$\{\langle x, \max(\nu_A(x), \min(1 - \nu_A(x), \mu_B(x))), 1 - \max(\nu_A(x), \mu_B(x)) \rangle x \in E\}$
→47	$\{\langle x, \overline{\text{sg}}(1 - \nu_A(x) - \mu_B(x)), (1 - \mu_B(x)) \cdot \text{sg}(1 - \nu_A(x) - \mu_B(x)) \rangle x \in E\}$
→48	$\{\langle x, 1 - (1 - \mu_B(x)) \cdot \text{sg}(1 - \nu_A(x) - \mu_B(x)), (1 - \mu_B(x)) \cdot \text{sg}(1 - \nu_A(x) - \mu_B(x)) \rangle x \in E\}$
→49	$\{\langle x, \min(1, \nu_A(x) + \mu_B(x)), \max(0, 1 - \nu_A(x) - \mu_B(x)) \rangle x \in E\}$
→50	$\{\langle x, \nu_A(x) + \mu_B(x) - \nu_A(x) \cdot \mu_B(x), 1 - \nu_A(x) - \mu_B(x) + \nu_A(x) \cdot \mu_B(x) \rangle x \in E\}$

→51	$\{ \langle x, \min(\max(\nu_A(x), \mu_B(x)), \max(1 - \nu_A(x), \nu_A(x)), \max(\mu_B(x), 1 - \mu_B(x))), \max(1 - \max(\nu_A(x), \mu_B(x)), \min(1 - \nu_A(x), \nu_A(x)), \min(\mu_B(x), 1 - \mu_B(x))) \rangle x \in E \}$
→52	$\{ \langle x, 1 - (1 - \min(\nu_A(x), \mu_B(x))).\text{sg}(1 - \nu_A(x) - \mu_B(x)), 1 - \min(\nu_A(x), \mu_B(x)).\text{sg}(1 - \nu_A(x) - \mu_B(x)) \rangle x \in E \}$
→53	$\{ \langle x, \nu_A(x) + (1 - \nu_A(x))^2 \cdot \mu_B(x), (1 - \nu_A(x)) \cdot \nu_A(x) + (1 - \nu_A(x))^2 \cdot (1 - \mu_B(x)) \rangle x \in E \}$
→54	$\{ \langle x, \mu_B(x) \cdot \overline{\text{sg}}(\nu_A(x)) + \text{sg}(\nu_A(x)) \cdot (\overline{\text{sg}}(1 - \mu_B(x)) + \nu_A(x) \cdot \text{sg}(1 - \mu_B(x))), (1 - \mu_B(x)) \cdot \overline{\text{sg}}(\nu_A(x)) + (1 - \nu_A(x)) \cdot \text{sg}(\nu_A(x)) \cdot \text{sg}(1 - \mu_B(x)) \rangle x \in E \}$
→55	$\{ \langle x, 1 - \text{sg}(1 - \nu_A(x) - \mu_B(x)), 1 - \overline{\text{sg}}(1 - \nu_A(x) - \mu_B(x)) \rangle x \in E \}$
→56	$\{ \langle x, \max(\overline{\text{sg}}(1 - \nu_A(x)), \mu_B(x)), \min(\text{sg}(1 - \nu_A(x)), 1 - \mu_B(x)) \rangle x \in E \}$
→57	$\{ \langle x, \max(\overline{\text{sg}}(1 - \nu_A(x)), \text{sg}(\mu_B(x))), \min(\text{sg}(1 - \nu_A(x)), \overline{\text{sg}}(\mu_B(x))) \rangle x \in E \}$
→58	$\{ \langle x, \max(\overline{\text{sg}}(1 - \nu_A(x)), \overline{\text{sg}}(1 - \mu_B(x))), 1 - \max(\nu_A(x), \mu_B(x)) \rangle x \in E \}$
→59	$\{ \langle x, \max(\overline{\text{sg}}(1 - \nu_A(x)), \mu_B(x)), (1 - \max(\nu_A(x), \mu_B(x))) \rangle x \in E \}$
→60	$\{ \langle x, \max(\overline{\text{sg}}(1 - \nu_A(x)), \overline{\text{sg}}(1 - \mu_B(x))), \min(1 - \nu_A(x), \overline{\text{sg}}(\mu_B(x))) \rangle x \in E \}$
→61	$\{ \langle x, \max(\mu_B(x), \min(\nu_B(x), \nu_A(x))), \min(\nu_B(x), \mu_A(x)) \rangle x \in E \}$
→62	$\{ \langle x, \overline{\text{sg}}(\nu_B(x) - \nu_A(x)), \mu_A(x) \cdot \text{sg}(\nu_B(x) - \nu_A(x)) \rangle x \in E \}$
→63	$\{ \langle x, 1 - (1 - \nu_A(x)) \cdot \text{sg}(\nu_B(x) - \nu_A(x)), \mu_A(x) \cdot \text{sg}(\nu_B(x) - \nu_A(x)) \rangle x \in E \}$
→64	$\{ \langle x, \mu_B(x) + \nu_B(x) \cdot \nu_A(x), \nu_B(x) \cdot \mu_A(x) \rangle x \in E \}$
→65	$\{ \langle x, 1 - (1 - \min(\mu_B(x), \nu_A(x))) \cdot \text{sg}(\nu_B(x) - \nu_A(x)), \max(\nu_B(x), \mu_A(x)) \cdot \text{sg}(\nu_B(x) - \nu_A(x)) \cdot \text{sg}(\mu_A(x) - \mu_B(x)) \rangle x \in E \}$
→66	$\{ \langle x, \mu_B(x) + \nu_B(x)^2 \cdot \nu_A(x), \nu_B(x) \cdot \mu_B(x) + \nu_B(x)^2 \cdot \mu_A(x) \rangle x \in E \}$
→67	$\{ \langle x, \nu_A(x) \cdot \overline{\text{sg}}(1 - \nu_B(x)) + \text{sg}(1 - \nu_B(x)) \cdot (\overline{\text{sg}}(1 - \nu_A(x)) + \mu_B(x) \cdot \text{sg}(1 - \nu_A(x))), \mu_A(x) \cdot \overline{\text{sg}}(1 - \nu_B(x)) + \nu_B(x) \cdot \text{sg}(1 - \nu_B(x)) \cdot \text{sg}(1 - \nu_A(x)) \rangle x \in E \}$
→68	$\{ \langle x, 1 - (1 - \nu_A(x)) \cdot \text{sg}(\nu_B(x) - \nu_A(x)), \mu_A(x) \cdot \text{sg}(\nu_B(x) - \nu_A(x)) \cdot \text{sg}(\mu_A(x) - \mu_B(x)) \rangle x \in E \}$

→69	$\{\langle x, 1 - (1 - \nu_A(x)).\text{sg}(\nu_B(x) - \nu_A(x)) - \mu_A(x).\overline{\text{sg}}(\nu_B(x) - \nu_A(x)).\text{sg}(\mu_A(x) - \mu_B(x)), \mu_A(x).\text{sg}(\mu_A(x) - \mu_B(x)) \rangle x \in E\}$
→70	$\{\langle x, \max(\overline{\text{sg}}(\nu_B(x)), \nu_A(x)), \min(\text{sg}(\nu_B(x)), \mu_A(x)) \rangle x \in E\}$
→71	$\{\langle x, \max(\mu_B(x), \nu_A(x)), \min(\nu_B(x).\mu_B(x) + \nu_B(x)^2, \mu_A(x)) \rangle x \in E\}$
→72	$\{\langle x, \max(\mu_B(x), \nu_A(x)), \min(1 - \mu_B(x), \mu_A(x)) \rangle x \in E\}$
→73	$\{\langle x, \max(1 - \max(\text{sg}(\nu_B(x)), \text{sg}(1 - \mu_B(x))), \nu_A(x)), \min(\text{sg}(1 - \mu_B(x)), \mu_A(x)) \rangle x \in E\}$
→74	$\{\langle x, \max(\overline{\text{sg}}(\nu_B(x)), \text{sg}(\nu_A(x))), \min(\text{sg}(\nu_B(x)), \overline{\text{sg}}(\nu_A(x))) \rangle x \in E\}$
→75	$\{\langle x, \max(\mu_B(x), \nu_A(x).(\nu_A(x) + \mu_A(x))), \min(\nu_B(x).(\nu_B(x) + \mu_B(x)), \mu_A(x).(\nu_A(x)^2 + \mu_A(x)) + \nu_A(x).\mu_A(x)) \rangle x \in E\}$
→76	$\{\langle x, \max(\mu_B(x), 1 - \mu_A(x)), \min(1 - \mu_B(x), \mu_A(x)) \rangle x \in E\}$
→77	$\{\langle x, (1 - \min(\text{sg}(1 - \mu_B(x)), \overline{\text{sg}}(1 - \mu_A(x))))), \min(\text{sg}(1 - \mu_B(x)), \overline{\text{sg}}(1 - \mu_A(x))) \rangle x \in E\}$
→78	$\{\langle x, \max(\overline{\text{sg}}(1 - \mu_B(x)), \nu_A(x)), \min(\text{sg}(\nu_B(x)), \mu_A(x)) \rangle x \in E\}$
→79	$\{\langle x, \max(\overline{\text{sg}}(1 - \mu_B(x)), \text{sg}(\nu_A(x))), \min(\text{sg}(\nu_B(x)), \overline{\text{sg}}(1 - \mu_A(x))) \rangle x \in E\}$
→80	$\{\langle x, \max(\overline{\text{sg}}(1 - \mu_B(x)), \nu_A(x)), \min(\nu_B(x), \mu_A(x)) \rangle x \in E\}$
→81	$\{\langle x, \max(\overline{\text{sg}}(1 - \mu_B(x)), \overline{\text{sg}}(1 - \nu_A(x))), \min(\nu_B(x), \overline{\text{sg}}(1 - \mu_A(x))) \rangle x \in E\}$
→82	$\{\langle x, \max(1 - \nu_B(x), \min(\nu_B(x), 1 - \mu_A(x))), \min(\nu_B(x), \mu_A(x)) \rangle x \in E\}$
→83	$\{\langle x, \overline{\text{sg}}(\nu_B(x) + \mu_A(x) - 1), \mu_A(x).\text{sg}(\nu_B(x) + \mu_A(x) - 1) \rangle x \in E\}$
→84	$\{\langle x, 1 - \mu_A(x).\text{sg}(\nu_B(x) + \mu_A(x) + 1), \mu_A(x).\text{sg}(\nu_B(x) + \mu_A(x) + 1) \rangle x \in E\}$
→85	$\{\langle x, 1 - \nu_B(x) + \nu_B(x)^2.(1 - \mu_A(x)), \nu_B(x).(1 - \nu_B(x) + \nu_B(x)^2) \rangle x \in E\}$
→86	$\{\langle x, (1 - \mu_A(x)).\overline{\text{sg}}(1 - \nu_B(x)) + \text{sg}(1 - \nu_B(x)).\overline{\text{sg}}(\mu_A(x) + \min(1 - \nu_B(x), \text{sg}(\mu_A(x))))), \mu_A(x).\overline{\text{sg}}(1 - \nu_B(x)) + \nu_B(x).\text{sg}(1 - \nu_B(x)).\text{sg}(\mu_A(x)) \rangle x \in E\}$
→87	$\{\langle x, \max(\overline{\text{sg}}(\nu_B(x)), 1 - \mu_A(x)), \min(\text{sg}(\nu_B(x)), \mu_A(x)) \rangle x \in E\}$
→88	$\{\langle x, \max(\overline{\text{sg}}(\nu_B(x)), \text{sg}(1 - \mu_A(x))), \min(\text{sg}(\nu_B(x)), \overline{\text{sg}}(1 - \mu_A(x))) \rangle x \in E\}$
→89	$\{\langle x, \max(\overline{\text{sg}}(\nu_B(x)), 1 - \mu_A(x)), \min(\nu_B(x), \mu_A(x)) \rangle x \in E\}$
→90	$\{\langle x, \max(\overline{\text{sg}}(\nu_B(x)), \overline{\text{sg}}(\mu_A(x))), \min(\nu_B(x), \overline{\text{sg}}(1 - \mu_A(x))) \rangle x \in E\}$

→91	$\{\langle x, \max(\mu_B(x), \min(1 - \mu_B(x), \nu_A(x))), 1 - \max(\mu_B(x), \nu_A(x)) \rangle x \in E\}$
→92	$\{\langle x, \overline{\text{sg}}(1 - \mu_B(x) - \nu_A(x)), \min(1 - \nu_A(x), \text{sg}(1 - \mu_B(x) - \nu_A(x))) \rangle x \in E\}$
→93	$\{\langle x, 1 - \min(1 - \nu_A(x), \text{sg}(1 - \mu_B(x) - \nu_A(x))), \min(1 - \nu_A(x), \text{sg}(1 - \mu_B(x) - \nu_A(x))) \rangle x \in E\}$
→94	$\{\langle x, \mu_B(x) + (1 - \mu_B(x))^2 \cdot \nu_A(x), (1 - \mu_B(x)) \cdot \mu_B(x) + (1 - \mu_B(x))^2 \cdot (1 - \nu_A(x)) \rangle x \in E\}$
→95	$\{\langle x, \min(\nu_A(x), \overline{\text{sg}}(\mu_B(x))) + (\text{sg}(\mu_B(x)) \cdot (\overline{\text{sg}}(1 - \nu_A(x)) + \min(\mu_B(x), \text{sg}(1 - \nu_A(x))))), (\min(1 - \nu_A(x), \overline{\text{sg}}(\mu_B(x))) + \min(\min(1 - \mu_B(x), \text{sg}(\mu_B(x))), \text{sg}(1 - \nu_A(x)))) \rangle x \in E\}$
→96	$\{\langle x, \max(\overline{\text{sg}}(1 - \mu_B(x)), \nu_A(x)), \min(\text{sg}(1 - \mu_B(x)), 1 - \nu_A(x)) \rangle x \in E\}$
→97	$\{\langle x, \max(\overline{\text{sg}}(1 - \mu_B(x)), \text{sg}(\nu_A(x))), \min(\text{sg}(1 - \mu_B(x)), \overline{\text{sg}}(\nu_A(x))) \rangle x \in E\}$
→98	$\{\langle x, \max(\overline{\text{sg}}(1 - \mu_B(x)), \nu_A(x)), (1 - \max(\mu_B(x), \nu_A(x))) \rangle x \in E\}$
→99	$\{\langle x, \max(\overline{\text{sg}}(1 - \mu_B(x)), \overline{\text{sg}}(1 - \nu_A(x))), \min(1 - \mu_B(x), \overline{\text{sg}}(\nu_A(x))) \rangle x \in E\}$
→100	$\{\langle x, \max(\min(\nu_A(x), \text{sg}(\mu_A(x))), \mu_B(x)), \min(\min(\mu_A(x), \text{sg}(\nu_A(x))), \nu_B(x)) \rangle x \in E\}$
→101	$\{\langle x, \max(\min(\nu_A(x), \text{sg}(\mu_A(x))), \min(\mu_B(x), \text{sg}(\nu_B(x))), \min(\min(\mu_A(x), \text{sg}(\nu_A(x))), \min(\nu_B(x), \text{sg}(\mu_B(x)))) \rangle x \in E\}$
→102	$\{\langle x, \max(\nu_A(x), \min(\mu_B(x), \text{sg}(\nu_B(x))), \min(\mu_A(x), \min(\nu_B(x), \text{sg}(\mu_B(x)))) \rangle x \in E\}$
→103	$\{\langle x, \max(\min(1 - \mu_A(x), \text{sg}(\mu_A(x))), 1 - \nu_B(x)), \min(\mu_A(x), \text{sg}(1 - \mu_A(x)), \nu_B(x)) \rangle x \in E\}$
→104	$\{\langle x, \max(\min(1 - \mu_A(x), \text{sg}(\mu_A(x))), \min(1 - \nu_B(x), \text{sg}(\nu_B(x))), \min(\min(\mu_A(x), \text{sg}(1 - \mu_A(x))), \min(\nu_B(x), \text{sg}(1 - \nu_B(x)))) \rangle x \in E\}$
→105	$\{\langle x, \max(1 - \mu_A(x), \min(1 - \nu_B(x), \text{sg}(\nu_B(x))), \min(\mu_A(x), \min(\nu_B(x), \text{sg}(1 - \nu_B(x)))) \rangle x \in E\}$
→106	$\{\langle x, \max(\min(\nu_A(x), \text{sg}(1 - \nu_A(x))), \mu_B(x)), \min(\min(1 - \nu_A(x), \text{sg}(\nu_A(x))), 1 - \mu_B(x)) \rangle x \in E\}$

\rightarrow_{107}	$\{\langle x, \max(\min(\nu_A(x), \text{sg}(1 - \nu_A(x))), \min(\mu_B(x), \text{sg}(1 - \mu_B(x))), \min(\min(1 - \nu_A(x), \text{sg}(\nu_A(x))), \min(1 - \mu_B(x), \text{sg}(\mu_B(x))))\rangle x \in E\}$
\rightarrow_{108}	$\{\langle x, \max(\nu_A(x), \min(\mu_B(x), \text{sg}(1 - \mu_B(x))), \min(1 - \nu_A(x), \min(1 - \mu_B(x), \text{sg}(\mu_B(x))))\rangle x \in E\}$
\rightarrow_{109}	$\{\langle x, \nu_A(x) + \min(\overline{\text{sg}}(1 - \mu_A(x)), \mu_B(x)), \mu_A(x) \cdot \nu_A(x) + \min(\overline{\text{sg}}(1 - \mu_A(x)), \nu_B(x))\rangle x \in E\}$
\rightarrow_{110}	$\{\langle x, \max(\nu_A(x), \mu_B(x)), \min(\mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \mu_A(x)), \nu_B(x))\rangle x \in E\}$
\rightarrow_{111}	$\{\langle x, \max(\nu_A(x), \mu_B(x) \cdot \nu_B(x) + \overline{\text{sg}}(1 - \mu_B(x))), \min(\mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \mu_A(x)), \nu_B(x) \cdot (\mu_B(x) \cdot \nu_B(x) + \overline{\text{sg}}(1 - \mu_B(x))) + \overline{\text{sg}}(1 - \nu_B(x)))\rangle x \in E\}$
\rightarrow_{112}	$\{\langle x, \nu_A(x) + \mu_B(x) - \nu_A(x) \cdot \mu_B(x), \mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \mu_A(x)) \cdot \nu_B(x)\rangle x \in E\}$
\rightarrow_{113}	$\{\langle x, \nu_A(x) + (\mu_B(x) \cdot \nu_B(x) - \nu_A(x) \cdot (\mu_B(x) \cdot \nu_B(x) + \overline{\text{sg}}(1 - \mu_B(x))), (\mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \mu_A(x))) \cdot (\nu_B(x) \cdot (\mu_B(x) \cdot \nu_B(x) + \overline{\text{sg}}(1 - \mu_B(x))) + \overline{\text{sg}}(1 - \nu_B(x)))\rangle x \in E\}$
\rightarrow_{114}	$\{\langle x, 1 - \mu_A(x) + \min(\overline{\text{sg}}(1 - \mu_A(x)), (1 - \nu_B(x))), (\mu_A(x) \cdot (1 - \mu_A(x)) + \min(\overline{\text{sg}}(1 - \mu_A(x)), \nu_B(x)))\rangle x \in E\}$
\rightarrow_{115}	$\{\langle x, 1 - \min(\mu_A(x), \nu_B(x)), \min((\mu_A(x) \cdot (1 - \mu_A(x)) + \overline{\text{sg}}(1 - \mu_A(x))), \nu_B(x))\rangle x \in E\}$
\rightarrow_{116}	$\{\langle x, \max(1 - \mu_A(x), (1 - \nu_B(x)) \cdot \nu_B(x) + \overline{\text{sg}}(\nu_B(x))), \min(\mu_A(x) \cdot (1 - \mu_A(x)) + \overline{\text{sg}}(1 - \mu_A(x)), \nu_B(x) \cdot ((1 - \nu_B(x)) \cdot \nu_B(x) + \overline{\text{sg}}(\nu_B(x))) + \overline{\text{sg}}(1 - \nu_B(x)))\rangle x \in E\}$
\rightarrow_{117}	$\{\langle x, 1 - \mu_A(x) - \nu_B(x) + \mu_A(x) \cdot \nu_B(x), (\mu_A(x) \cdot (1 - \mu_A(x)) + \overline{\text{sg}}(1 - \mu_A(x))) \cdot \nu_B(x)\rangle x \in E\}$
\rightarrow_{118}	$\{\langle x, (1 - \mu_A(x)) \cdot \text{sg}(\nu_B(x)) + \mu_A(x) \cdot \nu_B(x) \cdot (1 - \nu_B(x)), (\mu_A(x) - \mu_A(x)^2 + \overline{\text{sg}}(1 - \mu_A(x))) \cdot ((1 - \nu_B(x)) \cdot \nu_B(x)^2 + \overline{\text{sg}}(1 - \nu_B(x))) + \overline{\text{sg}}(1 - \nu_B(x))\rangle x \in E\}$
\rightarrow_{119}	$\{\langle x, \nu_A(x) + \min(\overline{\text{sg}}(\nu_A(x)), \mu_B(x)), (1 - \nu_A(x)) \cdot \nu_A(x) + \min(\overline{\text{sg}}(\nu_A(x)), 1 - \mu_B(x))\rangle x \in E\}$
\rightarrow_{120}	$\{\langle x, \max(\nu_A(x), \mu_B(x)), \min((1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}(\nu_A(x)), 1 - \mu_B(x))\rangle x \in E\}$
\rightarrow_{121}	$\{\langle x, \max(\nu_A(x), \mu_B(x) \cdot (1 - \mu_B(x)) + \overline{\text{sg}}(1 - \mu_B(x))), \min((1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}(\nu_A(x)), (1 - \mu_B(x)) \cdot (\mu_B(x) \cdot (1 - \mu_B(x)) + \overline{\text{sg}}(1 - \mu_B(x))) + \overline{\text{sg}}(\mu_B(x)))\rangle x \in E\}$

→122	$\{ \langle x, \nu_A(x) + \mu_B(x) - \nu_A(x) \cdot \mu_B(x), \rangle$ $\langle (1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}(\nu_A(x)) \cdot (1 - \mu_B(x)) \rangle x \in E \}$
→123	$\{ \langle x, \nu_A(x) + \mu_B(x) \cdot (1 - \mu_B(x)) - \nu_A(x)$ $\cdot (\mu_B(x) \cdot (1 - \mu_B(x)) + \overline{\text{sg}}(1 - \mu_B(x))),$ $\langle (1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}(\nu_A(x)) \cdot ((1 - \mu_B(x)) \cdot (\mu_B(x) \cdot (1 - \mu_B(x)))$ $+ \overline{\text{sg}}(1 - \mu_B(x))) + \overline{\text{sg}}(\mu_B(x)) \rangle x \in E \}$
→124	$\{ \langle x, \mu_B(x) + \min(\overline{\text{sg}}(1 - \nu_B(x)), \nu_A(x)),$ $\nu_B(x) \cdot \mu_B(x) + \min(\overline{\text{sg}}(1 - \nu_B(x)), \mu_A(x)) \rangle x \in E \}$
→125	$\{ \langle x, \max(\mu_B(x), \nu_A(x)),$ $\min(\nu_B(x) \cdot \mu_B(x) + \overline{\text{sg}}(1 - \nu_B(x)), \mu_A(x)) \rangle x \in E \}$
→126	$\{ \langle x, \max(\mu_B(x), \nu_A(x)) \cdot \mu_A(x) + \overline{\text{sg}}(1 - \nu_A(x)),$ $\min(\nu_B(x) \cdot \mu_B(x) + \overline{\text{sg}}(1 - \nu_B(x)), \mu_A(x) \cdot$ $(\nu_A(x) \cdot \mu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(1 - \mu_A(x)) \rangle x \in E \}$
→127	$\{ \langle x, \mu_B(x) + \nu_A(x) - \mu_B(x) \cdot \nu_A(x),$ $(\nu_B(x) \cdot \mu_B(x) + \overline{\text{sg}}(1 - \nu_B(x))) \cdot \mu_A(x) \rangle x \in E \}$
→128	$\{ \langle x, \mu_B(x) + \nu_A(x) \cdot \mu_A(x) - \mu_B(x) \cdot$ $(\nu_A(x) \cdot \mu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))),$ $(\nu_B(x) \cdot \mu_B(x) + \overline{\text{sg}}(1 - \nu_B(x))) \cdot (\mu_A(x) \cdot (\nu_A(x) \cdot \mu_A(x))$ $+ \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(1 - \mu_A(x)) \rangle x \in E \}$
→129	$\{ \langle x, 1 - \nu_B(x) + \min(\overline{\text{sg}}(1 - \nu_B(x)), 1 - \mu_A(x)),$ $\nu_B(x) \cdot (1 - \nu_B(x)) + \min(\overline{\text{sg}}(1 - \nu_B(x)), \mu_A(x)) \rangle x \in E \}$
→130	$\{ \langle x, 1 - \min(\nu_B(x), \mu_A(x)),$ $\min(\nu_B(x) \cdot (1 - \nu_B(x)) + \overline{\text{sg}}(1 - \nu_B(x)), \mu_A(x)) \rangle x \in E \}$
→131	$\{ \langle x, \max(1 - \nu_B(x), (1 - \mu_A(x)) \cdot \mu_A(x) + \overline{\text{sg}}(\mu_A(x))),$ $\min(\nu_B(x) \cdot (1 - \nu_B(x)) + \overline{\text{sg}}(1 - \nu_B(x)), \mu_A(x) \cdot ((1 - \mu_A(x))$ $\cdot \mu_A(x) + \overline{\text{sg}}(\mu_A(x))) + \overline{\text{sg}}(1 - \mu_A(x)) \rangle x \in E \}$
→132	$\{ \langle x, 1 - \mu_A(x) \cdot \nu_B(x),$ $(\nu_B(x) \cdot (1 - \nu_B(x)) + \overline{\text{sg}}(1 - \nu_B(x))) \cdot \mu_A(x) \rangle x \in E \}$
→133	$\{ \langle x, ((1 - \nu_B(x) + ((1 - \mu_A(x)) \cdot \mu_A(x)))$ $- ((1 - \nu_B(x)) \cdot ((1 - \mu_A(x)) \cdot \mu_A(x)) + \overline{\text{sg}}(\mu_A(x))))),$ $(\nu_B(x) \cdot (1 - \nu_B(x)) + \overline{\text{sg}}(1 - \nu_B(x))) \cdot (\mu_A(x) \cdot ((1 - \mu_A(x)) \cdot \mu_A(x))$ $+ \overline{\text{sg}}(\mu_A(x))) + \overline{\text{sg}}(1 - \mu_A(x)) \rangle x \in E \}$
→134	$\{ \langle x, \mu_B(x) + \min(\overline{\text{sg}}(\mu_B(x)), \nu_A(x)),$ $(1 - \mu_B(x)) \cdot \mu_B(x) + \min(\overline{\text{sg}}(\mu_B(x)), 1 - \nu_A(x)) \rangle x \in E \}$
→135	$\{ \langle x, \max(\mu_B(x), \nu_A(x)),$ $\min((1 - \mu_B(x)) \cdot \mu_B(x) + \overline{\text{sg}}(\mu_B(x)), 1 - \nu_A(x)) \rangle x \in E \}$

\rightarrow_{136}	$\{\langle x, \max(\mu_B(x), \nu_A(x) \cdot (1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x))), \min((1 - \mu_B(x)) \cdot \mu_B(x) + \overline{\text{sg}}(\mu_B(x)), (1 - \nu_A(x)) \cdot (\nu_A(x) \cdot (1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(\nu_A(x))) \rangle x \in E\}$
\rightarrow_{137}	$\{\langle x, \mu_B(x) + \nu_A(x) - \mu_B(x) \cdot \nu_A(x), ((1 - \mu_B(x)) \cdot \mu_B(x) + \overline{\text{sg}}(\mu_B(x))) \cdot (1 - \nu_A(x)) \rangle x \in E\}$
\rightarrow_{138}	$\{\langle x, \mu_B(x) + \nu_A(x) \cdot (1 - \nu_A(x)) - \mu_B(x) \cdot (\nu_A(x) \cdot (1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x))), ((1 - \mu_B(x)) \cdot \mu_B(x) + \overline{\text{sg}}(\mu_B(x))) \cdot ((1 - \nu_A(x)) \cdot (\nu_A(x) \cdot (1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x)) + \overline{\text{sg}}(\nu_A(x)))) \rangle x \in E\}$

Here, following [5], the full list of implications is given, because in the previous research of the author there are some misprints in the list of implications, that are corrected here .

Table 3: List of intuitionistic fuzzy negations

\neg_1	$\{\langle x, \nu_A(x), \mu_A(x) \rangle x \in E\}$
\neg_2	$\{\langle x, \overline{\text{sg}}(\mu_A(x)), \text{sg}(\mu_A(x)) \rangle x \in E\}$
\neg_3	$\{\langle x, \nu_A(x), \mu_A(x) \cdot \nu_A(x) + \mu_A(x)^2 \rangle x \in E\}$
\neg_4	$\{\langle x, \nu_A(x), 1 - \nu_A(x) \rangle x \in E\}$
\neg_5	$\{\langle x, \overline{\text{sg}}(1 - \nu_A(x)), \text{sg}(1 - \nu_A(x)) \rangle x \in E\}$
\neg_6	$\{\langle x, \overline{\text{sg}}(1 - \nu_A(x)), \text{sg}(\mu_A(x)) \rangle x \in E\}$
\neg_7	$\{\langle x, \overline{\text{sg}}(1 - \nu_A(x)), \mu_A(x) \rangle x \in E\}$
\neg_8	$\{\langle x, 1 - \mu_A(x), \mu_A(x) \rangle x \in E\}$
\neg_9	$\{\langle x, \overline{\text{sg}}(\mu_A(x)), \mu_A(x) \rangle x \in E\}$
\neg_{10}	$\{\langle x, \overline{\text{sg}}(1 - \nu_A(x)), 1 - \nu_A(x) \rangle x \in E\}$
\neg_{11}	$\{\langle x, \text{sg}(\nu_A(x)), \overline{\text{sg}}(\nu_A(x)) \rangle x \in E\}$
\neg_{12}	$\{\langle x, \nu_A(x) \cdot (\nu_A(x) + \mu_A(x)), \mu_A(x) \cdot (\nu_A(x)^2 + \mu_A(x) + \nu_A(x) \cdot \mu_A(x)) \rangle x \in E\}$
\neg_{13}	$\{\langle x, \text{sg}(1 - \mu_A(x)), \overline{\text{sg}}(1 - \mu_A(x)) \rangle x \in E\}$
\neg_{14}	$\{\langle x, \text{sg}(\nu_A(x)), \overline{\text{sg}}(1 - \mu_A(x)) \rangle x \in E\}$
\neg_{15}	$\{\langle x, \overline{\text{sg}}(1 - \nu_A(x)), \overline{\text{sg}}(1 - \mu_A(x)) \rangle x \in E\}$
\neg_{16}	$\{\langle x, \overline{\text{sg}}(\mu_A(x)), \overline{\text{sg}}(1 - \mu_A(x)) \rangle x \in E\}$
\neg_{17}	$\{\langle x, \overline{\text{sg}}(1 - \nu_A(x)), \overline{\text{sg}}(\nu_A(x)) \rangle x \in E\}$
\neg_{18}	$\{\langle x, \min(\nu_A(x), \text{sg}(\mu_A(x))), \min(\mu_A(x), \text{sg}(\nu_A(x))) \rangle x \in E\}$
\neg_{19}	$\{\langle x, \min(\nu_A(x), \text{sg}(\mu_A(x))), 0 \rangle x \in E\}$
\neg_{20}	$\{\langle x, \nu_A(x), 0 \rangle x \in E\}$

\neg_{21}	$\{\langle x, \min(1 - \mu_A(x), \text{sg}(\mu_A(x))), \min(\mu_A(x), \text{sg}(1 - \mu_A(x))) \rangle x \in E\}$
\neg_{22}	$\{\langle x, \min((1 - \mu_A(x)), \text{sg}(\mu_A(x))), 0) \rangle x \in E\}$
\neg_{23}	$\{\langle x, 1 - \mu_A(x), 0 \rangle x \in E\}$
\neg_{24}	$\{\langle x, \min(\nu_A(x), \text{sg}(1 - \nu_A(x))), \min(1 - \nu_A(x), \text{sg}(\nu_A(x))) \rangle x \in E\}$
\neg_{25}	$\{\langle x, \min(\nu_A(x), \text{sg}(1 - \nu_A(x))), 0 \rangle x \in E\}$
\neg_{26}	$\{\langle x, \nu_A(x), \mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \mu_A(x)) \rangle x \in E\}$
\neg_{27}	$\{\langle x, 1 - \mu_A(x), \mu_A(x) \cdot (1 - \mu_A(x)) + \overline{\text{sg}}(1 - \mu_A(x)) \rangle x \in E\}$
\neg_{28}	$\{\langle x, \nu_A(x), (1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}(\nu_A(x)) \rangle x \in E\}$
\neg_{29}	$\{\langle x, \nu_A(x) \cdot \mu_A(x) + \overline{\text{sg}}(1 - \nu_A(x)), \mu_A(x) \cdot (\nu_A(x) \cdot \mu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(1 - \mu_A(x)) \rangle x \in E\}$
\neg_{30}	$\{\langle x, \nu_A(x) \cdot \mu_A(x), \mu_A(x) \cdot (\nu_A(x) \cdot \mu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(1 - \mu_A(x)) \rangle x \in E\}$
\neg_{31}	$\{\langle x, (1 - \mu_A(x)) \cdot \mu_A(x) + \overline{\text{sg}}(\mu_A(x)), \mu_A(x) \cdot ((1 - \mu_A(x)) \cdot \mu_A(x) + \overline{\text{sg}}(\mu_A(x))) + \overline{\text{sg}}(1 - \mu_A(x)) \rangle x \in E\}$
\neg_{32}	$\{\langle x, (1 - \mu_A(x)) \cdot \mu_A(x), \mu_A(x) \cdot ((1 - \mu_A(x)) \cdot \mu_A(x) + \overline{\text{sg}}(\mu_A(x))) + \overline{\text{sg}}(1 - \mu_A(x)) \rangle x \in E\}$
\neg_{33}	$\{\langle x, \nu_A(x) \cdot (1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x)), (1 - \nu_A(x)) \cdot (\nu_A(x) \cdot (1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(\nu_A(x)) \rangle x \in E\}$
\neg_{34}	$\{\langle x, \nu_A(x) \cdot (1 - \nu_A(x)), (1 - \nu_A(x)) \cdot (\nu_A(x) \cdot (1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(\nu_A(x)) \rangle x \in E\}$

Table 4: Correspondence between intuitionistic fuzzy negations and implications

\neg_1	$\rightarrow_1, \rightarrow_4, \rightarrow_5, \rightarrow_6, \rightarrow_7, \rightarrow_{10}, \rightarrow_{13}, \rightarrow_{61}, \rightarrow_{63}, \rightarrow_{64}, \rightarrow_{66}, \rightarrow_{67}, \rightarrow_{68}, \rightarrow_{69}, \rightarrow_{70}, \rightarrow_{71}, \rightarrow_{72}, \rightarrow_{73}, \rightarrow_{78}, \rightarrow_{80}, \rightarrow_{124}, \rightarrow_{125}, \rightarrow_{127}$
\neg_2	$\rightarrow_2, \rightarrow_3, \rightarrow_8, \rightarrow_{11}, \rightarrow_{16}, \rightarrow_{20}, \rightarrow_{31}, \rightarrow_{32}, \rightarrow_{37}, \rightarrow_{40}, \rightarrow_{41}, \rightarrow_{42}$
\neg_3	$\rightarrow_9, \rightarrow_{17}, \rightarrow_{21}$
\neg_4	$\rightarrow_{12}, \rightarrow_{18}, \rightarrow_{22}, \rightarrow_{46}, \rightarrow_{49}, \rightarrow_{50}, \rightarrow_{51}, \rightarrow_{53}, \rightarrow_{54}, \rightarrow_{91}, \rightarrow_{93}, \rightarrow_{94}, \rightarrow_{95}, \rightarrow_{96}, \rightarrow_{98}, \rightarrow_{134}, \rightarrow_{135}, \rightarrow_{137}$
\neg_5	$\rightarrow_{14}, \rightarrow_{15}, \rightarrow_{19}, \rightarrow_{23}, \rightarrow_{47}, \rightarrow_{48}, \rightarrow_{52}, \rightarrow_{55}, \rightarrow_{56}, \rightarrow_{57}$
\neg_6	$\rightarrow_{24}, \rightarrow_{26}, \rightarrow_{27}, \rightarrow_{65}$
\neg_7	$\rightarrow_{25}, \rightarrow_{28}, \rightarrow_{29}, \rightarrow_{62}$
\neg_8	$\rightarrow_{30}, \rightarrow_{33}, \rightarrow_{34}, \rightarrow_{35}, \rightarrow_{36}, \rightarrow_{38}, \rightarrow_{39}, \rightarrow_{76}, \rightarrow_{82}, \rightarrow_{84}, \rightarrow_{85}, \rightarrow_{86}, \rightarrow_{87}, \rightarrow_{89}, \rightarrow_{129}, \rightarrow_{130}, \rightarrow_{132}$

\neg_9	$\rightarrow 43, \rightarrow 44, \rightarrow 45, \rightarrow 83$
\neg_{10}	$\rightarrow 58, \rightarrow 59, \rightarrow 60, \rightarrow 92$
\neg_{11}	$\rightarrow 74, \rightarrow 97$
\neg_{12}	$\rightarrow 75$
\neg_{13}	$\rightarrow 77, \rightarrow 88$
\neg_{14}	$\rightarrow 79$
\neg_{15}	$\rightarrow 81$
\neg_{16}	$\rightarrow 90$
\neg_{17}	$\rightarrow 99$
\neg_{18}	$\rightarrow 100$
\neg_{19}	$\rightarrow 101$
\neg_{20}	$\rightarrow 102, \rightarrow 108$
\neg_{21}	$\rightarrow 103$
\neg_{22}	$\rightarrow 104$
\neg_{23}	$\rightarrow 105$
\neg_{24}	$\rightarrow 106$
\neg_{25}	$\rightarrow 107$
\neg_{26}	$\rightarrow 109, \rightarrow 110, \rightarrow 111, \rightarrow 112, \rightarrow 113$
\neg_{27}	$\rightarrow 114, \rightarrow 115, \rightarrow 116, \rightarrow 117, \rightarrow 118$
\neg_{28}	$\rightarrow 119, \rightarrow 120, \rightarrow 121, \rightarrow 122, \rightarrow 123$
\neg_{29}	$\rightarrow 126$
\neg_{30}	$\rightarrow 128$
\neg_{31}	$\rightarrow 131$
\neg_{32}	$\rightarrow 133$
\neg_{33}	$\rightarrow 136$
\neg_{34}	$\rightarrow 138$

Now, we introduce the definitions of the new “subtraction” operations. As a basis of the new instances of this operation, we use the formula from classical set theory

$$A - B = \neg(A \rightarrow B),$$

where A and B are two IFSs. Therefore, 134 new “subtraction” operations can originate. This process is difficult, having in mind the very complex forms of some implications and negations from Tables 2 and 3. By this reason, here we introduce the definition of the first 14 new instances of the “subtraction” operation (see Table 5) and the rest definitions will be given in future.

Table 5: List of the intuitionistic fuzzy subtractions

-1	$\{\langle x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \min(\mu_A(x), \mu_B(x))) \rangle x \in E\}$
-2	$\{\langle x, \text{sg}(\mu_A(x) - \mu_B(x)), \overline{\text{sg}}(\mu_A(x) - \mu_B(x)) \rangle x \in E\}$
-3	$\{\langle x, \overline{\text{sg}}(\mu_A(x) - \mu_B(x)), \overline{\text{sg}}(\mu_A(x) - \mu_B(x)) \rangle x \in E\}$
-4	$\{\langle x, \max(\nu_A(x), \mu_B(x)), \min(\mu_A(x), \nu_B(x)) \rangle x \in E\}$
-5	$\{\langle x, \max(0, \mu_A(x) + \nu_B(x) - 1), \min(1, \nu_A(x) + \mu_B(x)) \rangle x \in E\}$
-6	$\{\langle x, \mu_A(x)\nu_B(x), \nu_A(x) + \mu_A(x)\mu_B(x) \rangle x \in E\}$
-7	$\{\langle x, \max(\min(\mu_A(x), \nu_B(x)), \min(\mu_A(x), \nu_A(x)), \min(\mu_B(x), \nu_B(x))), \min(\max(\nu_A(x), \mu_B(x)), \max(\mu_A(x), \nu_A(x)), \max(\mu_B(x), \nu_B(x))) \rangle x \in E\}$
-8	$\{\langle x, ((1 - \text{sg}(\min(\nu_A(x), \mu_B(x)))) \cdot \text{sg}(\mu_A(x) - \mu_B(x)), \overline{\text{sg}}(\mu_A(x) - \mu_B(x)) + \text{sg}(\min(\nu_A(x), \mu_B(x)))) \cdot \text{sg}(\mu_A(x) - \mu_B(x)) \rangle x \in E\}$
-9	$\{\langle x, \mu_A(x) \cdot \nu_A(x) + \mu_A(x)^2 \nu_B(x), \mu_A(x) \nu_A(x)^2 + \mu_A(x)^2 \nu_A(x) \nu_B(x) + \mu_A(x)^3 \nu_A(x) \mu_B(x) + \mu_A(x)^4 \mu_B(x) \nu_B(x) + \nu_A(x)^2 + 2 + \mu_A(x)^2 \nu_A(x) \mu_B(x) + \mu_A(x)^4 \mu_B(x)^2 \rangle x \in E\}$
-10	$\{\langle x, \nu_B(x) \cdot \overline{\text{sg}}(1 - \mu_A(x)) + \mu_A(x) \cdot \text{sg}(1 - \mu_A(x)) \cdot \text{sg}(1 - \mu_B(x)), \mu_B(x) \cdot \overline{\text{sg}}(1 - \mu_A(x)) + \text{sg}(1 - \mu_A(x)) \cdot (\overline{\text{sg}}(1 - \mu_B(x)) + \nu_A(x) \cdot \text{sg}(1 - \mu_B(x))) \rangle x \in E\}$
-11	$\{\langle x, \overline{\text{sg}}(\mu_A(x) - \mu_B(x)), \overline{\text{sg}}(\mu_A(x) - \mu_B(x)) \rangle x \in E\}$
-12	$\{\langle x, 1 - \max(\nu_A(x), \mu_B(x)), \max(\nu_A(x), \mu_B(x)) \rangle x \in E\}$
-13	$\{\langle x, \mu_A(x) \cdot \nu_B(x), \nu_A(x) + \mu_B(x) - \nu_A(x) \cdot \mu_B(x) \rangle x \in E\}$
-14	$\{\langle x, \overline{\text{sg}}(1 - \nu_B(x)) \cdot \text{sg}(\nu_B(x) - \nu_A(x)), \text{sg}(1 - \nu_B(x)) \cdot \text{sg}(\nu_B(x) - \nu_A(x)) \rangle x \in E\}$
-15	$\{\langle x, \overline{\text{sg}}((1 - \min(\mu_A(x), \nu_B(x))) \cdot \text{sg}(\overline{\text{sg}}(\mu_A(x) - \mu_B(x)) + \overline{\text{sg}}(\nu_B(x) - \nu_A(x))) - \max(\mu_A(x), \nu_B(x)) \cdot \overline{\text{sg}}(\mu_A(x) - \mu_B(x)) \cdot \overline{\text{sg}}(\nu_B(x) - \nu_A(x))), \text{sg}((1 - \min(\mu_A(x), \nu_B(x))) \cdot \text{sg}(\overline{\text{sg}}(\mu_A(x) - \mu_B(x)) + \overline{\text{sg}}(\nu_B(x) - \nu_A(x))) - \max(\mu_A(x), \nu_B(x)) \cdot \overline{\text{sg}}(\mu_A(x) - \mu_B(x)) \cdot \overline{\text{sg}}(\nu_B(x) - \nu_A(x))) \rangle x \in E\}$
-16	$\{\langle x, \overline{\text{sg}}(\max(\overline{\text{sg}}(\mu_A(x)), \mu_B(x))), \text{sg}(\max(\overline{\text{sg}}(\mu_A(x)), \mu_B(x))) \rangle x \in E\}$
-17	$\{\langle x, \min(\mu_A(x) \cdot \nu_A(x) + \mu_A(x)^2, \nu_B(x)), \max(\nu_A(x), \mu_B(x)) \cdot \min(\mu_A(x) \cdot \nu_A(x) + \mu_A(x)^2, \nu_B(x)) + \max(\nu_A(x), \mu_B(x))^2 \rangle x \in E\}$
-18	$\{\langle x, \min(1 - \nu_A(x), \nu_B(x)), \max(\nu_A(x), 1 - \nu_B(x)) \rangle x \in E\}$

-19	$\{\langle x, \overline{\text{sg}}(1 - \min(\text{sg}(1 - \nu_A(x)), \nu_B(x))), \text{sg}(1 - \min(\text{sg}(1 - \nu_A(x)), \nu_B(x))) \rangle x \in E\}$
-20	$\{\langle x, \overline{\text{sg}}(\max(\overline{\text{sg}}(\mu_A(x)), \text{sg}(\mu_B(x))), \text{sg}(\max(\overline{\text{sg}}(\mu_A(x)), \text{sg}(\mu_B(x)))) \rangle x \in E\}$
-21	$\{\langle x, \min(\mu_A(x) \cdot (\mu_A(x) + \nu_A(x)), \nu_B(x) \cdot (\mu_B(x)^2 + \nu_B(x) + \mu_B(x) \cdot \nu_B(x))), \max(\nu_A(x), \mu_B(x) \cdot (\mu_B(x) + \nu_B(x))) \cdot \min(\mu_A(x) \cdot (\mu_A(x) + \nu_A(x)), \nu_B(x) \cdot (\mu_B(x)^2 + \nu_B(x) + \mu_B(x) \cdot \nu_B(x))) + \max(\nu_A(x), \mu_B(x) \cdot (\mu_B(x) + \nu_B(x)))^2 \rangle x \in E\}$
-22	$\{\langle x, \min(1 - \nu_A(x), \nu_B(x)), \max(\nu_A(x), 1 - \nu_B(x)) \rangle x \in E\}$
-23	$\{\langle x, \overline{\text{sg}}(1 - \min(\text{sg}(1 - \nu_A(x)), \overline{\text{sg}}(1 - \nu_B(x))), \text{sg}(1 - \min(\text{sg}(1 - \nu_A(x)), \overline{\text{sg}}(1 - \nu_B(x)))) \rangle x \in E\}$
-24	$\{\langle x, \overline{\text{sg}}(1 - \text{sg}(\mu_A(x) - \mu_B(x)) \cdot \text{sg}(\nu_B(x) - \nu_A(x))), \text{sg}(\overline{\text{sg}}(\mu_A(x) - \mu_B(x)) \cdot \overline{\text{sg}}(\nu_B(x) - \nu_A(x))) \rangle x \in E\}$
-25	$\{\langle x, \overline{\text{sg}}(1 - \min(\mu_A(x), \nu_B(x))), \max(\nu_A(x), \overline{\text{sg}}(\mu_A(x)) \cdot \overline{\text{sg}}(1 - \nu_A(x)), \mu_B(x) \cdot \overline{\text{sg}}(\nu_B(x)) \cdot \overline{\text{sg}}(1 - \mu_B(x))) \rangle x \in E\}$
-26	$\{\langle x, \overline{\text{sg}}(1 - \min(\text{sg}(\mu_A(x)), \nu_B(x))), \text{sg}(\max(\overline{\text{sg}}(1 - \nu_A(x)), \mu_B(x))) \rangle x \in E\}$
-27	$\{\langle x, \overline{\text{sg}}(1 - \min(\text{sg}(\mu_A(x)), \overline{\text{sg}}(1 - \nu_B(x))), \text{sg}(\max(\overline{\text{sg}}(1 - \nu_A(x)), \text{sg}(\mu_B(x)))) \rangle x \in E\}$
-28	$\{\langle x, \overline{\text{sg}}(1 - \min(\mu_A(x), \nu_B(x))), \max(\overline{\text{sg}}(1 - \nu_A(x)), \mu_B(x)) \rangle x \in E\}$
-29	$\{\langle x, \overline{\text{sg}}(1 - \min(\mu_A(x), \overline{\text{sg}}(1 - \nu_B(x))), \max(\overline{\text{sg}}(1 - \nu_A(x)), \overline{\text{sg}}(1 - \mu_B(x))) \rangle x \in E\}$
-30	$\{\langle x, \min(\mu_A(x), \max(1 - \mu_A(x), \nu_B(x)), 1 - \max(1 - \mu_A(x), \min(\mu_A(x), 1 - \nu_B(x)))) \rangle x \in E\}$
-31	$\{\langle x, \text{sg}(\mu_A(x) + \nu_B(x) - 1), \overline{\text{sg}}(\mu_A(x) + \nu_B(x) - 1) \rangle x \in E\}$
-32	$\{\langle x, \overline{\text{sg}}(1 - \nu_B(x) \cdot \text{sg}(\mu_A(x) + \nu_B(x) - 1)), \text{sg}(1 - \nu_B(x) \cdot \text{sg}(\mu_A(x) + \nu_B(x) - 1)) \rangle x \in E\}$
-33	$\{\langle x, \min(\mu_A(x), \nu_B(x)), 1 - \min(\mu_A(x), \nu_B(x)) \rangle x \in E\}$
-34	$\{\langle x, 1 - \min(1, 2 - \mu_A(x) - \nu_B(x)), \min(1, 2 - \mu_A(x) - \nu_B(x)) \rangle x \in E\}$
-35	$\{\langle x, \mu_A(x) \cdot \nu_B(x), 1 - \mu_A(x) \cdot \nu_B(x) \rangle x \in E\}$
-36	$\{\langle x, 1 - \min(1 - \min(\mu_A(x), \nu_B(x)), \max(\mu_A(x), (1 - \mu_A(x)), \max(1 - \nu_B(x), \nu_B(x))), \min(1 - \min(\mu_A(x), \nu_B(x)), \max(\mu_A(x), (1 - \mu_A(x)), \max(1 - \nu_B(x), \nu_B(x)))) \rangle x \in E\}$

-37	$\{ \langle x, \max(\mu_A(x), \nu_B(x)).\text{sg}(\mu_A(x) + \nu_B(x) - 1), 1 - \max(\mu_A(x), \nu_B(x)).\text{sg}(\mu_A(x) + \nu_B(x) - 1) \rangle x \in E \}$
-38	$\{ \langle x, \mu_A(x) + (\mu_A(x))^2.(1 - \nu_B(x)), 1 - \mu_A(x) + (\mu_A(x))^2.(1 - \nu_B(x)) \rangle x \in E \}$
-39	$\{ \langle x, 1 - (1 - \nu_B(x)).\overline{\text{sg}}(1 - \mu_A(x)) + \text{sg}(1 - \mu_A(x)).(\overline{\text{sg}}(\nu_B(x)) + (1 - \mu_A(x)).\text{sg}(\nu_B(x))), (1 - \nu_B(x)).\overline{\text{sg}}(1 - \mu_A(x)) + \text{sg}(1 - \mu_A(x)).(\overline{\text{sg}}(\nu_B(x)) + (1 - \mu_A(x)).\text{sg}(\nu_B(x))) \rangle x \in E \}$
-40	$\{ \langle x, \overline{\text{sg}}(1 - \text{sg}(\mu_A(x) + \nu_B(x) - 1)), \text{sg}(1 - \text{sg}(\mu_A(x) + \nu_B(x) - 1)) \rangle x \in E \}$
-41	$\{ \langle x, \overline{\text{sg}}(\max(\overline{\text{sg}}(\mu_A(x)), (1 - \nu_B(x))))), \text{sg}(\max(\overline{\text{sg}}(\mu_A(x)), (1 - \nu_B(x)))) \rangle x \in E \}$
-42	$\{ \langle x, \overline{\text{sg}}(\max(\overline{\text{sg}}(\mu_A(x)), \text{sg}(1 - \nu_B(x))))), \text{sg}(\max(\overline{\text{sg}}(\mu_A(x)), \text{sg}(1 - \nu_B(x)))) \rangle x \in E \}$
-43	$\{ \langle x, \overline{\text{sg}}(\max(\overline{\text{sg}}(\mu_A(x)), 1 - \nu_B(x))), \max(\overline{\text{sg}}(\mu_A(x)), 1 - \nu_B(x)) \rangle x \in E \}$
-44	$\{ \langle x, \overline{\text{sg}}(\max(\overline{\text{sg}}(\mu_A(x)), 1 - \nu_B(x))), \max(\overline{\text{sg}}(\mu_A(x)), 1 - \nu_B(x)) \rangle x \in E \}$
-45	$\{ \langle x, \overline{\text{sg}}(\max(\overline{\text{sg}}(\mu_A(x)), \overline{\text{sg}}(\nu_B(x))))), \max(\overline{\text{sg}}(\mu_A(x)), \overline{\text{sg}}(\nu_B(x))) \rangle x \in E \}$
-46	$\{ \langle x, 1 - \max(\nu_A(x), \mu_B(x)), \max(\nu_A(x), \mu_B(x)) \rangle x \in E \}$
-47	$\{ \langle x, \overline{\text{sg}}(1 - (1 - \mu_B(x)).\text{sg}(1 - \nu_A(x) - \mu_B(x))), \text{sg}(1 - (1 - \mu_B(x)).\text{sg}(1 - \nu_A(x) - \mu_B(x))) \rangle x \in E \}$
-48	$\{ \langle x, \overline{\text{sg}}(1 - (1 - \mu_B(x)).\text{sg}(1 - \nu_A(x) - \mu_B(x))), \text{sg}(1 - (1 - \mu_B(x)).\text{sg}(1 - \nu_A(x) - \mu_B(x))) \rangle x \in E \}$
-49	$\{ \langle x, \max(0, 1 - \nu_A(x) - \mu_B(x)), \min(1, \nu_A(x) + \mu_B(x)) \rangle x \in E \}$
-50	$\{ \langle x, 1 - \nu_A(x) - \mu_B(x) + \nu_A(x).\mu_B(x), \nu_A(x) - \mu_B(x) + \nu_A(x).\mu_B(x) \rangle x \in E \}$
-51	$\{ \langle x, \max(1 - \max(\nu_A(x), \mu_B(x)), \min(1 - \nu_A(x), \nu_A(x)), \min(\mu_B(x), 1 - \mu_B(x))), 1 - \max(1 - \max(\nu_A(x), \mu_B(x)), \min(1 - \nu_A(x), \nu_A(x)), \min(\mu_B(x), 1 - \mu_B(x))) \rangle x \in E \}$
-52	$\{ \langle x, \overline{\text{sg}}(\min(\nu_A(x), \mu_B(x)).\text{sg}(1 - \nu_A(x) - \mu_B(x))), \text{sg}(\min(\nu_A(x), \mu_B(x)).\text{sg}(1 - \nu_A(x) - \mu_B(x))) \rangle x \in E \}$
-53	$\{ \langle x, (1 - \nu_A(x)).\nu_A(x) + (1 - \nu_A(x))^2.(1 - \mu_B(x)), 1 - (1 - \nu_A(x)).\nu_A(x) - (1 - \nu_A(x))^2.(1 - \mu_B(x)) \rangle x \in E \}$

-54	$\{\langle x, (1 - \mu_B(x)).\overline{\text{sg}}(\nu_A(x)) + (1 - \nu_A(x))$ $\cdot \text{sg}(\nu_A(x)).\text{sg}(1 - \mu_B(x)),$ $1 - (1 - \mu_B(x)).\overline{\text{sg}}(\nu_A(x)) + (1 - \nu_A(x))$ $\cdot \text{sg}(\nu_A(x)).\text{sg}(1 - \mu_B(x)) \mid x \in E\}$
-55	$\{\langle x, \overline{\text{sg}}(\overline{\text{sg}}(1 - \nu_A(x) - \mu_B(x))),$ $\text{sg}(\overline{\text{sg}}(1 - \nu_A(x) - \mu_B(x))) \mid x \in E\}$
-56	$\{\langle x, \overline{\text{sg}}(1 - \min(\text{sg}(1 - \nu_A(x)), 1 - \mu_B(x))),$ $\text{sg}(1 - \min(\text{sg}(1 - \nu_A(x)), 1 - \mu_B(x))) \mid x \in E\}$
-57	$\{\langle x, \overline{\text{sg}}(1 - \min(\text{sg}(1 - \nu_A(x)), \overline{\text{sg}}(\mu_B(x))),$ $\text{sg}(1 - \min(\text{sg}(1 - \nu_A(x)), \overline{\text{sg}}(\mu_B(x)))) \mid x \in E\}$
-58	$\{\langle x, \overline{\text{sg}}(\max(\nu_A(x), \mu_B(x))), \max(\nu_A(x), \mu_B(x)) \mid x \in E\}$
-59	$\{\langle x, \overline{\text{sg}}(\max(\nu_A(x), \mu_B(x))), \max(\nu_A(x), \mu_B(x)) \mid x \in E\}$
-60	$\{\langle x, \overline{\text{sg}}(1 - \min(1 - \nu_A(x), \overline{\text{sg}}(\mu_B(x))),$ $1 - \min(1 - \nu_A(x), \overline{\text{sg}}(\mu_B(x))) \mid x \in E\}$
-61	$\{\langle x, \max(\mu_B(x), \min(\nu_B(x), \nu_A(x))), \min(\nu_B(x), \mu_A(x)) \mid x \in E\}$
-62	$\{\langle x, \overline{\text{sg}}(1 - \mu_A(x)).\text{sg}(\nu_B(x) - \nu_A(x)),$ $\overline{\text{sg}}(\nu_B(x) - \nu_A(x)) \mid x \in E\}$
-63	$\{\langle x, \mu_A(x).\text{sg}(\nu_B(x) - \nu_A(x)),$ $1 - (1 - \nu_A(x)).\text{sg}(\nu_B(x) - \nu_A(x)) \mid x \in E\}$
-64	$\{\langle x, \nu_B(x).\mu_A(x), \mu_B(x) + \nu_B(x).\nu_A(x) \mid x \in E\}$
-65	$\{\langle x, \overline{\text{sg}}(1 - \max(\nu_B(x), \mu_A(x))$ $\cdot \text{sg}(\nu_B(x) - \nu_A(x)).\text{sg}(\mu_A(x) - \mu_B(x)),$ $\text{sg}(1 - (1 - \min(\mu_B(x), \nu_A(x))).\text{sg}(\nu_B(x) - \nu_A(x))), \mid x \in E\}$
-66	$\{\langle x, \nu_B(x).\mu_B(x) + \nu_B(x)^2.\mu_A(x),$ $\mu_B(x) + \nu_B(x)^2.\nu_A(x) \mid x \in E\}$
-67	$\{\langle x, \mu_A(x).\overline{\text{sg}}(1 - \nu_B(x)) + \nu_B(x).\text{sg}(1 - \nu_B(x)).\text{sg}(1 - \nu_A(x)),$ $\nu_A(x).\overline{\text{sg}}(1 - \nu_B(x)) + \text{sg}(1 - \nu_B(x)).(\overline{\text{sg}}(1 - \nu_A(x))$ $+ \mu_B(x).\text{sg}(1 - \nu_A(x))) \mid x \in E\}$
-68	$\{\langle x, \mu_A(x).\text{sg}(\nu_B(x) - \nu_A(x)).\text{sg}(\mu_A(x) - \mu_B(x)),$ $1 - (1 - \nu_A(x)).\text{sg}(\nu_B(x) - \nu_A(x)) \mid x \in E\}$
->69	$\{\langle x, \mu_A(x).\text{sg}(\mu_A(x) - \mu_B(x)),$ $1 - (1 - \nu_A(x)).\text{sg}(\nu_B(x) - \nu_A(x))$ $- \mu_A(x).\overline{\text{sg}}(\nu_B(x) - \nu_A(x)).\text{sg}(\mu_A(x) - \mu_B(x)) \mid x \in E\}$
->70	$\{\langle x, \min(\text{sg}(\nu_B(x)), \mu_A(x)), \max(\overline{\text{sg}}(\nu_B(x)), \nu_A(x)) \mid x \in E\}$
->71	$\{\langle x, \min(\nu_B(x).\mu_B(x) + \nu_B(x)^2, \mu_A(x)),$ $\max(\mu_B(x), \nu_A(x)) \mid x \in E\}$
->72	$\{\langle x, \min(1 - \mu_B(x), \mu_A(x)), \max(\mu_B(x), \nu_A(x)) \mid x \in E\}$
->73	$\{\langle x, \min(\text{sg}(1 - \mu_B(x)), \mu_A(x)),$ $\max(1 - \max(\text{sg}(\nu_B(x)), \text{sg}(1 - \mu_B(x))), \nu_A(x)) \mid x \in E\}$

-74	$\{ \langle x, \text{sg}(\min(\text{sg}(\nu_B(x)), \overline{\text{sg}}(\nu_A(x)))) \rangle, \overline{\text{sg}}(\min(\text{sg}(\nu_B(x)), \overline{\text{sg}}(\nu_A(x)))) \rangle x \in E \}$
-75	$\{ \langle x, \min(\nu_B(x).(\nu_B(x) + \mu_B(x)), \mu_A(x).(\nu_A(x)^2 + \mu_A(x)) + \nu_A(x).\mu_A(x)).(\min(\nu_B(x).(\nu_B(x) + \mu_B(x)), \mu_A(x).(\nu_A(x)^2 + \mu_A(x)) + \nu_A(x).\mu_A(x))) \rangle, \max(\mu_B(x), \nu_A(x).(\nu_A(x) + \mu_A(x)))) \rangle, \max(\mu_B(x), \nu_A(x).(\nu_A(x) + \mu_A(x))).(\min(\nu_B(x).(\nu_B(x) + \mu_B(x)), \mu_A(x).(\nu_A(x)^2 + \mu_A(x)) + \nu_A(x).\mu_A(x))^2 + \max(\mu_B(x), \nu_A(x).(\nu_A(x) + \mu_A(x))) + \min(\nu_B(x).(\nu_B(x) + \mu_B(x)), \mu_A(x).(\nu_A(x)^2 + \mu_A(x)) + \nu_A(x).\mu_A(x))) \rangle x \in E \}$
-76	$\{ \langle x, 1 - \max(\mu_B(x), 1 - \mu_A(x)), \max(\mu_B(x), 1 - \mu_A(x)) \rangle x \in E \}$
-77	$\{ \langle x, \text{sg}(\min(\text{sg}(1 - \mu_B(x)), \overline{\text{sg}}(1 - \mu_A(x)))) \rangle, \overline{\text{sg}}(\min(\text{sg}(1 - \mu_B(x)), \overline{\text{sg}}(1 - \mu_A(x)))) \rangle x \in E \}$
-78	$\{ \langle x, \min(\text{sg}(\nu_B(x)), \mu_A(x)), \max(\overline{\text{sg}}(1 - \mu_B(x)), \nu_A(x)) \rangle x \in E \}$
-79	$\{ \langle x, \text{sg}(\min(\text{sg}(\nu_B(x)), \overline{\text{sg}}(1 - \mu_A(x)))) \rangle, \overline{\text{sg}}(1 - \max(\overline{\text{sg}}(1 - \mu_B(x)), \text{sg}(\nu_A(x)))) \rangle x \in E \}$
-80	$\{ \langle x, \min(\nu_B(x), \mu_A(x)), \max(\overline{\text{sg}}(1 - \mu_B(x)), \nu_A(x)) \rangle x \in E \}$
-81	$\{ \langle x, \overline{\text{sg}}(1 - \min(\nu_B(x), \overline{\text{sg}}(1 - \mu_A(x)))) \rangle, \overline{\text{sg}}(1 - \max(\overline{\text{sg}}(1 - \mu_B(x)), \overline{\text{sg}}(1 - \nu_A(x)))) \rangle x \in E \}$
-82	$\{ \langle x, 1 - \max(1 - \nu_B(x), \min(\nu_B(x), 1 - \mu_A(x))), \max(1 - \nu_B(x), \min(\nu_B(x), 1 - \mu_A(x))) \rangle x \in E \}$
-83	$\{ \langle x, \overline{\text{sg}}(\overline{\text{sg}}(\nu_B(x) + \mu_A(x) - 1)) \rangle, \overline{\text{sg}}(\nu_B(x) + \mu_A(x) - 1) \rangle x \in E \}$
-84	$\{ \langle x, \mu_A(x).\text{sg}(\nu_B(x) + \mu_A(x) + 1), 1 - \mu_A(x).\text{sg}(\nu_B(x) + \mu_A(x) + 1) \rangle x \in E \}$
-85	$\{ \langle x, \nu_B(x) + \nu_B(x)^2.(1 - \mu_A(x)), 1 - \nu_B(x) + \nu_B(x)^2.(1 - \mu_A(x)) \rangle x \in E \}$
-86	$\{ \langle x, 1 - (1 - \mu_A(x)).\overline{\text{sg}}(1 - \nu_B(x)), (1 - \mu_A(x)).\overline{\text{sg}}(1 - \nu_B(x)) \rangle x \in E \}$
-87	$\{ \langle x, 1 - \max(\overline{\text{sg}}(\nu_B(x)), 1 - \mu_A(x)), \max(\overline{\text{sg}}(\nu_B(x)), 1 - \mu_A(x)) \rangle x \in E \}$
-88	$\{ \langle x, \text{sg}(1 - \max(\overline{\text{sg}}(\nu_B(x)), \text{sg}(1 - \mu_A(x)))) \rangle, \overline{\text{sg}}(1 - \max(\overline{\text{sg}}(\nu_B(x)), \text{sg}(1 - \mu_A(x)))) \rangle x \in E \}$
-89	$\{ \langle x, 1 - \max(\overline{\text{sg}}(\nu_B(x)), 1 - \mu_A(x)), \max(\overline{\text{sg}}(\nu_B(x)), 1 - \mu_A(x)) \rangle x \in E \}$
-90	$\{ \langle x, \overline{\text{sg}}(\max(\overline{\text{sg}}(\nu_B(x)), \overline{\text{sg}}(\mu_A(x)))) \rangle, \overline{\text{sg}}(1 - \max(\overline{\text{sg}}(\nu_B(x)), \overline{\text{sg}}(\mu_A(x)))) \rangle x \in E \}$

-91	$\{\langle x, 1 - \max(\mu_B(x), \nu_A(x)), \max(\mu_B(x), \nu_A(x)) \rangle x \in E\}$
-92	$\{\langle x, \overline{\text{sg}}(1 - \min(1 - \nu_A(x), \text{sg}(1 - \mu_B(x) - \nu_A(x))))), 1 - \min(1 - \nu_A(x), \text{sg}(1 - \mu_B(x) - \nu_A(x))) \rangle x \in E\}$
-93	$\{\langle x, \min(1 - \nu_A(x), \text{sg}(1 - \mu_B(x) - \nu_A(x))), 1 - \min(1 - \nu_A(x), \text{sg}(1 - \mu_B(x) - \nu_A(x))) \rangle x \in E\}$
-94	$\{\langle x, (1 - \mu_B(x)) \cdot \mu_B(x) + (1 - \mu_B(x))^2 \cdot (1 - \nu_A(x)), 1 - ((1 - \mu_B(x)) \cdot \mu_B(x) + (1 - \mu_B(x))^2 \cdot (1 - \nu_A(x))) \rangle x \in E\}$
-95	$\{\langle x, (\min(1 - \nu_A(x), \overline{\text{sg}}(\mu_B(x))) + \min(\min(1 - \mu_B(x), \text{sg}(\mu_B(x))), \text{sg}(1 - \nu_A(x))))), 1 - (\min(1 - \nu_A(x), \overline{\text{sg}}(\mu_B(x))) + \min(\min(1 - \mu_B(x), \text{sg}(\mu_B(x))), \text{sg}(1 - \nu_A(x)))) \rangle x \in E\}$
-96	$\{\langle x, \min(\text{sg}(1 - \mu_B(x)), 1 - \nu_A(x)), 1 - \min(\text{sg}(1 - \mu_B(x)), 1 - \nu_A(x)) \rangle x \in E\}$
-97	$\{\langle x, \text{sg}(\min(\text{sg}(1 - \mu_B(x)), \overline{\text{sg}}(\nu_A(x))))), \overline{\text{sg}}(\min(\text{sg}(1 - \mu_B(x)), \overline{\text{sg}}(\nu_A(x)))) \rangle x \in E\}$
-98	$\{\langle x, 1 - \max(\mu_B(x), \nu_A(x)), \max(\mu_B(x), \nu_A(x)) \rangle x \in E\}$
-99	$\{\langle x, \overline{\text{sg}}(1 - \min(1 - \mu_B(x), \overline{\text{sg}}(\nu_A(x))))), \overline{\text{sg}}(\min(1 - \mu_B(x), \overline{\text{sg}}(\nu_A(x)))) \rangle x \in E\}$
-100	$\{\langle x, \min(\min(\min(\mu_A(x), \text{sg}(\nu_A(x))), \nu_B(x)), \text{sg}(\max(\min(\nu_A(x), \text{sg}(\mu_A(x))), \mu_B(x))))), \min(\max(\min(\nu_A(x), \text{sg}(\mu_A(x))), \mu_B(x)), \text{sg}(\min(\min(\mu_A(x), \text{sg}(\nu_A(x))), \nu_B(x)))) \rangle x \in E\}$
-101	$\{\langle x, \min(\min(\min(\mu_A(x), \text{sg}(\nu_A(x))), \min(\nu_B(x), \text{sg}(\mu_B(x))))), \text{sg}(\max(\min(\nu_A(x), \text{sg}(\mu_A(x))), \min(\mu_B(x), \text{sg}(\nu_B(x)))))), 0 \rangle x \in E\}$
-102	$\{\langle x, \min(\mu_A(x), \min(\nu_B(x), \text{sg}(\mu_B(x))))), 0 \rangle x \in E\}$
-103	$\{\langle x, \min(1 - \max(\min(1 - \mu_A(x), \text{sg}(\mu_A(x))), 1 - \nu_B(x)), \text{sg}(\max(\min(1 - \mu_A(x), \text{sg}(\mu_A(x))), 1 - \nu_B(x))))), \min(\max(\min(1 - \mu_A(x), \text{sg}(\mu_A(x))), 1 - \nu_B(x)), \text{sg}(1 - \max(\min(1 - \mu_A(x), \text{sg}(\mu_A(x))), 1 - \nu_B(x)))) \rangle x \in E\}$
-104	$\{\langle x, \min(1 - \max(\min(1 - \mu_A(x), \text{sg}(\mu_A(x))), \min(1 - \nu_B(x), \text{sg}(\nu_B(x))))), \text{sg}(\max(\min(1 - \mu_A(x), \text{sg}(\mu_A(x))), \min(1 - \nu_B(x), \text{sg}(\nu_B(x))))), 0 \rangle x \in E\}$
-105	$\{\langle x, 1 - \max(1 - \mu_A(x), \min(1 - \nu_B(x), \text{sg}(\nu_B(x))))), 0 \rangle x \in E\}$
-106	$\{\langle x, \min(\min(\mu_A(x), \min(\nu_B(x), \text{sg}(1 - \nu_B(x))))), \text{sg}(1 - \min(\mu_A(x), \min(\nu_B(x), \text{sg}(1 - \nu_B(x))))), \min(1 - \min(\mu_A(x), \min(\nu_B(x), \text{sg}(1 - \nu_B(x))))), \text{sg}(\min(\mu_A(x), \min(\nu_B(x), \text{sg}(1 - \nu_B(x)))))) \rangle x \in E\}$

-107	$\{\langle x, \min(\max(\min(\nu_A(x), \text{sg}(1 - \nu_A(x))), \min(\mu_B(x), \text{sg}(1 - \mu_B(x))))), \text{sg}(1 - \max(\min(\nu_A(x), \text{sg}(1 - \nu_A(x))), \min(\mu_B(x), \text{sg}(1 - \mu_B(x))))), 0) x \in E \}$
-108	$\{\langle x, \min(1 - \nu_A(x), \min(1 - \mu_B(x), \text{sg}(\mu_B(x))), 0) x \in E \}$
-109	$\{\langle x, \mu_A(x) \cdot \nu_A(x) + \min(\overline{\text{sg}}(1 - \mu_A(x)), \nu_B(x)), \nu_A(x) + \min(\overline{\text{sg}}(1 - \mu_A(x)), \mu_B(x)) \cdot \mu_A(x) \cdot \nu_A(x) + \min(\overline{\text{sg}}(1 - \mu_A(x)), \nu_B(x)) + \overline{\text{sg}}(1 - \nu_A(x) + \min(\overline{\text{sg}}(1 - \mu_A(x)), \mu_B(x))) \rangle x \in E \}$
-110	$\{\langle x, \min(\mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \mu_A(x)), \nu_B(x)), \max(\nu_A(x), \mu_B(x)) \cdot \min(\mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \mu_A(x)), \nu_B(x)) + \overline{\text{sg}}(1 - \max(\nu_A(x), \mu_B(x))) \rangle x \in E \}$
-111	$\{\langle x, \min(\mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \mu_A(x)), \nu_B(x) \cdot (\mu_B(x) \cdot \nu_B(x) + \overline{\text{sg}}(1 - \mu_B(x))) + \overline{\text{sg}}(1 - \nu_B(x))), \max(\nu_A(x), \mu_B(x) \cdot \nu_B(x) + \overline{\text{sg}}(1 - \mu_B(x))) \cdot \min(\mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \mu_A(x)), \nu_B(x) \cdot (\mu_B(x) \cdot \nu_B(x) + \overline{\text{sg}}(1 - \mu_B(x))) + \overline{\text{sg}}(1 - \nu_B(x))) + \overline{\text{sg}}(1 - \max(\nu_A(x), \mu_B(x) \cdot \nu_B(x) + \overline{\text{sg}}(1 - \mu_B(x)))) \rangle x \in E \}$
-112	$\{\langle x, \mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \mu_A(x)) \cdot \nu_B(x), \nu_A(x) + \mu_B(x) - \nu_A(x) \cdot \mu_B(x) \cdot \mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \mu_A(x)) \cdot \nu_B(x) + \overline{\text{sg}}(1 - \nu_A(x) + \mu_B(x) - \nu_A(x) \cdot \mu_B(x)) \rangle x \in E \}$
-113	$\{\langle x, (\mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \mu_A(x))) \cdot (\nu_B(x) \cdot (\mu_B(x) \cdot \nu_B(x) + \overline{\text{sg}}(1 - \mu_B(x))) + \overline{\text{sg}}(1 - \nu_B(x))), \nu_A(x) + (\mu_B(x) \cdot \nu_B(x) - \nu_A(x) \cdot (\mu_B(x) \cdot \nu_B(x) + \overline{\text{sg}}(1 - \mu_B(x)))) \cdot (\mu_A(x) \cdot \nu_A(x) + \overline{\text{sg}}(1 - \mu_A(x))) \cdot (\nu_B(x) \cdot (\mu_B(x) \cdot \nu_B(x) + \overline{\text{sg}}(1 - \mu_B(x))) + \overline{\text{sg}}(1 - \nu_B(x))) + \overline{\text{sg}}(1 - \nu_A(x) + (\mu_B(x) \cdot \nu_B(x) - \nu_A(x) \cdot (\mu_B(x) \cdot \nu_B(x) + \overline{\text{sg}}(1 - \mu_B(x)))) \rangle x \in E \}$
-114	$\{\langle x, \mu_A(x) + \min(\overline{\text{sg}}(1 - \mu_A(x)), (1 - \nu_B(x))), (1 - \mu_A(x) + \min(\overline{\text{sg}}(1 - \mu_A(x)), (1 - \nu_B(x)))) \cdot \mu_A(x) + \min(\overline{\text{sg}}(1 - \mu_A(x)), (1 - \nu_B(x))) + \overline{\text{sg}}(\mu_A(x) + \min(\overline{\text{sg}}(1 - \mu_A(x)), (1 - \nu_B(x)))) \rangle x \in E \}$
-115	$\{\langle x, \min(\mu_A(x), \nu_B(x)), (1 - \min(\mu_A(x), \nu_B(x))) \cdot \min(\mu_A(x), \nu_B(x)) + \overline{\text{sg}}(\min(\mu_A(x), \nu_B(x))) \rangle x \in E \}$

-116	$\{ \langle x, 1 - \max(1 - \mu_A(x), (1 - \nu_B(x)) \cdot \nu_B(x) + \overline{\text{sg}}(\nu_B(x))), \max(1 - \mu_A(x), (1 - \nu_B(x)) \cdot \nu_B(x) + \overline{\text{sg}}(\nu_B(x))) \cdot (1 - \max(1 - \mu_A(x), (1 - \nu_B(x)) \cdot \nu_B(x) + \overline{\text{sg}}(\nu_B(x)))) + \overline{\text{sg}}(1 - \max(1 - \mu_A(x), (1 - \nu_B(x)) \cdot \nu_B(x) + \overline{\text{sg}}(\nu_B(x)))) \rangle x \in E \}$
-117	$\{ \langle x, \mu_A(x) - \nu_B(x) + \mu_A(x) \cdot \nu_B(x), ((1 - \mu_A(x) - \nu_B(x) + \mu_A(x) \cdot \nu_B(x)) \cdot (\mu_A(x) - \nu_B(x) + \mu_A(x) \cdot \nu_B(x)) + \overline{\text{sg}}(\mu_A(x) - \nu_B(x) + \mu_A(x) \cdot \nu_B(x))) \rangle x \in E \}$
-118	$\{ \langle x, 1 - (1 - \mu_A(x)) \cdot \text{sg}(\nu_B(x)) + \mu_A(x) \cdot \nu_B(x) \cdot (1 - \nu_B(x)), (1 - \mu_A(x)) \cdot \text{sg}(\nu_B(x)) + \mu_A(x) \cdot \nu_B(x) \cdot (1 - \nu_B(x)) \cdot (1 - (1 - \mu_A(x)) \cdot \text{sg}(\nu_B(x)) + \mu_A(x) \cdot \nu_B(x) \cdot (1 - \nu_B(x))) + \overline{\text{sg}}(1 - (1 - \mu_A(x)) \cdot \text{sg}(\nu_B(x)) + \mu_A(x) \cdot \nu_B(x) \cdot (1 - \nu_B(x))) \rangle x \in E \}$
-119	$\{ \langle x, (1 - \nu_A(x)) \cdot \nu_A(x) + \min(\overline{\text{sg}}(\nu_A(x)), 1 - \mu_B(x)), (1 - (1 - \nu_A(x)) \cdot \nu_A(x) + \min(\overline{\text{sg}}(\nu_A(x)), 1 - \mu_B(x))) \cdot (1 - \nu_A(x)) \cdot \nu_A(x) + \min(\overline{\text{sg}}(\nu_A(x)), 1 - \mu_B(x)) + \overline{\text{sg}}((1 - \nu_A(x)) \cdot \nu_A(x) + \min(\overline{\text{sg}}(\nu_A(x)), 1 - \mu_B(x))) \rangle x \in E \}$
-120	$\{ \langle x, \min((1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}(\nu_A(x)), 1 - \mu_B(x)), (1 - \min((1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}(\nu_A(x)), 1 - \mu_B(x))) \cdot \min((1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}(\nu_A(x)), 1 - \mu_B(x)) + \overline{\text{sg}}(\min((1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}(\nu_A(x)), 1 - \mu_B(x))) \rangle x \in E \}$
-121	$\{ \langle x, \min((1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}(\nu_A(x)), (1 - \mu_B(x)) \cdot (\mu_B(x) \cdot (1 - \mu_B(x)) + \overline{\text{sg}}(1 - \mu_B(x)) + \overline{\text{sg}}(\mu_B(x))), (1 - \min((1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}(\nu_A(x)), (1 - \mu_B(x)) \cdot (\mu_B(x) \cdot (1 - \mu_B(x)) + \overline{\text{sg}}(1 - \mu_B(x)) + \overline{\text{sg}}(\mu_B(x)))) \cdot \min((1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}(\nu_A(x)), (1 - \mu_B(x)) \cdot (\mu_B(x) \cdot (1 - \mu_B(x)) + \overline{\text{sg}}(1 - \mu_B(x)) + \overline{\text{sg}}(\mu_B(x))) + \overline{\text{sg}}(\min((1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}(\nu_A(x)), (1 - \mu_B(x)) \cdot (\mu_B(x) \cdot (1 - \mu_B(x)) + \overline{\text{sg}}(1 - \mu_B(x)) + \overline{\text{sg}}(\mu_B(x)))) \rangle x \in E \}$
-122	$\{ \langle x, ((1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}(\nu_A(x))) \cdot (1 - \mu_B(x)), (1 - ((1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}(\nu_A(x)))) \cdot (1 - \mu_B(x)) \cdot ((1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}(\nu_A(x))) \cdot (1 - \mu_B(x)) + \overline{\text{sg}}(((1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}(\nu_A(x))) \cdot (1 - \mu_B(x))) \rangle x \in E \}$
-123	$\{ \langle x, ((1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}(\nu_A(x))) \cdot (((1 - \mu_B(x)) \cdot (\mu_B(x) \cdot (1 - \mu_B(x)) + \overline{\text{sg}}(1 - \mu_B(x))) + \overline{\text{sg}}(\mu_B(x))), (1 - ((1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}(\nu_A(x))) \cdot (((1 - \mu_B(x)) \cdot (\mu_B(x) \cdot (1 - \mu_B(x)) + \overline{\text{sg}}(1 - \mu_B(x))) + \overline{\text{sg}}(\mu_B(x))) \rangle x \in E \}$
	continues

	$ \begin{aligned} & +\overline{\text{sg}}(1 - \mu_B(x))) + \overline{\text{sg}}(\mu_B(x))) \\ & \cdot ((1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}(\nu(x))) \\ & \cdot (((1 - \mu_B(x)) \cdot (\mu_B(x) \cdot (1 - \mu_B(x))) \\ & + \overline{\text{sg}}(1 - \mu_B(x))) + \overline{\text{sg}}(\mu_B(x))) \\ & + \overline{\text{sg}}(((1 - \nu_A(x)) \cdot \nu_A(x) + \overline{\text{sg}}(\nu(x))) \\ & \cdot (((1 - \mu_B(x)) \cdot (\mu_B(x) \cdot (1 - \mu_B(x))) \\ & + \overline{\text{sg}}(1 - \mu_B(x))) + \overline{\text{sg}}(\mu_B(x)))) x \in E \} \end{aligned} $
-124	$ \{ \langle x, \nu_B(x) \cdot \mu_B(x) + \min(\overline{\text{sg}}(1 - \nu_B(x)), \mu_A(x)), \mu_B(x) + \min(\overline{\text{sg}}(1 - \nu_B(x)), \nu_A(x)) \rangle x \in E \} $
-125	$ \{ \langle x, \min(\nu_B(x) \cdot \mu_B(x) + \overline{\text{sg}}(1 - \nu_B(x)), \mu_A(x)), \max(\mu_B(x), \nu_A(x)) \rangle x \in E \} $
-126	$ \begin{aligned} & \{ \langle x, \min(\nu_B(x) \cdot \mu_B(x) + \overline{\text{sg}}(1 - \nu_B(x)), \mu_A(x) \cdot \\ & (\nu_A(x) \cdot \mu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(1 - \mu_A(x))) \\ & \cdot \max(\mu_B(x), \nu_A(x) \cdot \mu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))) \\ & + \overline{\text{sg}}(1 - \min(\nu_B(x) \cdot \mu_B(x) + \overline{\text{sg}}(1 - \nu_B(x)), \mu_A(x) \cdot \\ & (\nu_A(x) \cdot \mu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(1 - \mu_A(x))))), \\ & \max(\mu_B(x), \nu_A(x) \cdot \mu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))) \\ & \cdot (\min(\nu_B(x) \cdot \mu_B(x) + \overline{\text{sg}}(1 - \nu_B(x)), \mu_A(x) \cdot \\ & (\nu_A(x) \cdot \mu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(1 - \mu_A(x))) \\ & \cdot \max(\mu_B(x), \nu_A(x) \cdot \mu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))) \\ & + \overline{\text{sg}}(1 - \min(\nu_B(x) \cdot \mu_B(x) + \overline{\text{sg}}(1 - \nu_B(x)), \mu_A(x) \cdot \\ & (\nu_A(x) \cdot \mu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(1 - \mu_A(x)))))) \\ & + \overline{\text{sg}}(1 - \max(\mu_B(x), \nu_A(x) \cdot \mu_A(x) \\ & + \overline{\text{sg}}(1 - \nu_A(x)))) \rangle x \in E \} \end{aligned} $
-127	$ \{ \langle x, (\nu_B(x) \cdot \mu_B(x) + \overline{\text{sg}}(1 - \nu_B(x))) \cdot \mu_A(x), \mu_B(x) + \nu_A(x) - \mu_B(x) \cdot \nu_A(x) \rangle x \in E \} $
-128	$ \begin{aligned} & \{ \langle x, (\nu_B(x) \cdot \mu_B(x) + \overline{\text{sg}}(1 - \nu_B(x))) \cdot (\mu_A(x) \cdot (\nu_A(x) \cdot \mu_A(x) \\ & + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(1 - \mu_A(x))) \cdot (\mu_B(x) + \nu_A(x) \cdot \mu_A(x) \\ & - \mu_B(x) \cdot (\nu_A(x) \cdot \mu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))))), \\ & (\mu_B(x) + \nu_A(x) \cdot \mu_A(x) - \mu_B(x) \cdot \\ & (\nu_A(x) \cdot \mu_A(x) + \overline{\text{sg}}(1 - \nu_A(x)))) \cdot (\nu_B(x) \cdot \mu_B(x) + \overline{\text{sg}}(1 - \nu_B(x))) \\ & \cdot (\mu_A(x) \cdot (\nu_A(x) \cdot \mu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(1 - \mu_A(x))) \\ & ((\nu_B(x) \cdot \mu_B(x) + \overline{\text{sg}}(1 - \nu_B(x))) \cdot (\mu_A(x) \cdot (\nu_A(x) \cdot \mu_A(x) \\ & + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(1 - \mu_A(x))) \cdot (\mu_B(x) + \nu_A(x) \cdot \mu_A(x) \\ & - \mu_B(x) \cdot (\nu_A(x) \cdot \mu_A(x) + \overline{\text{sg}}(1 - \nu_A(x)))) \\ & + \overline{\text{sg}}(1 - (\nu_B(x) \cdot \mu_B(x) + \overline{\text{sg}}(1 - \nu_B(x))) \\ & \cdot (\mu_A(x) \cdot (\nu_A(x) \cdot \mu_A(x) + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(1 - \mu_A(x)))))) \\ & + \overline{\text{sg}}(1 - \mu_B(x) - \nu_A(x) \cdot \mu_A(x) + \mu_B(x) \cdot \\ & (\nu_A(x) \cdot \mu_A(x) + \overline{\text{sg}}(1 - \nu_A(x)))) \rangle x \in E \} \end{aligned} $

	$+\overline{\text{sg}}(\min((1 - \mu_B(x)) \cdot \mu_B(x) + \overline{\text{sg}}(\mu_B(x)), (1 - \nu_A(x)) \cdot (\nu_A(x) \cdot (1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x))) + \overline{\text{sg}}(\nu_A(x))))) x \in E\}$
-137	$\{ \langle x, ((1 - \mu_B(x)) \cdot \mu_B(x) + \overline{\text{sg}}(\mu_B(x))) \cdot (1 - \nu_A(x)), 1 - ((1 - \mu_B(x)) \cdot \mu_B(x) + \overline{\text{sg}}(\mu_B(x))) \cdot (1 - \nu_A(x)) \rangle x \in E\}$
-138	$\{ \langle x, ((1 - \mu_B(x)) \cdot \mu_B(x) + \overline{\text{sg}}(\mu_B(x))) \cdot (1 - \nu_A(x)) \cdot (\nu_A(x) \cdot (1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x)) + \overline{\text{sg}}(\nu_A(x))) \cdot (1 - ((1 - \mu_B(x)) \cdot \mu_B(x) + \overline{\text{sg}}(\mu_B(x)))) \cdot (1 - \nu_A(x)) \cdot (\nu_A(x) \cdot (1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x)) + \overline{\text{sg}}(\nu_A(x))))), (1 - ((1 - \mu_B(x)) \cdot \mu_B(x) + \overline{\text{sg}}(\mu_B(x)))) \cdot (1 - \nu_A(x)) \cdot (\nu_A(x) \cdot (1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x)) + \overline{\text{sg}}(\nu_A(x)))) \cdot (((1 - \mu_B(x)) \cdot \mu_B(x) + \overline{\text{sg}}(\mu_B(x))) \cdot (1 - \nu_A(x)) \cdot (\nu_A(x) \cdot (1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x)) + \overline{\text{sg}}(\nu_A(x)))) \cdot (1 - ((1 - \mu_B(x)) \cdot \mu_B(x) + \overline{\text{sg}}(\mu_B(x)))) \cdot (1 - \nu_A(x)) \cdot (\nu_A(x) \cdot (1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x)) + \overline{\text{sg}}(\nu_A(x)))) + \overline{\text{sg}}(1 - ((1 - \mu_B(x)) \cdot \mu_B(x) + \overline{\text{sg}}(\mu_B(x)))) \cdot (1 - \nu_A(x)) \cdot (\nu_A(x) \cdot (1 - \nu_A(x)) + \overline{\text{sg}}(1 - \nu_A(x)) + \overline{\text{sg}}(\nu_A(x)))) + \overline{\text{sg}}(\nu_A(x)))) \rangle x \in E\}$

Some of the most important properties of the subtractions are:

- (a) $A - E^* = O^*$,
- (b) $A - O^* = A$,
- (c) $E^* - A = \neg A$,
- (d) $O^* - A = O^*$,
- (e) $(A - B) \cap C = (A \cap C) - B = A \cap (C - B)$,
- (f) $(A \cap B) - C = (A - C) \cap (B - C)$,
- (g) $(A \cup B) - C = (A - C) \cup (B - C)$,
- (h) $(A - B) - C = (A - C) - B$,
- (i) $(A - C) \cap B = A \cap (B - C)$,
- (j) $O^* - U^* = O^*$,
- (k) $O^* - E^* = O^*$,
- (l) $U^* - O^* = U^*$,
- (m) $U^* - E^* = O^*$,
- (n) $E^* - O^* = E^*$,
- (o) $E^* - U^* = O^*$.

In Table 6 are given these subtractions that satisfy these properties.

Table 6: Properties of the “subtraction” operations

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
-1	-	+	+	+	-	-	-	-	-	+	+	+	-	+	-
-2	+	-	-	+	-	+	+	-	-	+	+	-	+	+	-
-3	+	-	+	+	-	+	+	+	-	+	+	-	+	+	-
-4	+	+	+	+	+	+	+	+	+	+	+	+	+	+	-
-5	+	+	+	+	-	+	+	+	-	+	+	+	+	+	-
-6	-	+	+	+	-	-	-	-	-	+	+	+	-	+	-
-7	-	+	+	-	-	-	-	-	-	-	+	+	-	+	-
-8	+	-	-	+	-	+	-	-	-	+	+	-	+	+	-
-9	-	-	+	+	-	-	-	-	-	+	+	+	-	+	-
-10	+	+	+	+	-	-	-	-	-	+	+	+	+	+	-
-11	+	-	+	+	-	+	+	+	-	+	+	-	+	+	-
-12	+	-	-	+	-	+	+	+	-	+	+	-	+	+	-
-13	+	+	+	+	-	+	+	+	-	+	+	+	+	+	-
-14	+	-	+	+	-	+	+	+	-	+	+	-	+	+	+
-15	+	-	-	+	-	+	-	-	-	+	+	-	+	+	-
-16	+	-	+	+	-	+	+	+	-	+	+	-	+	+	-
-17	+	-	+	+	-	-	-	-	-	+	+	+	+	+	-
-18	+	-	+	+	-	+	+	+	-	+	+	-	+	+	+
-19	+	-	+	+	-	+	+	+	-	+	+	-	+	+	+
-20	+	-	+	+	-	+	+	+	-	+	+	-	+	+	-
-21	+	-	-	+	-	-	-	-	-	+	+	+	+	+	-
-22	+	-	+	+	-	+	+	+	-	+	+	-	+	+	+
-23	+	-	+	+	-	+	+	+	-	+	+	-	+	+	+
-24	+	-	-	+	-	-	-	-	-	+	+	+	+	+	-
-25	+	-	-	+	-	+	+	+	-	+	+	+	+	+	-
-26	+	-	+	+	-	+	+	+	-	+	+	+	+	+	-
-27	+	-	+	+	-	+	+	+	-	+	+	+	+	+	-
-28	+	-	+	+	-	+	+	-	-	+	+	+	+	+	-
-29	+	-	-	+	-	+	+	+	-	+	+	+	+	+	-
-30	-	-	-	+	-	-	-	-	-	+	+	-	+	+	+
-31	+	-	-	+	-	+	+	-	-	+	+	-	+	+	+
-32	+	-	-	+	-	+	+	+	-	+	+	-	+	+	+
-33	+	-	-	+	-	+	+	+	-	+	+	-	+	+	+
-34	+	-	-	+	-	+	+	+	-	+	+	-	+	+	+
-35	+	-	-	+	-	+	+	+	-	+	+	-	+	+	+

-36	-	-	-	-	-	-	-	+	-	+	+	-	+	+	+
-37	+	-	-	+	-	+	+	-	-	+	+	-	+	+	+
-38	-	-	-	+	-	-	-	-	-	+	+	-	+	+	+
-39	+	-	-	+	-	-	-	-	-	+	+	-	+	+	+
-40	+	-	-	+	-	+	+	-	-	+	+	-	+	+	+
-41	+	-	-	+	-	+	+	+	-	+	+	-	+	+	+
-42	+	-	-	+	-	+	+	+	-	+	+	-	+	+	+
-43	+	-	-	+	-	+	+	+	-	+	+	-	+	+	+
-44	+	-	-	+	-	+	+	-	-	+	+	-	+	+	+
-45	+	-	-	+	-	+	+	+	-	+	+	-	+	+	+
-46	+	-	-	+	-	+	+	+	-	+	+	-	+	+	-
-47	+	-	-	+	-	+	+	+	-	+	+	-	+	+	-
-48	+	-	-	+	-	+	+	+	-	+	+	-	+	+	-
-49	+	-	-	+	-	+	+	+	-	+	+	-	+	+	-
-50	+	-	-	+	-	+	+	+	-	+	+	-	+	+	-
-51	-	-	-	-	-	-	-	+	-	+	+	-	+	+	-
-52	+	-	-	+	-	+	+	-	-	+	+	-	+	+	-
-53	-	-	-	+	-	-	-	-	-	+	+	-	+	+	-
-54	+	-	-	+	-	-	-	-	-	+	+	-	+	+	-
-55	+	-	-	+	-	+	+	-	-	+	+	-	+	+	-
-56	+	-	-	+	-	+	+	+	-	+	+	-	+	+	-
-57	+	-	-	+	-	+	+	+	-	+	+	-	+	+	-
-58	+	-	-	+	-	+	+	+	-	+	+	-	+	+	-
-59	+	-	-	+	-	+	+	+	-	+	+	-	+	+	-
-60	+	-	-	+	-	+	+	+	-	+	+	-	+	+	-
-61	+	+	+	-	-	+	+	-	-	-	+	+	+	+	-
-62	+	-	-	+	-	+	+	-	-	+	+	+	+	+	+
-63	+	+	-	+	-	+	-	+	-	+	+	+	+	+	+
-64	+	+	+	-	-	+	+	-	-	-	+	+	+	+	-
-65	+	-	-	+	-	+	+	-	-	+	+	+	+	+	+
-66	+	+	-	-	-	+	+	-	-	-	+	+	+	+	-
-67	+	+	+	+	-	+	+	-	-	+	+	+	+	+	-
-68	+	+	-	+	-	+	-	+	-	+	+	+	+	+	+
-69	+	+	-	+	-	-	+	+	-	+	+	+	+	+	-
-70	+	+	-	+	+	+	+	+	+	+	+	+	+	+	+
-71	+	+	-	+	+	+	+	+	+	+	+	+	+	+	-
-72	+	+	-	+	+	+	+	+	+	+	+	+	+	+	-
-73	+	+	-	+	+	+	+	+	+	+	+	+	+	+	-

-74	+	-	+	+	-	+	+	+	-	+	+	-	+	+	+
-75	+	-	-	+	-	-	-	-	-	+	+	+	+	+	-
-76	+	-	+	+	-	+	+	+	-	+	+	-	+	+	-
-77	+	-	+	+	-	+	+	+	-	+	+	-	+	+	-
-78	+	+	-	+	+	+	+	+	+	+	+	+	+	+	-
-79	+	-	+	+	-	+	+	+	-	+	+	+	+	+	-
-80	+	+	-	+	+	+	+	+	+	+	+	+	+	+	-
-81	+	-	+	+	-	+	+	+	-	+	+	+	+	+	-
-82	+	-	-	-	-	+	+	-	-	+	+	-	+	+	+
-83	+	-	-	+	-	+	+	-	-	+	+	-	+	+	+
-84	+	-	-	+	-	+	+	+	-	+	+	-	+	+	+
-85	+	-	-	-	-	+	+	-	-	+	+	-	+	+	+
-86	+	-	-	+	-	+	+	-	-	+	+	-	+	+	+
-87	+	-	-	+	-	+	+	+	-	+	+	-	+	+	+
-88	+	-	-	+	-	+	+	+	-	+	+	-	+	+	+
-89	+	-	-	+	-	+	+	+	-	+	+	-	+	+	+
-90	+	-	-	+	-	+	+	+	-	+	+	-	+	+	+
-91	+	-	-	+	-	+	+	+	-	+	+	-	+	+	-
-92	+	-	-	+	-	+	+	+	-	+	+	-	+	+	-
-93	+	-	-	+	-	+	+	+	-	+	+	-	+	+	-
-94	+	-	-	-	-	+	+	-	-	+	+	-	+	+	-
-95	+	-	-	+	-	+	+	-	-	+	+	-	+	+	-
-96	+	-	-	+	-	+	+	+	-	+	+	-	+	+	-
-97	+	-	-	+	-	+	+	+	-	+	+	-	+	+	-
-98	+	-	-	+	-	+	+	+	-	+	+	-	+	+	-
-99	+	-	-	+	-	+	+	-	-	+	+	-	+	+	-
-100	-	-	-	-	-	-	-	+	-	-	-	+	-	-	-
-101	-	-	-	-	-	-	-	+	-	-	-	+	-	-	-
-102	-	-	-	-	-	+	+	+	-	-	-	+	-	-	-
-103	-	-	-	-	-	-	-	-	-	-	-	+	-	-	-
-104	-	-	-	-	-	-	-	+	-	-	-	+	-	-	-
-105	-	-	-	-	-	+	+	+	-	-	-	+	-	+	-
-106	-	-	-	-	-	-	-	+	-	-	-	+	-	-	-
-107	-	-	-	-	-	-	-	+	-	-	-	+	-	-	-
-108	-	-	-	-	-	+	+	-	-	-	-	+	-	-	-
-109	-	-	+	+	-	-	-	-	-	+	+	+	-	+	-
-110	+	-	+	+	-	-	-	-	-	+	+	+	+	+	-
-111	+	-	-	+	-	-	-	-	-	+	+	+	+	+	-

-112	+	-	+	+	-	-	-	-	-	+	+	+	+	+	-
-113	-	-	-	-	-	-	-	-	-	+	-	+	-	+	-
-114	-	-	-	+	-	-	-	-	-	+	+	-	+	+	+
-115	+	-	-	+	-	-	-	+	-	+	+	-	+	+	+
-116	+	-	-	+	-	-	-	+	-	+	+	-	+	+	+
-117	+	-	-	+	-	-	-	+	-	+	+	-	+	+	+
-118	-	-	-	-	-	-	-	-	-	-	-	-	-	+	-
-119	-	-	-	+	-	-	-	-	-	+	+	-	+	+	-
-120	+	-	-	+	-	-	-	-	-	+	+	-	+	+	-
-121	+	-	-	+	-	-	-	-	-	+	+	-	+	+	-
-122	+	-	-	+	-	-	-	-	-	+	+	-	+	+	-
-123	+	-	-	+	-	-	-	-	-	+	+	-	+	+	-
-124	+	+	-	-	-	+	+	-	-	-	+	+	+	+	-
-125	+	+	-	+	+	+	+	+	+	+	+	+	+	+	-
-126	+	-	-	+	-	-	-	-	-	+	+	+	+	+	-
-127	+	+	-	+	-	+	+	+	-	+	+	+	+	+	-
-128	-	-	-	-	-	-	-	-	-	-	-	+	+	-	-
-129	+	-	-	-	-	+	+	-	-	+	+	-	+	+	+
-130	+	-	-	+	-	+	+	+	-	+	+	-	+	+	+
-131	+	-	-	+	-	-	-	-	-	+	+	-	+	+	+
-132	+	-	-	+	-	+	+	+	-	+	+	-	+	+	+
-133	-	-	-	-	-	-	-	-	-	-	-	+	-	-	+
-134	+	-	-	-	-	+	+	-	-	+	+	-	+	+	-
-135	+	-	-	+	-	+	+	+	-	+	+	-	+	+	-
-136	+	-	-	+	-	-	-	-	-	+	+	-	+	+	-
-137	+	-	-	+	-	+	+	+	-	+	+	-	+	+	-
-138	+	-	-	+	-	-	-	-	-	+	+	+	+	-	-

In a next research we will continue to study the definitions and properties of the new subtractions based on the intuitionistic fuzzy implications.

An **OPEN PROBLEM** is to find another approach to introducing variants of the “subtraction” operation over IFSs. If this is possible, the behaviour of the new operations must be studied, also.

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.

It may be viewed as a result of fruitful discussions held during the Tenth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2011) organized in Warsaw on September 30, 2011 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

[Http://www.ibspan.waw.pl/ifs2011](http://www.ibspan.waw.pl/ifs2011)

The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Tenth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2011) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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