Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

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A new intuitionistic fuzzy implication. Part 2

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Abstract

A new intuitionistic fuzzy implication is constructed. Some of its basic properties, as its relations with Modus Ponens, with intuitionistic logic axioms and Georg Klir and Bo Yuan's axioms are studied.

Keywords: implication, intuitionistic fuzzy logic.

1 Introduction

The concept of *"intuitionistic fuzzy propositional calculus"* was introduced about 20 years ago (see, e.g., [1, 2]). Initially, it contained only one form of conjunction, disjunction and two forms of implication. In a series of papers, a lot of new implications were defined in the frames of the intuitionistic fuzzy logic – see, e.g. [3, 4, 5, 6, 7, 8].

Here, we shall study a new implication and its basic properties.

In intuitionistic fuzzy propositional calculus, if x is a variable, then its truth-value is represented by the ordered couple

$$V(x) = \langle a, b \rangle,$$

so that $a, b, a+b \in [0,1]$, where a and b are degrees of validity and of non-validity of x.

Below, we shall assume that for the three variables x, y and z the equalities: $V(x) = \langle a, b \rangle, V(y) = \langle c, d \rangle, V(z) = \langle e, f \rangle$ $(a, b, c, d, e, f, a+b, c+d, e+f \in [0,1])$ hold.

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For the needs of the discussion below, following the definition from [1], we shall use the notion of Intuitionistic Fuzzy Tautology (IFT) by:

$$x \text{ is an IFT, if and only if for } V(x) = \langle a, b\rangle \text{ holds: } a \geq b,$$

while x will be a tautology iff $a = 1$ and $b = 0$. As in the case of ordinary logics, x is a tautology, if $V(x) = \langle 1, 0\rangle$.

For two variables x and y the operations "conjunction" (&) and "disjunction" ($\vee$) are defined (see [1]) by:

$$V(x\&y) = \langle \min(a, c), \max(b, d)\rangle,$$

$$V(x \vee y) = \langle \max(a, c), \min(b, d)\rangle.$$

2 Main results

Here, the following new implication will be defined:

$$V(x \to y) = \langle \max(\overline{sg}(a-c), \mathrm{sg}(a).\min(1, \frac{c}{a})),$$

$$\min(sg(a-c), \mathrm{sg}(a).\mathrm{sg}(d).\max(0, \min(\frac{b}{d}, 1-\frac{c}{a})))\rangle.$$

First, we check that it is defined correctly, i.e.,

$$0 \leq \max(\overline{sg}(a-c), \mathrm{sg}(a).\min(1, \frac{c}{a})) \leq 1,$$

$$0 \leq \min(sg(a-c), \mathrm{sg}(a).\mathrm{sg}(d).\max(0, \min(\frac{b}{d}, 1-\frac{c}{a}))) \leq 1,$$

and

$$0 \leq \max(\overline{sg}(a-c), \mathrm{sg}(a).\min(1, \frac{c}{a}))$$

$$+\min(sg(a-c), \mathrm{sg}(a).\mathrm{sg}(d).\max(0, \min(\frac{b}{d}, 1-\frac{c}{a}))) \leq 1.$$

The first two inequalities are obvious. Let us check the third one. Let

$$X \equiv \max(\overline{sg}(a-c), \mathrm{sg}(a).\min(1, \frac{c}{a}))$$

$$+\min(sg(a-c), \mathrm{sg}(a).\mathrm{sg}(d).\max(0, \min(\frac{b}{d}, 1-\frac{c}{a}))).$$

Obviously, $X \geq 0$.

If $a \le c$, then

$$X = \max(1, \mathrm{sg}(a).\min(1, \frac{c}{a}))$$

$$+\min(0, \mathrm{sg}(a).\mathrm{sg}(d).\max(0, \min(\frac{b}{d}, 1 - \frac{c}{a}))) = 1 + 0 = 1.$$

If $a > c$, then

$$X = \max(0, \mathrm{sg}(a).\min(1, \frac{c}{a})) + \min(1, \mathrm{sg}(a).\mathrm{sg}(d).\max(0, \min(\frac{b}{d}, 1 - \frac{c}{a}))).$$

$$= \mathrm{sg}(a).\min(1, \frac{c}{a}) + \mathrm{sg}(a).\mathrm{sg}(d).\max(0, \min(\frac{b}{d}, 1 - \frac{c}{a})).$$

If $a = 0$, then, obviously, $X = 0$. If $a > 0$, then

$$X = \min(1, \frac{c}{a}) + \mathrm{sg}(d).\max(0, \min(\frac{b}{d}, 1 - \frac{c}{a})).$$

If $a \ge c$, then $1 - \frac{c}{a} \ge 0$ and

$$X = \frac{c}{a} + \mathrm{sg}(d).\max(0, \min(\frac{b}{d}, 1 - \frac{c}{a})).$$

If $d = 0$, then

$$X = \frac{c}{a} \le 1.$$

If $d > 0$, then

$$X = \frac{c}{a} + \max(0, \min(\frac{b}{d}, 1 - \frac{c}{a}))$$

$$= \frac{c}{a} + \min(\frac{b}{d}, 1 - \frac{c}{a})$$

$$\le \frac{c}{a} + 1 - \frac{c}{a} = 1.$$

If $a < c$, then $\frac{c}{a} > 1$, $1 - \frac{c}{a} < 0$ and

$$X = 1 + \mathrm{sg}(d).\max(0, 1 - \frac{c}{a}) = 1.$$

Therefore, the definition is correct.

From the new implication, using formula

$$\neg x = x \to \langle 0, 1 \rangle,$$

we generate the following negation:

$$V(\neg x) = \langle \overline{sg}(a), \mathrm{sg}(a).b \rangle$$

It does not have analogues among the other intuitionistic fuzzy negations.

It is well known (see, e.g. [10]) that Modus Ponens has the form

$$\frac{x, \;\; x \to y}{y}.$$

Theorem 1. The new implication

(a) satisfies Modus Ponens in the case of tautology,

(b) does not satisfy Modus Ponens in the IFT-case.

Proof: (a). Let x and $x \to y$ be tautologies. Then

$$a = 1, b = 0,$$

$$\max(\overline{sg}(1 - c), \min(1, c)) = 1,$$

$$\min(sg(1 - c), \mathrm{sg}(d). \max(0, \min(\frac{b}{d}, 1 - c)) = 0.$$

Therefore, $c = 1$ and hence $d = 0$, i.e. y is a tautology.

(b) It is seen easy that from $a \geq b$ and

$$\max(\overline{sg}(a - c), \mathrm{sg}(a). \min(1, \frac{c}{a}))$$

$$\geq \min(sg(a - c), \mathrm{sg}(a).\mathrm{sg}(d). \max(0, \min(\frac{b}{d}, 1 - \frac{c}{a})))$$

the inequality $c \geq d$ does not follow.

In [9], the following properties of the implications and negations are givens:

Property P1: $A \to \neg\neg A$,

Property P2: $\neg\neg A \to A$,

Property P3: $\neg\neg\neg A = \neg A$.

Theorem 2. The new implication and negation satisfy only Property P1.

The checks of Properties P1 and P2 are similar to the above ones. The check of Property 3 is based on the equalities

$$\neg\neg\neg\langle a, b\rangle = \neg\neg\langle \overline{sg}(a), \mathrm{sg}(a).b\rangle = \neg\langle \mathrm{sg}(a), 0\rangle = \langle \overline{sg}(a), 0\rangle.$$

Some variants of fuzzy implications (marked by $I(x, y)$) are described in book [9] by George Klir and Bo Yuan, and the following nine axioms are discussed, where

$$I(x, y) \equiv x \to y$$

and

$$N(x) \equiv I(x, 0).$$

Axiom 1: $(\forall x, y)(x \leq y \to (\forall z)(I(x, z) \geq I(y, z))$.
Axiom 2: $(\forall x, y)(x \leq y \to (\forall z)(I(z, x) \leq I(z, y))$.
Axiom 3: $(\forall y)(I(\overline{0}, y) = \overline{1})$.
Axiom 4: $(\forall y)(I(\overline{1}, y) = y)$.
Axiom 5: $(\forall x)(I(x, x) = \overline{1})$.
Axiom 6: $(\forall x, y, z)(I(x, I(y, z)) = I(y, I(x, z)))$.
Axiom 7: $(\forall x, y)(I(x, y) = \overline{1}$ iff $x \leq y)$.
Axiom 8: $(\forall x, y)(I(x, y) = I(N(y), N(x)))$.
Axiom 9: I is a continuous function,
where

$$V(\overline{0}) = \langle 0, 1 \rangle,$$

$$V(\overline{1}) = \langle 1, 0 \rangle.$$

Theorem 3: The new implication satisfies axioms 3 and 5.

The validity of this assertion follows form the definition of the new implication.

Now, we shall modify two of the above axioms:
Axiom 3': $(\forall y)(I(0, y)$ is an IFT).
Axiom 5': $(\forall x)(I(x, x)$ is an IFT).
Theorem 4: The new implication satisfies axioms 3' and 5'.

The proofs are analogous of the above ones.

3 Conclusion

The new implication and negation have some unique properties with respect to the other already existing intuitionistic fuzzy logic implications and negations.

In a next research intuitionistic fuzzy set analogues of the new intuitionistic fuzzy logic implication and negation will be discussed.

References

[1] Atanassov K., Two variants of intuitionistc fuzzy propositional calculus. *Preprint IM-MFAIS-5-88,* Sofia, 1988.

[2] Atanassov, K. *Intuitionistic Fuzzy Sets.* Springer Physica-Verlag, Heidelberg, 1999.

[3] Atanassov, K. Intuitionistic fuzzy implications and Modus Ponens. *Notes on Intuitionistic Fuzzy Sets,* Vol. 11, 2005, No. 1, 1-5.

[4] Atanassov, K., On some types of intuitionistic fuzzy negations. *Notes on Intuitionistic Fuzzy Sets,* Vol. 11, 2005, No. 4, 170-172.

[5] Atanassov, K. A new intuitionistic fuzzy implication from a modal type. *Advanced Studies in Contemporary Mathematics,* Vol. 12, 2006, No. 1, 117-122.

[6] Atanassov, K. On some intuitionistic fuzzy implications. *Comptes Rendus de l'Academie bulgare des Sciences,* Tome 59, 2006 , No. 1, 19-24.

[7] Dimitrov, D. On intuitionistic logic axioms and intuitionistic fuzzy implications and negations. *Issues in Intuitionistic Fuzzy Sets and Generalized Nets.* Vol. 7, Warsaw School of Information Technology (in press).

[8] Dimitrov, D., K. Atanassov. On intuitionistic logic axioms and intuitionistic fuzzy implications and negations. *Issues in Intuitionistic Fuzzy Sets and Generalized Nets.* Vol. 7, Warsaw School of Information Technology (in press).

[9] Klir, G. and Bo Yuan, *Fuzzy Sets and Fuzzy Logic.* Prentice Hall, New Jersey, 1995.

[10] Rasiova H., R. Sikorski. *The Mathematics of Metamathematics,* Warszawa, Pol. Acad. of Sci., 1963.

The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Tenth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2011) organized in Warsaw on September 30, 2011 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Tenth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2011) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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