Recent Advances in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics
Volume II: Applications

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ISBN 9788389475367

Design of interval type-2 fuzzy systems

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Abstract

Type-2 fuzzy sets are used for modeling uncertainty and imprecision in a better way. These type-2 fuzzy sets were originally presented by Zadeh in 1975 and are essentially "fuzzy fuzzy" sets where the fuzzy degree of membership is a type-1 fuzzy set. The new concepts were introduced by Mendel and Liang allowing the characterization of a type-2 fuzzy set with an upper membership function and a lower membership function; and these two functions can be represented each by a type-1 fuzzy set membership function. The interval between these two functions represents the footprint of uncertainty (FOU), which is used to characterize a type-2 fuzzy set.

Keywords: interval type-2 fuzzy logic, type-2 fuzzy models, high order fuzzy sets.

1 Introduction

On the past decade, fuzzy systems have displaced conventional technology in different scientific and system engineering applications, especially in pattern recognition and control systems. The same fuzzy technology, in approximation reasoning form, is resurging also in the information technology, where it is now giving support to decision making and expert systems with powerful reasoning capacity and a limited quantity of rules.

The fuzzy sets were presented by L.A. Zadeh in 1965 [10,11,12] to process/manipulate data and information affected by unprobabilistic uncertainty/imprecision. These were designed to mathematically represent the vagueness and uncertainty of linguistic problems; thereby obtaining formal tools to work with intrinsic imprecision in different type of problems; it is considered a generalization of the classic set theory.

Recent Advances in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume II: Applications *(K.T. Atanassow, W.Homenda, O.Hryniewicz, J.Kacprzyk, M.Krawczak, Z.Nahorski, E. Szmidt, S. Zadrożny, Eds.), IBS PAN - SRI PAS, Warsaw, 2010.*

Intelligent Systems based on fuzzy logic are fundamental tools for nonlinear complex system modeling. The fuzzy sets and fuzzy logic are the base for fuzzy systems, where their objective has been to model how the brain manipulates inexact information [4].

Type-2 fuzzy sets are used for modeling uncertainty and imprecision in a better way. These type-2 fuzzy sets were originally presented by Zadeh in 1975 and are essentially "fuzzy fuzzy" sets where the fuzzy degree of membership is a type-1 fuzzy set [5, 13]. The new concepts were introduced by Mendel and Liang [6, 7] allowing the characterization of a type-2 fuzzy set with a superior membership function and an inferior membership function; these two functions can be represented each one by a type-1 fuzzy set membership function. The interval between these two functions represents the footprint of uncertainty (FOU), which is used to characterize a type-2 fuzzy set.

The uncertainty is the imperfection of knowledge about the natural process or natural state. The statistical uncertainty is the randomness or error that comes from different sources as we use it in a statistical methodology. There are different sources of uncertainty in the evaluation and calculus process. The five types of uncertainty that emerge from the imprecise knowledge natural state are:

- Measurement uncertainty. It is the error on observed quantities.
- Process uncertainty. It is the dynamic randomness.
- Model uncertainty. It is the wrong specification of the model structure.
- Estimate uncertainty. It is the one that can appear from any of the previous uncertainties or a combination of them, and it is called inexactness and imprecision.
- Implementation uncertainty. It is the consequence of the variability that results from sorting politics, i.e. incapacity to reach the exact strategic objective.

2 Interval type-2 fuzzy set theory

A type-2 fuzzy set [5,15] expresses the non-deterministic truth degree with imprecision and uncertainty for an element that belongs to a set. A type-2 fuzzy set denoted by $\tilde{\tilde{A}}$, is characterized by a type-2 membership function $\mu_{\tilde{\tilde{A}}}(x,u)$, where x$\in$X, u $\in J_x^u \subseteq [0,1]$ and $0 \leq \mu_{\tilde{\tilde{A}}}(x,u) \leq 1$ defined in equation (1).

$$\tilde{\tilde{A}} = \left\{ (x, \mu_{\tilde{\tilde{A}}}(x)) \mid x \in X \right\}$$
$$= \left\{ \left(x, u, \mu_{\tilde{\tilde{A}}}(x,u) \right) \mid \forall x \in X, \forall u \in J_x^u \subseteq [0,1] \right\} \quad (1)$$

An example of a type-2 membership function constructed in the IT2FLS toolbox was composed by a Pi primary and a Gbell secondary type-1 membership functions, these are depicted in Fig.1.

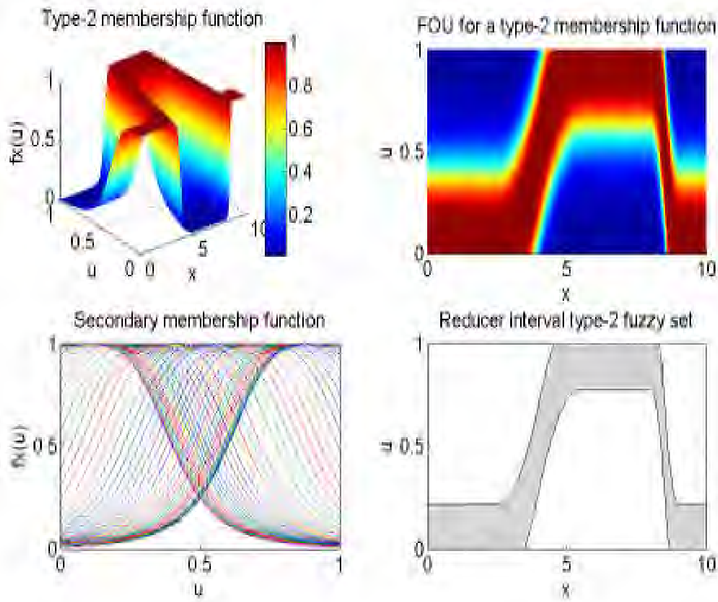

Figure 1: FOU for Type-2 Membership Functions

If $\tilde{\tilde{A}}$ is continuous it is denoted in equation (2).

$$\tilde{\tilde{A}} = \left\{ \int_{x \in X} \left[\int_{u \in J_x^u \subseteq [0,1]} f_x(u) / u \right] / x \right\} \quad (2)$$

where $\int\int$ denotes the union of x and u. If $\tilde{\tilde{A}}$ is discrete then it is denoted by equation (3).

$$\tilde{\tilde{A}} = \left\{ \sum_{x \in X} \mu_{\tilde{\tilde{A}}}(x) / x \right\} = \left\{ \sum_{i=1}^{N} \left[\sum_{k=1}^{M_i} f_{x_i}(u_{ik}) / u_{ik} \right] / x_i \right\} \quad (3)$$

where $\sum\sum$ denotes the union of x and u.

If $f_x(u)=1, \forall u \in [\underline{J}_x^u, \overline{J}_x^u] \subseteq [0,1]$, the type-2 membership function $\mu_{\tilde{A}}(x,u)$ is expressed by one type-1 inferior membership function, $\underline{J}_x^u \equiv \underline{\mu}_A(x)$ and one type-1 superior, $\overline{J}_x^u \equiv \overline{\mu}_A(x)$ (Fig. 2), then it is called an interval type-2 fuzzy set [6] denoted by equations (4) and (5).

$$\tilde{\tilde{A}} = \begin{cases} (x,u,1) \mid \forall x \in X, \\ \qquad \forall u \in [\underline{\mu}_{\tilde{A}}(x), \overline{\mu}_{\tilde{A}}(x)] \subseteq [0,1] \end{cases} \tag{4}$$

or

$$\begin{aligned} \tilde{\tilde{A}} &= \left\{ \int_{x \in X} \left[\int_{u \in [\underline{J}_x^u, \overline{J}_x^u] \subseteq [0,1]} 1/u \right] / x \right\} \\ &= \left\{ \int_{x \in X} \left[\int_{u \in [\underline{\mu}_{\tilde{A}}(x), \overline{\mu}_{\tilde{A}}(x)] \subseteq [0,1]} 1/u \right] / x \right\} \end{aligned} \tag{5}$$

If $\tilde{\tilde{A}}$ is a type-2 fuzzy Singleton, the membership function is defined by equation (6).

$$\mu_{\tilde{\tilde{A}}}(x) = \begin{cases} 1/1 & si\ x = x' \\ 1/0 & si\ x \neq x' \end{cases} \tag{6}$$

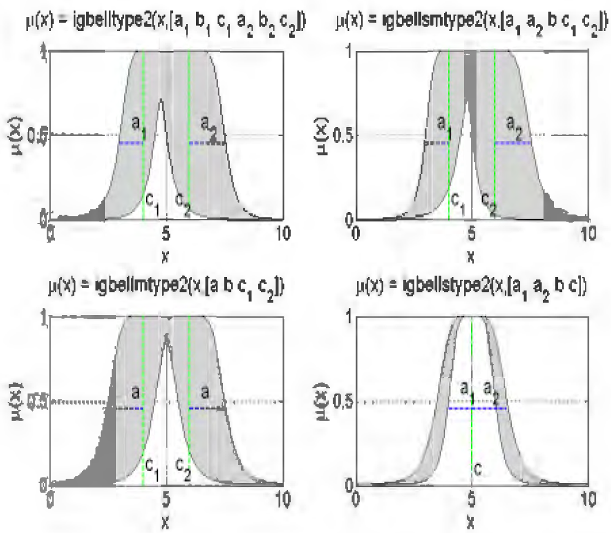

Figure 2: FOU for Gbell Primary Interval Type-2 Membership Functions

We can apply some operators to the fuzzy sets, or we can make some operations between them [7,8,13]. When we apply an operator to one fuzzy set we obtain another fuzzy set; by the same manner when we combine an operation with two or more sets we obtain another fuzzy set. If we have two type-2 fuzzy subsets identified by the letters $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$, associated to a linguistic variable, we can define three basic operations: complement, union and intersection (Table 1).

Table 1. Interval Type-2 Fuzzy Set Operations.

NAME	OPERATOR	OPERATION
Union	$\sqcup$ = join	$\tilde{\tilde{A}} \sqcup \tilde{\tilde{B}} = \{\int_{x\in X} \mu_{\tilde{\tilde{A}}}(x) \sqcup \mu_{\tilde{\tilde{B}}}(x)/x\}$ $= \left\{ \int_{x\in X} \left[\int_{\alpha\in[\underline{\mu}_{\tilde{A}(x)} \vee \underline{\mu}_{\tilde{B}(x)}, \overline{\mu}_{\tilde{A}(x)} \vee \overline{\mu}_{\tilde{B}(x)}]} 1/\alpha \right] / x \right\}$
Intersection	$\sqcap$= meet	$\tilde{\tilde{A}} \sqcap \tilde{\tilde{B}} = \{\int_{x\in X} \mu_{\tilde{\tilde{A}}}(x) \sqcap \mu_{\tilde{\tilde{B}}}(x)/x\}$ $= \left\{ \int_{x\in X} \left[\int_{\alpha\in[\underline{\mu}_{\tilde{A}(x)} \wedge \underline{\mu}_{\tilde{B}(x)}, \overline{\mu}_{\tilde{A}(x)} \wedge \overline{\mu}_{\tilde{B}(x)}]} 1/\alpha \right] / x \right\}$
Negation	$\neg$	$\neg\tilde{\tilde{A}} = \left\{ \int_{x\in X} \mu_{\neg\tilde{\tilde{A}}}(x)/x \right\}$ $= \left\{ \int_{x\in X} \left[\int_{\alpha\in[1-\overline{\mu}_{\tilde{A}}(x), 1-\underline{\mu}_{\tilde{A}}(x)]} 1/\alpha \right] / x \right\}$

The human knowledge is expressed in fuzzy rule terms with the next syntax:

IF *<fuzzy proposition>* **THEN** *<fuzzy proposition>*.

The fuzzy propositions are divided in two types, the first one is named **atomic: x is A**, where x is a linguistic variable and A is a linguistic value; the second one is called **compounded: x is A AND y is B OR z is NOT C**, this is a compounded atomic fuzzy proposition with the “AND”, “OR” and “NOT” connectors, representing fuzzy intersection, union and complement respectively. The compounded fuzzy propositions are fuzzy relationships. The membership function of the rule IF-THEN is a fuzzy relation determined by a fuzzy implication operator. The fuzzy rules combine one or more fuzzy sets of entry, called antecedent, and are associated with one output fuzzy set, called consequents. The Fuzzy Sets of the antecedent are associated by fuzzy operators AND, OR, NOT and linguistic modifiers. The fuzzy rules permit expressing the available knowledge about the relationship between antecedent and consequents. To express this knowledge completely we normally have several rules, grouped to form what it is known a rule base, that is, a set of rules that express the known relationships between antecedent and consequents. The fuzzy rules are basically IF <Antecedent> THEN <Consequent> and expresses a fuzzy relationship or proposition.

In fuzzy logic the reasoning is imprecise, it is approximated, that means that we can infer from one rule a conclusion even if the antecedent doesn't comply completely. We can count on two basic inference methods between rules and inference laws, Generalized Modus Ponens (GMP) [5,6,13,14] and Generalized Modus Tollens (GMT), that represent the extensions or generalizations of classic reasoning. The GMP inference method is known as direct reasoning and is resumed as:

Rule	*IF x is A THEN y is B*
Fact	*x is A'*
Conclusion	*yes B'*

Where A, A', B and B' are fuzzy sets of any kind. This relationship is expressed as $B' = A' \circ (A \rightarrow B)$. Fig. 3 shows an example of Interval Type-2 direct reasoning with Interval Type-2 Fuzzy Inputs.

An Inference Fuzzy System is a rule base system that uses fuzzy logic, instead of Boolean logic utilized in data analysis [3, 7, 13]. Its basic structure includes four components (Fig. 4):

- ***Fuzzifier.*** Translates inputs (real values) to fuzzy values.
- ***Inference System.*** Applies a fuzzy reasoning mechanism to obtain a fuzzy output.
- ***Type Defuzzifier/Reducer.*** The defuzzifier traduces one output to precise values; the type reducer transforms a Type-2 Set into a Type-1 Fuzzy Set.
- ***Knowledge Base.*** Contains a set of fuzzy rules, and a membership functions set known as the database.

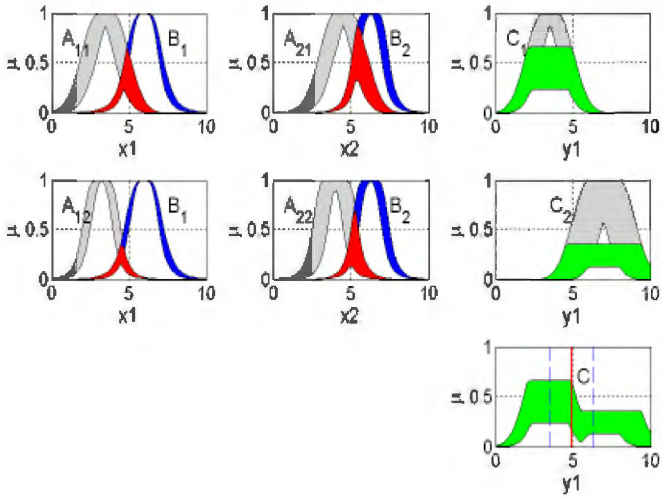

Figure 3: Interval Type-2 Fuzzy Reasoning

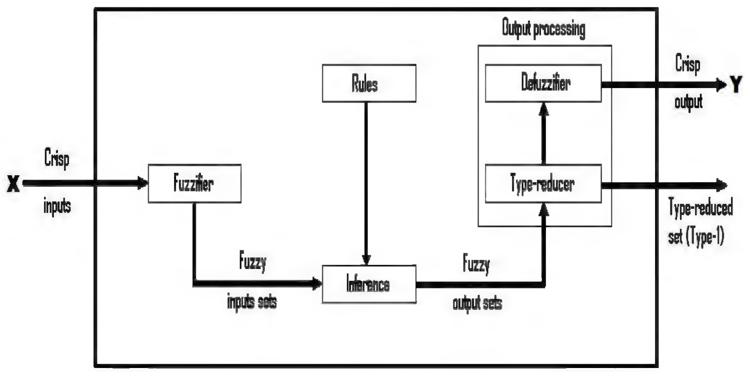

Figure 4: Type-2 Inference Fuzzy System Structure

The decision process is a task that identifies parameters by the inference system using the rules of the rule base data. These fuzzy rules define the connection between the input and output fuzzy variables. A fuzzy rule has the form: IF <Antecedent> THEN <Consequent>, where antecedent is a compound fuzzy logic expression of one or more simple fuzzy expressions connected with fuzzy operators; and the consequent is an expression that assigns fuzzy values to output variables. The inference system evaluates all the rules of the rule base and combines the weights of the consequents of all relevant rules in one fuzzy set using the aggregate operation. This operation is analog in fuzzy logic to the S-norm operator.

Fuzzy modeling is a task for parameter identification in a fuzzy inference system to obtain an adequate behavior. A fuzzy model with the direct view is constructed with the knowledge of an expert. This task becomes more difficult when the available knowledge is incomplete or when space is a problem, then the use of automatic views are recommended for the fuzzy model. It can be considered different point of views for fuzzy modeling, based on neural networks, genetic algorithms and hybrid methods. The selection of relevant variables and adequate rules is critical for generating a good system. One of the biggest problems occurring in fuzzy modeling is dimensionality, that is, when the computational requirements grow exponentially in relation of the quantity of variables.

3 Interval type-2 fuzzy system design

The Mamdani and Takagi-Sugeno-Kang (TSK) Interval Type-2 Fuzzy Inference Models [1] and the design of Interval Type-2 membership functions and operators are implemented in the IT2FLS Toolbox (Interval Type-2 Fuzzy Logic Systems) reused from the Matlab® commercial Fuzzy Logic Toolbox [3].

The Interval Type-2 Fuzzy Inference Systems (IT2FIS) structure is the MATLAB object that contains all the interval type-2 fuzzy inference system information. This structure is stored inside each GUI tool. Access functions such as getifistype2 and setifistype2 make it easy to examine this structure.

The implementation of the IT2FLS GUI is analogous to the GUI used for Type-1 FLS in the Matlab® Fuzzy Logic Toolbox, thus permitting the experienced user to adapt easily to the use of IT2FLS GUI. Figures 5 and 6 show the main screen of the Interval Type-2 Fuzzy Inference Systems Structure Editor called IT2FIS (Interval Type-2 Fuzzy Inference Systems).

The Mamdani IT2FIS, is designed with **n** inputs, **m** outputs and **r** rules. The *k*th rule with interval type-2 fuzzy antecedents $\tilde{\tilde{A}}_{k,i} \in \{\mu_{i,l_{k,i}}\}$, interval type-2 fuzzy consequent $\tilde{\tilde{C}}_{k,j} \in \{\sigma_{j,l_{k,j}}\}$ and interval type-2 fuzzy facts $\tilde{\tilde{A}}_i'$ are inferred as a direct reasoning [1, 7]. The evaluation of this type of reasoning is as follows:

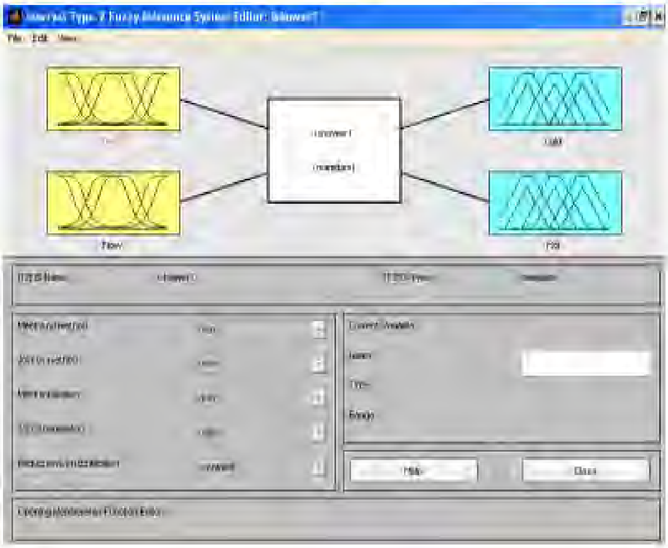

Figure 5: IT2FIS Editor

Figure 6: Interval Type-2 MF's Editor

$\tilde{\tilde{R}}_{k,j} = \tilde{\tilde{A}}_{k,1} \times \ldots \times \tilde{\tilde{A}}_{k,n} \to \tilde{\tilde{C}}_{k,j} = (\tilde{\tilde{A}}_{k,1} \to \tilde{\tilde{C}}_{k,j}) \times \ldots \times (\tilde{\tilde{A}}_{k,n} \to \tilde{\tilde{C}}_{k,j})$, kth rule.

$\tilde{\tilde{H}} = \tilde{\tilde{A}}'_1 \times \ldots \times \tilde{\tilde{A}}'_n$, facts.

$$\tilde{\tilde{C}}''_{k,j} = \tilde{\tilde{H}} \circ \tilde{\tilde{R}}_{k,j} = \sqcap_{i=1}^{n} [\tilde{\tilde{A}}'_i \circ (\tilde{\tilde{A}}_{k,i} \to \tilde{\tilde{C}}_{k,j})] \tag{7}$$

$$= \left\{ \int_Y \left[\int_{\alpha \in [\underline{\mu}_{\tilde{\tilde{C}}''_{k,j}}(y_j), \overline{\mu}_{\tilde{\tilde{C}}''_{k,j}}(y_j)] \subseteq [0,1]} 1/\alpha \right] / y_j \right\}$$

$$\underline{\mu}_{\tilde{\tilde{C}}''_{k,j}}(y_j) = \left[\mathop{\tilde{*}}_{i=1}^{n} \left(\underline{\mu}_{\tilde{\tilde{A}}'_i}(x_i) \,\tilde{*}\, \underline{\mu}_{\tilde{\tilde{A}}_{k,i}}(x_i) \right) \right] \tilde{*}\, \underline{\mu}_{\tilde{\tilde{C}}_{k,j}}(y_j) \tag{8}$$

$$\overline{\mu}_{\tilde{\tilde{C}}''_{k,j}}(y_j) = \left[\mathop{\tilde{*}}_{i=1}^{n} \left(\overline{\mu}_{\tilde{\tilde{A}}'_i}(x_i) \,\tilde{*}\, \overline{\mu}_{\tilde{\tilde{A}}_{k,i}}(x_i) \right) \right] \tilde{*}\, \overline{\mu}_{\tilde{\tilde{C}}_{k,j}}(y_j) \tag{9}$$

$$\tilde{\tilde{C}}'_{j} = \sqcup_{k=1}^{r} \tilde{\tilde{C}}''_{k,j} = \sqcup_{k=1}^{r} [\sqcap_{i=1}^{n} [\tilde{\tilde{A}}'_i \circ (\tilde{\tilde{A}}_{k,i} \to \tilde{\tilde{C}}_{k,j})]]$$

$$= \left\{ \int_Y \left[\int_{\alpha \in [\underline{\mu}_{\tilde{\tilde{C}}'_{j}}(y_j), \overline{\mu}_{\tilde{\tilde{C}}'_{j}}(y_j)] \subseteq [0,1]} 1/\alpha \right] / y_j \right\} \tag{10}$$

$$\underline{\mu}_{\tilde{\tilde{C}}'_{j}}(y_j) = \bigvee_{k=1}^{r} \left(\underline{\mu}_{\tilde{\tilde{C}}''_{k,j}}(y_j) \right)$$

$$= \bigvee_{k=1}^{r} \left(\left[\mathop{\tilde{*}}_{i=1}^{n} \left(\underline{\mu}_{\tilde{\tilde{A}}'_i}(x_i) \,\tilde{*}\, \underline{\mu}_{\tilde{\tilde{A}}_{k,i}}(x_i) \right) \right] \tilde{*}\, \underline{\mu}_{\tilde{\tilde{C}}_{k,j}}(y_j) \right)$$

$$\overline{\mu}_{\tilde{\tilde{C}}'_{j}}(y_j) = \bigvee_{k=1}^{r} \left(\overline{\mu}_{\tilde{\tilde{C}}''_{k,j}}(y_j) \right)$$

$$= \bigvee_{k=1}^{r} \left(\left[\mathop{\tilde{*}}_{i=1}^{n} \left(\overline{\mu}_{\tilde{\tilde{A}}'_i}(x_i) \,\tilde{*}\, \overline{\mu}_{\tilde{\tilde{A}}_{k,i}}(x_i) \right) \right] \tilde{*}\, \overline{\mu}_{\tilde{\tilde{C}}_{k,j}}(y_j) \right)$$

The defuzzification of the interval type-2 fuzzy aggregated output set $\tilde{\tilde{C}}_j'$ is: $\hat{y}_j = idefuzztype2(\mu_{\tilde{\tilde{C}}_j'}(y_j), 'type')$ where *type* is the name of the defuzzification technique. If $\tilde{\tilde{A}}_i'$ are interval type-2 fuzzy singletons then:

$$\mu_{\tilde{\tilde{C}}_{k,j}''}(y_j) = \left[\sqcap_{i=1}^{n} [\mu_{\tilde{\tilde{A}}_{k,i}}(x_i)] \right] \sqcap \mu_{\tilde{\tilde{C}}_{k,j}}(y_j) \tag{11}$$

$$= \left\{ \int_Y \left[\int_{\alpha \in [\underline{\mu}_{\tilde{\tilde{C}}_{k,j}''}(y_j), \overline{\mu}_{\tilde{\tilde{C}}_{k,j}''}(y_j)] \subseteq [0,1]} 1/\alpha \right] / y_j \right\}$$

$$\underline{\mu}_{\tilde{C}_{k,j}''}(y_j) = [\tilde{*}_{i=1}^{n} \underline{\mu}_{\tilde{A}_{k,i}}(\hat{x}_i)] \tilde{*} \underline{\mu}_{\tilde{C}_{k,j}}(y_j) \tag{12}$$

$$\overline{\mu}_{\tilde{C}_{k,j}''}(y_j) = [\tilde{*}_{i=1}^{n} \overline{\mu}_{\tilde{A}_{k,i}}(\hat{x}_i)] \tilde{*} \overline{\mu}_{\tilde{C}_{k,j}}(y_j) \tag{13}$$

$$\mu_{\tilde{\tilde{C}}_j'}(y_j) = \sqcup_{k=1}^{r} \mu_{\tilde{\tilde{C}}_{k,j}''}(y_j) = \sqcup_{k=1}^{r} [\sqcap_{i=1}^{n} [\mu_{\tilde{\tilde{A}}_{k,i}}(x_i)] \sqcap \mu_{\tilde{\tilde{C}}_{k,j}}(y_j)]] \tag{14}$$

$$= \left\{ \int_Y \left[\int_{\alpha \in [\underline{\mu}_{\tilde{\tilde{C}}_j'}(y_j), \overline{\mu}_{\tilde{\tilde{C}}_j'}(y_j)] \subseteq [0,1]} 1/\alpha \right] / y_j \right\}$$

$$\underline{\mu}_{\tilde{\tilde{C}}_j'}(y_j) = \bigvee_{k=1}^{r} [\underline{\mu}_{\tilde{\tilde{C}}_{k,j}''}(y_j)] = \bigvee_{k=1}^{r} \left([\tilde{*}_{i=1}^{n} \underline{\mu}_{\tilde{\tilde{A}}_{k,i}}(\hat{x}_i)] \tilde{*} \underline{\mu}_{\tilde{\tilde{C}}_{k,j}}(y_j) \right)$$

$$\overline{\mu}_{\tilde{\tilde{C}}_j'}(y_j) = \bigvee_{k=1}^{r} [\overline{\mu}_{\tilde{\tilde{C}}_{k,j}''}(y_j)] = \bigvee_{k=1}^{r} \left([\tilde{*}_{i=1}^{n} \overline{\mu}_{\tilde{\tilde{A}}_{k,i}}(\hat{x}_i)] \tilde{*} \overline{\mu}_{\tilde{\tilde{C}}_{k,j}}(y_j) \right) \tag{15}$$

The IT2FIS de Takagi-Sugeno-Kang system is designed with **n** inputs, **m** outputs and **r** rules. The *k*th rule with interval type-2 fuzzy antecedents $\tilde{\tilde{A}}_{k,i} \in \{\mu_{i,l_{k,i}}\}$, interval type-1 fuzzy set are used for the consequents sets, $f_{j,k} = \theta_{0,j}^k + \sum_{i=1}^{n} \theta_{i,j}^k \cdot x_i$ and real facts are inferred as a direct reasoning [9, 10]. The evaluation of this reasoning is:

$$\alpha_k = [\underline{\alpha}_k, \bar{\alpha}_k] = \sqcup_{i=1}^{n} [\ \mu_{\tilde{A}_{k,i}}(\hat{x}_i)]$$
$$= \left[\tilde{*}_{i=1}^{n} \left(\underline{\mu}_{\tilde{A}_{k,i}}(\hat{x}_i) \right), \tilde{*}_{i=1}^{n} \left(\bar{\mu}_{\tilde{A}_{k,i}}(\hat{x}_i) \right) \right] \tag{16}$$

where $\alpha_k = [\underline{\alpha}_k, \bar{\alpha}_k]$ is the firing set of the interval type-1 fuzzy antecedent of the *k*th rule.

$$f_{j,k} = \theta_{0,j}^{k} + \sum_{i=1}^{n} \theta_{i,j}^{k} \cdot \hat{x}_i$$

where $f_{j,k} = [{}^{l}f_{j,k}, {}^{r}f_{j,k}]$ is a real function of the interval consequents of the *k*th rule. If $\theta_{i,j}^{k} = [c_{i,j}^{k} - s_{i,j}^{k}, c_{i,j}^{k} + s_{i,j}^{k}]\ \ \forall\, i = 0,...,n$, where $c_{i,j}^{k}$ is the center and $s_{i,j}^{k}$ denotes the spread, then ${}^{l}f_{j,k}, {}^{r}f_{j,k}$ is expressed as:

$${}^{l}f_{j,k} = \sum_{i=1}^{n} c_{i,j}^{k} \cdot x_i + c_{0,j}^{k} - \sum_{i=1}^{n} s_{i,j}^{k} \cdot |x_i| - s_{0,j}^{k}$$
$${}^{r}f_{j,k} = \sum_{i=1}^{n} c_{i,j}^{k} \cdot x_i + c_{0,j}^{k} + \sum_{i=1}^{n} s_{i,j}^{k} \cdot |x_i| + s_{0,j}^{k} \tag{17}$$

With the Karnik and Mendel algorithm [10] the ${}^{l}\alpha_k$ and ${}^{r}\alpha_k$ are evaluated to obtain the FIS output variables, these are expressed as

$$\hat{y}_j^{l} = \frac{\sum_{k=1}^{r} {}^{l}\alpha_k \cdot {}^{l}f_{j,k}}{\sum_{k=1}^{r} {}^{l}\alpha_k} = \frac{\sum_{k=1}^{L} \bar{\alpha}_k \cdot {}^{l}f_{j,k} + \sum_{k=L+1}^{r} \underline{\alpha}_k \cdot {}^{l}f_{j,k}}{\sum_{k=1}^{L} \bar{\alpha}_k + \sum_{k=L+1}^{r} \underline{\alpha}_k}$$
$$\hat{y}_j^{r} = \frac{\sum_{k=1}^{r} {}^{r}\alpha_k \cdot {}^{r}f_{j,k}}{\sum_{k=1}^{r} {}^{r}\alpha_k} = \frac{\sum_{k=1}^{R} \underline{\alpha}_k \cdot {}^{r}f_{j,k} + \sum_{k=R+1}^{r} \bar{\alpha}_k \cdot {}^{r}f_{j,k}}{\sum_{k=1}^{R} \underline{\alpha}_k + \sum_{k=R+1}^{r} \bar{\alpha}_k} \tag{18}$$
$$\hat{y}_j = \frac{\hat{y}_j^{l} + \hat{y}_j^{r}}{2}$$

There are also other methods to obtain the previous upper and lower values of the output variables, which include approximate methods based on genetic algorithms or neural networks [1, 2].

4 Experimental results

The experimental results are devoted to show comparisons in the system's response in a feedback controller when using a type-1 FLC or a type-2 FLC. A set of five experiments is described in this section. The first two experiments were performed in ideal conditions, i.e., without any kind of disturbance. In the last three experiments, Gaussian noise was added to the feedback loop with the purpose of simulating, in a global way, the effects of uncertainty from several sources. The complete system was simulated in Matlab, and the controller was designed to follow the input as closely as possible. The plant is a nonlinear system modeled with equation:

$$y(i)=0.2\cdot y(i-3)\cdot 0.07y(i-2)+0.9\cdot y(i-1)+0.05\cdot u(i-1)+0.5\cdot u(i-2) \tag{19}$$

To illustrate the dynamics of the system, two different inputs are applied, first the input of equation:

Going back to the control problem, this system given by equation (19) was used in Figure 1, under the name of plant or process, in this figure we can see that the controller's output is applied directly to the plant's input. Since we are interested in comparing the performance between type-1 and type-2 FLC systems, the controller was tested in two ways:

1. One is considering the system as ideal, i.e., not introducing in the modules of the control system any source of uncertainty (experiments 1 and 2).
2. The other one is simulating the effects of uncertain modules (subsystems) response introducing some uncertainty (experiments 3, 4 and 5).

For both cases, the system's output is directly connected to the summing junction, but in the second case, the uncertainty was simulated introducing random noise with normal distribution. We added noise to the system's output $y(i)$ using the Matlab's function "randn", which generates random numbers with Gaussian distribution. The signal and the added noise in turn, were obtained with the programmer's expression (22), the result $y(i)$ was introduced to the summing junction of the controller system. Note that in expression (20) we are using the value 0.05, for experiments 3 and 4, but in the set of tests for experiment 5, we varied this value to obtain different SNR values.

$$y(i) = y(i) + 0.05 \cdot randn \tag{20}$$

The system was tested using as input, a unit step sequence free of noise, $r(i)$. For evaluating the system's response and comparing between type 1 and type 2 fuzzy controllers, the performance criteria of Integral of Squared Error (ISE), Integral of Absolute Value of Error (IAE), and Integral of Time per Absolute Value of Error (ITAE) were used. In Table 3, we summarize the values obtained in an ideal system for each criterion considering 400 units of time. For calculating ITAE a sampling time of $T_s = 0.1$ sec. was considered. In Experiment 5, we tested the systems, type-1 and type-2 FLCs, introducing different values of noise η, this was done by modifying the signal to noise ratio SNR [22],

$$SNR = \frac{\sum |s|^2}{\sum |\eta|^2} = \frac{P_{signal}}{P_{noise}} \tag{21}$$

Because many signals have a very wide dynamic range, SNRs are usually expressed in terms of the logarithmic decibel scale, SNR(db),

$$SNR(db) = 10 \log_{10} \left(\frac{P_{signal}}{P_{noise}} \right) \tag{22}$$

In Table 4, we show, for different values of SNR(db), the behavior of the errors ISE, IAE, ITAE for type-1 and type-2 FLCs. In all the cases the results for type-2 FLC are better than type-1 FLC. In the type-1 FLC, Gaussian membership functions (Gaussian MFs) for the inputs and for the output were used. A Gaussian MF is specified by two parameters $\{c,\sigma\}$:

$$\mu_A(x) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2} \tag{23}$$

c represents the MFs center and σ determines the MFs standard deviation.

For each of the inputs of the type-1 FLC, three Gaussian MFs were defined as: negative, zero, positive. The universe of discourse for these membership functions is in the range [-10 10]. For the output of the type-1 FLC, we have five Gaussian MFs: NG, N, Z, P and PG. Table 1 illustrates the characteristics of the MFs of the inputs and output of the type-1 FLC.

Table 1. Characteristics of the Inputs and Output of the Type-1 FLC

Variable	**Term**	**Center** c	**Standard deviation** σ
Input e	negative	-10	4.2466
	zero	0	4.2466
	positive	10	4.2466
Input Δe	Negative	-10	4.2466
	Zero	0	4.2466
	positive	10	4.2466
Output cde	NG	-10	2.1233
	N	-5	2.1233
	Z	0	2.1233
	P	5	2.1233
	PG	10	2.1233

In experiments 2, 4, and 5, for the type-2 FLC, as in type-1 FLC, we also selected Gaussian MFs for the inputs and for the output, but in this case we have interval type-2 Gaussian MFs with a fixed center, c, and an uncertain standard deviation, σ, i.e.,

$$\mu_A(x) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2} \tag{24}$$

In terms of the upper and lower membership functions, we have for $\overline{\mu}_{\tilde{A}}(x)$,

$$\overline{\mu}_{\tilde{\mathrm{A}}}(\mathrm{x}) = \mathrm{N}(\mathrm{c}, \sigma_2; \mathrm{x}) \tag{25}$$

and for the lower membership function $\underline{\mu}_{\tilde{A}}(x)$,

$$\underline{\mu}_{\tilde{\mathrm{A}}}(\mathrm{x}) = \mathrm{N}(\mathrm{c}, \sigma_1; \mathrm{x}) \tag{26}$$

where $\mathrm{N}(\mathrm{c}, \sigma_2, \mathrm{x}) \equiv \mathrm{e}^{-\frac{1}{2}\left(\frac{\mathrm{x}-\mathrm{c}}{\sigma_2}\right)^2}$, and $\mathrm{N}(\mathrm{c}, \sigma_1, \mathrm{x}) \equiv \mathrm{e}^{-\frac{1}{2}\left(\frac{\mathrm{x}-\mathrm{c}}{\sigma_1}\right)^2}$, [18]. Hence, in the type-2 FLC, for each input we defined three-interval type-2 fuzzy Gaussian MFs: negative, zero, positive in the interval [-10 10], as illustrated in Figures 7 and 8. For computing the output we have five interval type-2 fuzzy Gaussian MFs, which are NG, N, Z, P and PG, in the interval [-10 10], as can be seen in Figure 9. Table 2 shows the characteristics of the inputs and output of the type-2 FLC.

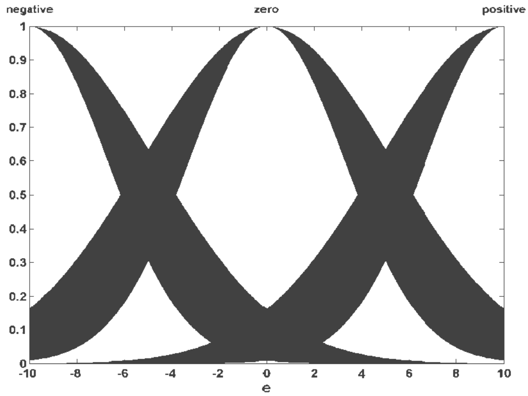

Figure 7: Input e membership functions for the type-2 FLC

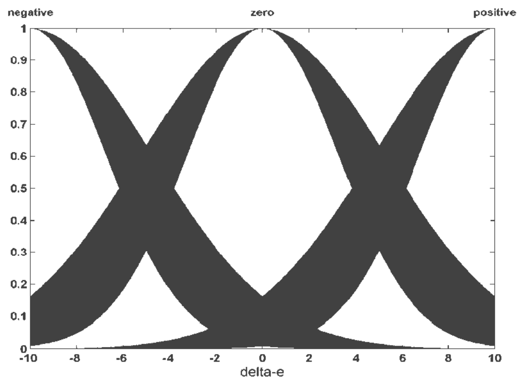

Figure 8: Input Δe membership functions for the type-2 FLC

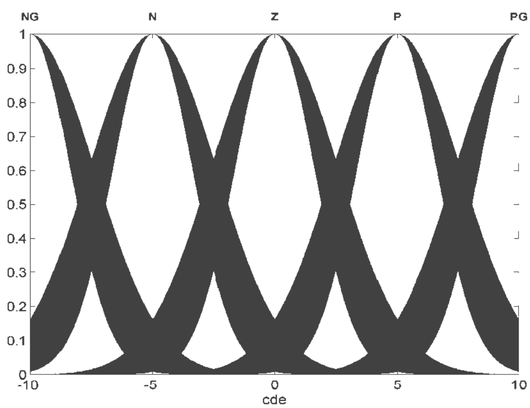

Figure 9: Output cde membership functions for the type-2 FLC

Table 2. Input and Output Parameters of the Type-2 FLC.

Variable	**Term**	**Center** c	**Standard deviation** σ_1	**Standard deviation** σ_2
Input e	negative	-10	5.2466	3.2466
	zero	0	5.2466	3.2466
	positive	10	5.2466	3.2466
Input Δe	Negative	-10	5.2466	3.2466
	Zero	0	5.2466	3.2466
	positive	10	5.2466	3.2466
Output *cde*	NG	-10	2.6233	1.6233
	N	-5	2.6233	1.6233
	Z	0	2.6233	1.6233
	P	5	2.6233	1.6233
	PG	10	2.6233	1.6233

Experiment 1: Simulation of an ideal system with a type-1 FLC.

In this experiment, uncertainty data was not added to the system, and the system response produced a settling time of about 140 units of time; i.e., the system tends to stabilize with time and the output will follow accurately the input. In Table 3, we listed the values of ISE, IAE, and ITAE for this experiment.

Table 3. Performance Criteria for Type-1 and Type-2 Fuzzy Controllers for 20 db Signal to Noise Ratio (After 200 Samples)

Performance Criteria	**Type-1 FLC**		**Type-2 FLC**	
	Ideal System	Syst. with uncertainty	Ideal System	Syst. with uncertainty
ISE	7.65	19.4	6.8	18.3
IAE	17.68	49.5	16.4	44.8
ITAE	62.46	444.2	56.39	402.9

Experiment 2: Simulation of an ideal system using the type-2 FLC.

Here, the same test conditions of Experiment 1 were used, but in this case, we implemented the controller's algorithm with type-2 fuzzy logic. The corresponding performance criteria are listed in Table 3, and we can observe that using a type-2 FLC we obtained the lower errors.

Experiment 3: System with uncertainty using a type-1 FLC.

In this case, equation (25) was used to simulate the effects of uncertainty introduced to the system by transducers, amplifiers, and any other element that in real world applications affects expected values. In this experiment the noise level was simulated in the range of 20 db of SNR ratio.

Experiment 4: System with uncertainty using a type-2 FLC.

In this experiment, uncertainty was introduced in the system, in the same way as in Experiment 3. In this case, a type-2 FLC was used and the results obtained with a type-1 FLC (Experiment 3) were improved.

Experiment 5. Varying the Signal to Noise Ratio (SNR) in type-1 and type-2 FLCs.

To test the robustness of the type-1 and type-2 FLCs, we repeated experiments 3 and 4 giving different noise levels, going from 30 db to 8 db of SNR ratio in each experiment. In Table 4, we summarized the values for ISE, IAE, and ITAE considering 200 units of time with a P_{signal} of 22.98 db in all cases. As it can be seen in Table 4, in presence of different noise levels, the behavior of type-2 FLC is in general better than type-1 FLC.

Table 4. Behavior of Type-1 and Type-2 Fuzzy Logic Controllers after Variation of Signal to Noise Ratio (Values Obtained for 200 Samples)

Noise variation				**Type-1 FLC**			**Type-2 FLC**		
SNR (db)	SNR	SumNoise	SumNoise	ISE	IAE	ITAE	ISE	IAE	ITAE
8	6.4	187.42	22.72	321.1	198.1	2234.1	299.4	194.1	2023.1
10	10.05	119.2	20.762	178.1	148.4	1599.4	168.7	142.2	1413.5
12	15.86	75.56	18.783	104.7	114.5	1193.8	102.1	108.8	1057.7
14	25.13	47.702	16.785	64.1	90.5	915.5	63.7	84.8	814.6
16	39.88	30.062	14.78	40.9	72.8	710.9	40.6	67.3	637.8
18	63.21	18.967	12.78	27.4	59.6	559.1	26.6	54.2	504.4
20	100.04	11.984	10.78	19.4	49.5	444.2	18.3	44.8	402.9
22	158.54	7.56	8.78	14.7	42	356.9	13.2	37.8	324.6
24	251.3	4.77	6.78	11.9	36.2	289	10.3	32.5	264.2
26	398.2	3.01	4.78	10.1	31.9	236.7	8.5	28.6	217.3
28	631.5	1.89	2.78	9.1	28.5	196.3	7.5	25.5	180.7
30	1008	1.19	0.78	8.5	25.9	164.9	7	23.3	152.6

5 Conclusions

We have presented in this paper the basic concepts of interval type-2 fuzzy logic. Also, the use of a toolbox for type-2 fuzzy logic, developed by our group, to apply the theory in solving real-world problems is illustrated. Simulation results in intelligent control show the feasibility of the approach. Future researh work includes applying interval type-2 fuzzy logic to other applications areas, and also considering generalized or non-singleton type-2 fuzzy sets.

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The papers presented in this Volume 2 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) organized in Warsaw on October 8, 2010 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

ISBN-13 9788389475367
ISBN 838947536-7
9 788389 475367