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Uncertainty analysis of greenhouse gases inventories using a probabilistic-possibilistic approach

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Abstract

In the paper we propose a more general methodology for the description of uncertainty related to the estimates of national greenhouse gas (GHG) inventories. We show that in some cases expert opinions should be described using rather fuzzy sets than probability distributions. In such cases we propose to use fuzzy random variables for the description of GHG emissions and removals. When the proposed probabilistic-possibilistic model is applied additional requirements should be formulated for the verification of commitments to the Kyoto Protocol.

Keywords: uncertainty, emission inventories, random fuzzy variables, possibility theory.

1 Introduction

Reduction of the emission of greenhouse gases is a subject of the hottest discussions both of scientists and politicians. The Intergovernmental Panel on Climate Change established in 1989 by the United Nations Environmental Programme (UNEP) and World Meteorological Organization (WMO) provides a framework for a collective work of scientists representing different scientific disciplines who try do describe a current state of climate change and its potential impact on world's environment and economy. An important part of the IPCC work, assigned to the Task Force on National Greenhouse Gas Inventories (TFI), is to establish a commonly agreed methodology for the calculation and reporting of national greenhouse gas (GHG) emissions and removals. TFI worked out five volumes of guidelines [1] with the methodology that should used by countries, parties of the Kyoto Protocol, for the calculation of GHG emissions and removals. A part of this methodology is dedicated to the problem of the evaluation of

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uncertainties related to GHG emissions and removals estimate. In establishing GHG inventories many different sources of input data are used. What is extremely important in the context of this paper, only a small part of GHG inventories is estimated directly from measurements of emissions and removals. Predominant part of these inventories is estimated indirectly from estimates of certain activities using so called emission factors. The values of these factors are either calculated from mathematical models of processes or provided by experts. Therefore, the total estimates of countries GHG emissions and removals, used for the verification of the compliance to the Kyoto Protocol agreements, heavily depend on the imprecise knowledge of experts.

An important part of the GHG assessment is the estimation of uncertainties related to each greenhouse gas inventory. The knowledge of these uncertainties is necessary not only for the evaluation of GHG inventories. According to many researchers, see e.g. [8] or [10], this knowledge should be taken into account in establishing rules of emissions trading. In the second section of this paper we present a short description of different type of uncertainties that are present in estimates of GHG inventories. We focus our attention on those uncertainties which are related to human assessments. These uncertainties are, as for now, described using the language of the theory of probability. How it is done, it is presented in the third section of the paper. We claim, however, that for certain types of such uncertainties the models of probability distributions might not be appropriate. In the fourth section of the paper we show some pitfalls of the probabilistic modeling of imprecisely known quantities when this imprecision is not related to variability of a random type. In the next section we present a fuzzy-probabilistic methodology which allows estimation of GHG inventories in presence of uncertainties of different nature. Then, in the fifth and sixth sections of the paper we show how to use this methodology for the estimation of the GHG inventories, and for the comparison of such quantities that are both random and fuzzy. These methods should be, in our opinion, used for the verification of compliances to the Kyoto Protocol, and in establishing rules for emissions trading. The paper is summarized at its end in a section with short conclusions.

2 Sources of uncertainty in emission inventories

The emission of greenhouse gases is considered by many scientists as an important factor that influences global climate. They have motivated politicians to sign the Kyoto Protocol which is an international agreement in which over one hundred countries agreed to mitigate the emission of greenhouse gases. The Parties who accept the Kyoto Protocol agreed to reduce the national emissions of greenhouse gases by specified percents given in the Annex I to this Protocol. In order to verify the compliance to agreed commitments it is necessary to create national greenhouse gas inventories which are used for the evaluation of the amount of greenhouse gases (GHG) which are emitted or removed by natural and non-natural (i.e. related to human's activities) sources. GHG inventories constitute the source of basic data that are used for building policies whose ultimate goal is to mitigate GHG emissions. They are also used for the determination of the nation compliance with commitments like the Kyoto Protocol or EU agreements.

National GHG inventories are, in general, not based only on direct measurements of emissions and removals. In many cases it based on measures of some activities and emission factors provided by experts. IPCC recommends a very simple model according to which the total emission of a certain greenhouse gas x_i is estimated by a linear function

$$x_i = \sum_{j=1}^{m} y_j a_{ij}, \quad i = 1, \dots, \tag{1}$$

where y_j are measures of m activities who are responsible for the emission of x_i, and a_{ij} are respective emission factors. For example, if x_i is the emission of CO_2 from national electric coal power plants the values of y_j may represent the amounts of coal burned at different power plants, and emission factors a_{ij} depend on the type of coal, technology of burning etc. Such calculations are usually performed for a specific year or a given period of time. Emissions evaluated according to (1) are calculated for all potential sources of GHG, and their sum represents the total estimated emission of a particular greenhouse gas. Moreover, it is a common practice to represent emissions of different greenhouse gases in terms of equivalent emissions of CO_2. The second component of the GHG inventory, i.e. GHG removals by different (natural and non-natural) sinks is evaluated in a similar way. However, in this case the concept of “activity” is understood in a rather general way.

From the very beginning it has been known and widely accepted fact that the estimates of GHG inventories are highly uncertain. This uncertainty is typical for estimations based on direct measurements of GHG emissions but to much greater extent for estimations based on indirect estimations that use estimations of activities and emission factors. The authors of the IPCC Guidelines [1] consider eight basic sources of uncertainty:

- Lack of completeness (e.g. lack of measurement methods or incomplete recognition of emission and removal processes);
- Model (e.g. oversimplified models, interpolation or extrapolation of used data, existence of alternative models, etc.;
- Lack of data (e.g. usage of proxy data obtained for analogous problems or seemingly similar situations);
- Lack of representativeness of data (e.g. usage of data obtained under different conditions);
- Statistical random sampling error (when data are obtained using methods of statistical sampling of populations);
- Measurement error (random or systematic errors related to the used measurement procedures);
- Misreporting or misclassification (uncertainty due to imprecise or incomplete description of data);
- Missing data (e.g. when measurements were attempted but no values were available).

Gillenwater *et al.* [8] divide all sources of uncertainty into two general categories: *scientific* uncertainty, and *estimation* uncertainty. The first category consists of all types of uncertainty that is related to incomplete knowledge of processes. They give an example of the process of indirect N_2O emissions associated with nitrogen-containing compounds that are emitted to the atmosphere and then deposited. The estimation uncertainty, according to [8], is either a *model* uncertainty (associated with mathematical models used in the processing of data) or a *parameter* uncertainty which can be evaluated using statistical methods. A more comprehensive discussion on the problem of uncertainty and its sources can be found in many publications. The authors of [1] suggest reading, for example, the book by Cullen and Frey [104].

When we are talking about uncertainties related to the estimates of GHG inventories we usually restrict ourselves to uncertainties related to direct or indirect "measurements". In real situation the total uncertainty depends also on random variability of analyzed quantities. For example, in case of lacking current measurement data we use extrapolated data from time series that describe historical data. These historical data are obviously random because of random variability of circumstances in which they have been measured. For example, depositions of chemical compounds depend heavily on randomly varying weather conditions. Therefore, the results estimations based on the analysis of trends in such data are definitely uncertain. In general, the problem of variability, which differs from uncertainty, is crucial in all cases when we have to forecast future GHG emissions or removals.

3 Quantitative assessment of uncertainty for GHG inventories

Emissions and removals of GHG calculated from (1) are highly uncertain. This uncertainty is due both to uncertain values of activities y_i, and – especially – emission (removal) factors a_{ij}. It has been noticed by many authors that the uncertainty related to activities are usually smaller in comparison to that related to emission (removal) factors. This is due to a more objective ways of measurement. In many cases measures of activities come from the results of censuses or comprehensive evaluations provided by governmental statistical institutions. Therefore, there is usually enough information to estimate uncertainty using classical statistical methods.

If we assume that the estimate of a certain activity *Y* represents an observed outcome of a random variable it is quite natural to express uncertainty in terms of the standard deviation of this variable. Let μ_Y be the expected (mean) value of such a random variable *Y*, and σ_Y its standard deviation. Usually, uncertainty of *X* is expressed in a form of interval $\mu_Y \pm 2\sigma_Y$. The width of this interval is usually reported in percents of the expected value. Thus, this representation looks like $\mu_Y \pm U_Y[\%]$, where $U_Y = (2\sigma_Y / \mu_Y) \bullet 100$. It is quite obvious that this representation is appropriate when the probability distribution function (pdf) of *Y* is symmetric. When this distribution is normal the "two-sigma" interval contains nearly 95% of probability mass of *Y*. Therefore, for probability distributions other than the normal uncertainty can be represented by an interval, not necessarily symmetric, that contains 95% probability mass of *Y*, distributed symmetrically around its expected (mean) value.

When the mean value of *Y* is estimated from a simple random sample the respective uncertainty is calculated in a form of a confidence interval. This methodology can be also used when the mean value of *Y* is forecasted from a time series. In all other cases the probability distribution of *Y* should be evaluated by experts.

In the case of emission (removal) factors their uncertainties are described in the same way, i.e. by treating an emission (removal) factor *a* as a random variable *A* described by certain probability distribution function. However, in contrast to the case of activities, the values of emission (removal) factors are evaluated using information from different sources which sometimes do not have statistical interpretation. Even if a given factor is evaluated subjectively by one expert its uncertainty has to be given, according to the IPCC Guidelines [1] by the same expert in terms of a probability density function. In such cases it is expected that the expert either provides the interval containing approximately 95% of the probability mass of the pdf that describes his/hers estimated value or indicates the type of the respective probability distribution (e.g. normal, log-normal, triangular or uniform), and provides additional information that allows to calculate such interval. This is a commonly agreed way of elicitation of expert opinions when we model such opinions in terms of probability distributions.

Let us notice that even if we are able to provide all necessary probability distributions that are necessary to calculate the probability distribution of the total GHG emission (removal) the problem of the evaluation of its pdf becomes difficult. Winiwarter and Rypdal [15] present the results of a comprehensive study in which they attributed certain probability distributions to the values of activities and emission factors. These pdf's should be used for the evaluation of uncertainty of the total GHG emission. This can be done using Monte Carlo simulation methods. The results of such simulation experiments which give much more accurate estimate of the uncertainty of the GHG inventories are known for a very few countries like Austria, Norway, and the Netherlands (see [15], [12], [11], [14]).

The evaluation of the uncertainty of the total GHG inventory depends upon the approach used for the combination of uncertainties. Monte Carlo simulation methods are labeled in the IPCC Guidelines [1] as "Approach 2". When the available information is insufficient for the design of Monte Carlo experiments the IPCC Guidelines recommend to use the "Approach 1" – the propagation of error methodology. According to this methodology the uncertainty of each component of the function that is used for the calculation of the GHG inventory is fully described by its variance. Then, the rules for the computation of variances of products, sums, and differences of random variables are used for the computation of the variance of the whole inventory. When uncertainty is expressed in percentages U_i of uncertain quantities x_i then the combined uncertainty of the product of n uncertain quantities is given by [1]:

$$U_{total} = \sqrt{U_1^2 + U_2^2 + \ldots + U_n^2} \,. \tag{2}$$

In the case of sum of n uncertain quantities we have [1]:

$$U_{total} = \frac{\sqrt{(U_1 x_1)^2 + (U_2 x_2)^2 + \ldots + (U_n x_n)^2}}{|x_1 + x_2 + \ldots + x_n|} \,. \tag{3}$$

This methodology is very simple, but has its important limitations. First of all, uncertainties attributed to all components should be relatively small (in terms of the quotient of the standard deviation and the mean value). Moreover, the inputs should be statistically independent or at most weakly correlated.

4 Pitfalls of probabilistic modeling of imprecise or incomplete knowledge

Thorough analysis of practical implementations of the methodology proposed in the IPCC Guidelines [1] shows that the estimation of the GHG inventories depends heavily on expert opinions. If we consider all eight main sources of uncertainty listed in Section 2 only two of them ("Lack of completeness" and "Statistical random sampling error") are not influenced by experts' opinions (see [1] for the explanation). In all other cases uncertain emissions (removals) are estimated using experts' opinions. Moreover, the uncertainty of such opinions is expressed in terms of probability distributions elicited from the experts. We claim that in some cases this *is not an appropriate method*.

Probability distributions describe random variables and *always* have interpretation in terms of frequencies of certain *random* observations. We believe that the existence of real or imaginary underlying random experiment is a prerequisite for using probabilistic models. We claim that experts' opinions do not necessarily posses this property, and thus should not be always modeled by probability distributions. This opinion has been also expressed in Gillenwater *et al.* [8] in a somewhat different context. Let take, for example, the case when emission processes are not fully understood, and there exist two competing models of such processes whose application leads to two completely different estimates of emission. IPCC Guidelines [1] suggest that in such a case an expert should assign his/hers own two-point probability distribution for the averaging of two different results of computations. We may ask however if he/she is entitled to do so. There is no hidden random experiment behind this estimate. Only one of these models may be correct, so we are not entitled to compute any particular weighted average. Both computed versions should be considered in further calculations, possibly with assigned subjective certainty measures.

It is agreed in many communities (not only in IPCC) that uncertainty of experts' opinions should be expressed in the form of probability distribution function. It has to be noted, however, that this assumption leads to very precisely defined consequences. First, by establishing a certain probability distribution as a model of expert's opinion we precisely define probabilities of all possible subsets of the sample space. Let's consider the example of a uniform distribution defined on an interval $[a,a+d]$. Such probability distributions are recommended as probabilistic models of expert opinions when experts are sure than the value of an assessed quantity belongs to a certain interval. Such information is considered as incomplete and very uncertain. It is usually not seen, however, that the assumption of uniform probability distribution leads, for example, to the assertion that the probability of having the value of this quantity in the interval $[a,a+d/3]$ is *exactly* two times smaller than this probability for the interval

[$a+d/3, a+d$]. Thus, despite very uncertain information expressed by an expert we have introduced a very specific probability structure on the set of all possible values of evaluated quantity. This structure determines all other information that can be derived from the initial one, such as averages, standard deviations, confidence intervals, etc.

There is also another consequence of modeling uncertain expert opinions using probability distributions. If we do so, we must follow the rules of probability when we process available information. When this information is incomplete and uncertain it may lead to unexpected consequences. Consider, for example, an activity y which produces two types of greenhouse gases. Suppose that we want to estimate emissions of these gases in terms of equivalent emissions of CO_2. Let's assume that emission factors are supplied by two *independent* experts in a form of intervals [a,b] and [c,d], respectively. If we use uniform distributions as models for emission factors the total emission should be modeled by the triangular probability distribution defined on the interval [$y(a+c), y(b+d)$] with a maximum at $y(a+b+c+d)/2$. However, if we ask experts about their opinion about the total emission they will usually indicate the interval [$y(a+c), y(b+d)$].

Similar examples have been given by many researchers who have claimed that probability distributions are not the only models that can be used for the description of imprecise and uncertain information. Many different approaches have been proposed for coping with this problem (see an excellent paper of Walley [13] for more information). Possibility distributions, introduced by Zadeh [16], in our opinion seem to be a good mathematical model for the description of information of that type. We introduce them in the next section of the paper.

5 Modeling of imprecisely reported data using the fuzzy set theory

In the analysis of data related to complex problems which may be delivered by human beings we often face the problem of imprecision. In many cases such data are provided by experts who are not able or not willing to present their expertise in terms of precise numbers. There are many examples of cases where such imprecise data are very common in practice. In many cases data and opinions about values of certain quantities are reported by people who use imprecise expressions like "about 5", "much larger than 5, but surely smaller than 10", etc. The attempt do describe such lack of precision in terms of probability seems to be very questionable, as it has been noted in the previous section of the paper. The main reason of this stems from a fact that these imprecise notions do not have interpretation in terms of frequencies. However, it has been noted that the fuzzy sets theory proposed by Lotfi A. Zadeh is especially useful for the formal description of such imprecise linguistic data. Moreover, if the imprecise data are also of a random character, then the theory of fuzzy random variables can be used for the mathematical description of imprecise (vague) statistical data.

A fuzzy set is defined on a non-fuzzy (crisp) set ***X*,** called the universe of discourse, using ordered pairs $(x, \mu(x))$, where $x \in \boldsymbol{X}$, and $\mu : \boldsymbol{X} \rightarrow [0,1]$ is the so called membership function. For more information about the properties of membership functions and methods of their elicitation see e.g. [2]. In the context of this paper we are rather interested in a subset of fuzzy sets called fuzzy numbers.

Definition 1 (Dubois and Prade [5])
The fuzzy subset A of the real line $\boldsymbol{R}$, with the membership function $\mu : \boldsymbol{R} \rightarrow [0,1]$, is a *fuzzy number* if

- is normal, i.e. there exists an element $x_0 \in \boldsymbol{R}$ such that $\mu(x_0) = 1$;
- is fuzzy convex, i.e. $\mu(\lambda x + (1-\lambda)y) \geq \mu(x) \wedge \mu(y)$ $\forall x, y \in \boldsymbol{R}$ and $\forall 0 \leq \lambda \leq 1$, and operator $\wedge$ stands for minimum;
- is upper semi-continuous;
- $\mathrm{supp}(\mu)$ is bounded.

A useful concept used for the description of fuzzy numbers is the α-cut. The α-cut A_α of a fuzzy number A is a non-fuzzy set defined as $A_\alpha = \{x \in R: \mu(x) \geq \alpha\}$. The family $\{A_\alpha : \alpha \in [0,1]\}$ is a set representation of the fuzzy number A. The α-cuts may be used for the alternative description of fuzzy numbers:

$$\mu(x) = \sup_{\alpha \in [0,1]} \{\alpha I_{A_\alpha}(x)\},$$

where $I_{A_\alpha}(x)$ denotes the characteristic function of A_α. Definition 1 implies that every α-cut of a fuzzy number is a closed interval. Hence, we have

$$A_\alpha = [A_\alpha^L, A_\alpha^U],$$

where

$$A_\alpha^L = inf\{x \in \mathbf{R}: \mu(x) \geq \alpha\},$$
$$A_\alpha^U = sup\{x \in \mathbf{R}: \mu(x) \geq \alpha\}.$$

The space of all fuzzy numbers will be denoted by $\boldsymbol{F}(\boldsymbol{R})$.

A fuzzy random variable may be defined by analogy to the definition of a real-valued random variable as a mapping that assigns to a random event an imprecise fuzzy number. The notion of a fuzzy random variable has been defined independently by many authors (see Gil and Hryniewicz [7] for more information). In general, a fuzzy random variable *X* is considered as a perception of an unknown usual random variable $V: \Omega \to \mathbf{R}$, called an original of *X*.

Formally, a fuzzy random variable can be defined using the following definition:

Definition 2 (Grzegorzewski and Hryniewicz [9])
A mapping $X: \Omega \to \boldsymbol{F}(\boldsymbol{R})$ is called a *fuzzy random variable* if it satisfies the following properties:

(1) $\{X_\alpha(\omega): \alpha \in [0,1]\}$ is a set representation of $X(\omega)$ for all $\omega \in \Omega$;

(2) for each $\alpha \in [0,1]$ both X_α^L and X_α^U defined as

$$X_\alpha^L = X_\alpha^L(\omega) = inf\ X_\alpha$$
$$X_\alpha^U = X_\alpha^U(\omega) = sup\ X_\alpha$$

are real-valued random variables on (Ω, F, P).

Let χ denotes a set of all possible originals of *X*. If only vague data are available, it is of course impossible to show which of the possible originals is true. Therefore, we can define a fuzzy set of χ, with a membership function $\nu: \chi \to \boldsymbol{F}(\boldsymbol{R})$ given as follows:

$$\nu(V) = inf\{\mu_{X(\omega)}(V(\omega)): \omega \in \Omega\}$$

which corresponds to the grade of acceptability that a fixed random variable *V* is the original of the fuzzy random variable in question.

Fuzzy random variables have been used for the description of many practical problems where stochastic randomness is present together with fuzzy imprecision.

6 Fuzzy-random modeling of GHG inventories

The existence of different types of uncertainty that influence the estimated value of total greenhouse gases emission has been explained in the second section of this paper. In existing literature it is usually assumed that probability distributions are sufficient for their description. In this paper we also assume that all activity measures y_{ij} can be assessed with specific degrees of accuracy described in terms of probability distributions. Thus, we assume that uncertainties related to the assessments of y_{ij} are of random character. In fact they consist of many components. One type of uncertainty is related to measurement procedures and is important when the measures of activities are estimated from random samples. Another important uncertainty is related to random variability of real (unobserved) values of all activity measures. This type of uncertainty is known in the literature as the trend uncertainty, and has to be taken into account when measures of activities are estimated from time series. However, in order to make our model simple, we assume that all components of uncertainty are indistinguishable, and the overall uncertainty related to the assessments of a certain activity measure is described by a probability distribution estimated from existing statistical data.

The nature of uncertainty assigned to the associated emission factors $a_{ij}, i=1,\ldots,n; j=1,\ldots,m$ is much more difficult for precise evaluation. This uncertainty contains undoubtedly a random factor (for example, for an electric coal power plant the emission rate varies randomly with randomly varying quality of burned coal), but may also contain another factor, related to imprecise opinions of experts. The results of the assessment for Austria and Norway (see the papers Winiwarter & Rypdal [15], and Rypdal & Winiwarter [12]) show that imprecise expert opinions may contribute from 10% to 20% of total uncertainty of the total assessment. Therefore, correct description and interpretation of uncertainty related to this source is of the greatest practical importance.

Let us assume that each emission (removal) factor can be expressed as a product

$$a_{ij} = R_{ij} f_{ij} \tag{4}$$

where R_{ij} is *a random variable* representing a random part described by a *probability distribution*. Non-random part of a_{ij}, representing imprecise information provided by experts is described by a *possibility distribution* represented by a membership function of a *fuzzy variable* f_{ij}.

The representation of emission (removal) factors in a form given by (4) is equivalent to the representation by a *fuzzy random variable*. The type of the underlying probability distribution of the emission factor is determined by its random part R_{ij}. Non-random information provided by experts modifies parameters of the probability distribution of a_{ij}. It seems to us that this representation of the available information about the values of emission (removal) factors is more informative and less controversial than the classical representation using exclusively probability distribution functions.

Taking into account a double nature of uncertainty related to estimated values of the emission (removal) factors, we have shown that these quantities should be represented by fuzzy random variables. Therefore, each observed value of the emission factor should be given as a fuzzy number represented by a set of its α-cuts: $\left[a_{ij,L}^{\alpha}, a_{ij,R}^{\alpha}\right], \alpha \in (0,1]$. The existing nowadays area-specific information usually does not allow us to build complicated possibility distributions representing the fuzzy values of $f_{ij}, i = 1,\ldots,n; j = 1,\ldots,m$. In lack of specific information the possibility distribution of the imprecise part of the emission factor may be approximated by the following trapezoidal membership function (indices *ij* are omitted):

$$\mu(f) = \begin{cases} 0 & \text{if} \quad f < f_{0,min} \\ \dfrac{f - f_{0,min}}{f_{1,min} - f_{0,min}} & \text{if} \quad f_{0,min} \le f < f_{1,min} \\ 1 & \text{if} \quad f_{1,min} \le f < f_{1,max} \\ \dfrac{f_{0,max} - f}{f_{0,max} - f_{1,max}} & \text{if} \quad f_{1,max} \le f < f_{0,max} \\ 0 & \text{if} \quad f_{0,max} \le f \end{cases} \tag{5}$$

where $\left[f_{0,min}, f_{0,max}\right]$ is an interval of all possible values of the emission (removal) factor *f*, and $\left[f_{1,min}, f_{1,max}\right]$ is the interval containing those values of *f* that are fully possible. Note that this form of the possibility distribution describes, as special cases, two important practical situations: rectangular function (when the interval of possible values is given by an expert) and triangular function (when an expert additionally indicates the most plausible value). We must stress again that the possibility distribution defined by (4) does *not* have probabilistic interpretation in terms of frequencies.

Now we can write the formula in which we describe the total emission (removal) as the following fuzzy random variable

$$X_{total} = \sum_{i=1}^{n} \sum_{j=1}^{m} x_i R_{ij} f_{ij} \tag{6}$$

Observed values of this random variable are trapezoidal fuzzy numbers, as they are given as weighted sums of trapezoidal fuzzy numbers with weights given by values of products $x_i R_{ij}$.

Formula (6) shows how to evaluate probability distribution of the inventory X. It can be done using Monte Carlo simulation methods in exactly the same way as it has been done by many researchers who try to estimate the uncertainty attributed to GHG inventories. Simulation has to be limited only to these quantities which have probabilistic interpretation. The results of those simulations have to be used for the determination of the type of the probability distribution of X. Then, methods of fuzzy statistics [7] have to be used for the calculation of fuzzy parameters of interest (e.g. standard deviations, limits of confidence intervals, estimators of quantiles etc.). All these fuzzy numbers will be described by trapezoidal membership functions, and their calculation will not be dramatically more difficult than that which is performed in a fully probabilistic setting (see e.g. [14]).

7 Making decisions with imprecisely reported data

According to the Annex I to Kyoto Protocol a country fulfils its commitment if in the compliance year k its emission does not exceed the value $x_k = rx_0$, where x_0 is the emission in the base year, and r is a coefficient of reduction agreed upon in the Kyoto Protocol. According to the methodology proposed in this paper both the estimated total emission in the compliance year and the estimated emission in the base year are given in the form of trapezoidal fuzzy numbers $\tilde{x}$ and $\tilde{x}_k$, respectively. Thus, we have to verify if $\tilde{x} \le \tilde{x}_k$. Unfortunately, from the theory of fuzzy sets we know that the unique method for the verification of this inequality does not exist. In order to compare fuzzy numbers many practitioners propose to use the following procedure: first to defuzzify these numbers, and then to compare non-fuzzy results of the defuzzification process.

There exist many methods used for defuzzification of fuzzy numbers. The simplest one consists in the calculation of the center of gravity (the expected value) of the respective membership function. However, in the context of decision making we claim that the λ-average ranking method proposed in Campos and Gonzalez [3] is especially useful. Let $\widetilde{X}$ be a fuzzy number (fuzzy set) described by the set of its α-cuts $\left[X_L^\alpha, X_U^\alpha\right]$, and S be an additive measure on [0,1]. Moreover, let us assume that the support of $\widetilde{X}$ is a closed interval. The λ-average value of such a fuzzy number $\widetilde{X}$ is defined in [3] as

$$V_S^{\lambda}\left(\widetilde{X}\right)=\int_0^1\left[\lambda X_U^{\alpha}+(1-\lambda)X_L^{\alpha}\right]dS(\alpha),\lambda\in[0,1]. \tag{7}$$

In the case of continuous membership functions this integral is calculated with respect to $d\alpha$. Thus, the λ-average value of $\widetilde{X}$ can be viewed as its defuzzified representation. The parameter λ in the above integral is a subjective degree of the decision-maker's optimism (pessimism). In the case of fuzzy risks we have the following interpretation of the λ-average. If we take $\lambda = 0$ it means that the decision-maker is highly optimistic (or even over-optimistic), as it means that for the calculation of the defuzzified value of the fuzzy expected risk reflects only the minimal values of all α-cuts (representing the lowest possible risks) are taken into consideration. On the other hand, by taking $\lambda = 1$ the decision-maker demonstrates his total pessimism, as only the maximal values of all α-cuts (representing the highest possible risks) are considered. If the decision maker takes $\lambda = 0{,}5$ his attitude may be described as neutral. Thus, by varying the value of λ the decision maker is able to take into account the level of his optimism (pessimism) which may arise e.g. from having some additional information about the consequences of decisions to be made.

Another approach, which in our opinion gives a decision-maker a more deep insight into a decision-making process, comes from the theory of possibility. In this approach we use *Possibility of Dominance* and *Necessity of Strict Dominance* indices proposed by Dubois and Prade [6].

For two fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$ the *Possibility of Dominance* (*PD*) index is calculated from the formula

$$PD = Poss\left(\widetilde{A} \geq \widetilde{B}\right)= \sup_{x,y:x\geq y} \min\left\{\mu_{\widetilde{A}}(x),\mu_{\widetilde{B}}(y)\right\}. \tag{8}$$

The *PD* index gives the measure of *possibility* that the fuzzy number $\widetilde{A}$ is not smaller than the fuzzy number $\widetilde{B}$. Positive value of this index tells the decision maker that there exists even slightly evidence that the relation $\widetilde{A} \geq \widetilde{B}$ is true. In practice a positive value of the *PD* index means that supports of both fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$ are overlapping.

The degree of conviction that the relation $\widetilde{A} > \widetilde{B}$ is true is reflected by the *Necessity of Strict Dominance* (*NSD*) index defined as

$$\begin{aligned} NSD = Ness\left(\widetilde{A} > \widetilde{B}\right) &= 1- \sup_{x,y:x\leq y} min\left\{\mu_{\widetilde{A}}(x),\mu_{\widetilde{B}}(y)\right\}= \\ &= 1-Poss\left(\widetilde{B} \geq \widetilde{A}\right). \end{aligned} \tag{9}$$

The *NSD* index gives the measure of *necessity* that the fuzzy number $\widetilde{A}$ is greater than the fuzzy number $\widetilde{B}$. Positive value of this index tells the decision maker that there exists rather strong evidence that the relation $\widetilde{A} > \widetilde{B}$ is true. Note that in such a case the *PD* index for this relation is always equal to one. These possibilistic indices, and other similar indices, may be used for choosing the best option while solving complex decision problems.

Graphical interpretation of the *NSD* index of the relation $\widetilde{A} > \widetilde{B}$ is given on Fig. 1.

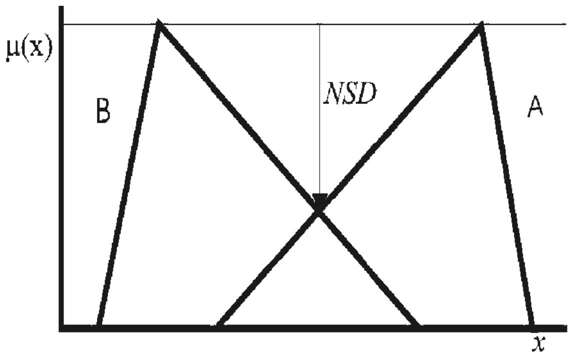

Figure1: Graphical interpretation of the *NSD* index for the relation $\widetilde{A} > \widetilde{B}$

Taking into account the last part of (9) we can easily find a simple graphical interpretation of the *PD* index of the relation $\widetilde{B} \geq \widetilde{A}$.

In the possibilistic approach to the problem of the verification of commitments to the Kyoto Protocol we need to set additional requirement. When only slight evidence of complying to the commitments is sufficient, the requirement on the value of the *PD* index is needed. However, when we need rather strong evidence that this commitment is fulfilled we should set the requirement in terms of the *NSD* index. For example, if we want our decision to be independent upon imprecise expert opinions we should set the required value of the *NSD* index to one.

4 Conclusions

Estimates of greenhouse gas inventories are very uncertain. This uncertainty has many sources, and to great extent depends on uncertain and imprecise experts opinions. In the paper we show that probabilistic description of certain experts opinions seem to inappropriate. Instead, we propose to use a possibilistic approach for the formal description of such non-probabilistic uncertainties where uncertain and imprecise experts opinions are described formally in terms of fuzzy sets. To model simultaneously uncertainties of different type we propose to use fuzzy random variables described by fuzzy probability distributions. When non-probabilistic uncertainty is present we propose to use indices of possibility and necessity in such decision processes like the verification of the commitments to the Kyoto Protocol.

In recent papers ([8], [10]) devoted to the problem of the verification of the commitments to the Kyoto Protocol new rules for the trading of the emission quotas have been proposed. In these newly proposed rules uncertainty of national GHG inventories has been taken into account. The probabilistic-possibilistic methodology which has been proposed in this paper can be incorporated into such proposals. This might be the subject of future work on the objective methods for the verification of the compliance to the Kyoto Protocol or similar international agreements.

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The papers presented in this Volume 2 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) organized in Warsaw on October 8, 2010 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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