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Volume II: Applications

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Construction of decision quality measure in IF-set-based MCDM environment

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Abstract

Multicriteria decision making (MCDM) methods provide optimal decision regardless of the quality of data involved in the process of decision making. Since in some circumstances this approach may be misleading for the decision maker, we construct a measure of quality for such decisions in IF-Set-based MCDM environment. This measure enables the decision maker to evaluate the reliability of the optimal decision indicated by MCDM methods.

Keywords: decision-making, decision quality measure, Atanassov's intuitionistic fuzzy sets.

1 Introduction

Attributes of low data quality such as incompleteness, uncertainty and lack of data accuracy are common phenomena in many real-life applications of multicriteria decision making (MCDM) models (cf. [5, 10]). These data quality attributes make a large contribution to the quality of the final decision resulting from the MCDM process. As has been clearly presented in a simple example in [3], decision lacking its quality context may be misleading for the decision maker, especially if based

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on incomplete and inaccurate data. Therefore the influence of data quality on the decision's quality should be properly measured and known to the decision maker when utilizing the MCDM method.

In this paper we consider an MCDM method based on Atanassov's IF-sets. The method is briefly described in Section 2. In Section 3 we define the notion of decision quality in MCDM and state general requirements for the method of determining it. A construction of such a quality measure is presented in Section 4. Section 5 contains an example of application of the presented method to sample decisions. We conclude with final remarks in Section 6.

2 Multicriteria decision making in IF-Set environment

A fuzzy set A' in X may be given as $A' = \{\langle x,\, \mu_{A'}(x)\rangle \,|\, x \in X\}$, where $\mu_{A'} : X \to [0,1]$ is the membership function of A' ([11]).

An Atanassov's intuitionistic fuzzy set (IF-Set) A is given by

$$A = \{\langle x,\, \mu_A(x),\, \nu_A(x)\rangle \,|\, x \in X\}, \tag{1}$$

where $0 \le \mu_A(x) + \nu_A(x) \le 1$ for all $x \in X$ ([1]). $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ are membership function and non-membership function of A, respectively. For IF-Set A we will call

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \tag{2}$$

an intuitionistic fuzzy index of x in A. The meaning of this function is a measure of hesitation of x in A.

We use the following formulation of the MCDM problem in an IF-Set environment (cf. [4, 8]). Let $X = \{x_1, \ldots, x_m\}$ be a set of alternatives, $A = \{a_1, \ldots, a_n\}$ be a set of local criteria, $W = \{w_1, \ldots, w_n\}$ be the weights of local criteria. We denote by μ_{ji} the degree to which alternative x_j satisfies the criterion a_i, and by ν_{ji} the degree to which alternative x_j does not satisfy the criterion a_i, where $0 \le \mu_{ji} + \nu_{ji} \le 1$, for $i = 1, \ldots, n$, $j = 1, \ldots, m$. By $\pi_{ji} = 1 - \mu_{ji} - \nu_{ji}$ we denote the intuitionistic fuzzy index of alternative x_j satisfying criterion a_i.

To provide the final ranking of alternatives with the top-ranked one as the optimum, the evaluation function $E : X \to [0,1] \times [0,1]$ is used. It utilizes the membership and non-membership degrees of fulfillment of given criteria by every alternative. Then the ranking function $R : [0,1] \times [0,1] \to \mathbb{R}$ creates an ordered list of alternatives with respect to R. Thus we say that $x^* \in X$ is an optimal decision iff $R(E(x^*)) = \max_{x \in X} R(E(x))$. Both the evaluation and the ranking functions of different forms are widely considered in literature (cf. [2, 6, 9]). For further considerations we use the following notation:

$$E(x_i) = (e_i^l, e_i^u), \tag{3}$$

$$R(e_i^l, e_i^u) = r_i. \tag{4}$$

3 Requirements for the decision quality measure

The goal of the MCDM is to provide the optimal decision to the decision maker. This goal is achieved even when data quality is very low, and then the decision cannot be considered to be a high quality one. In such conditions, real-life decision of an actual decision maker may often be to abstain from deciding, to postpone the decision or to take other actions dependent on the decision maker and decision context. Thus this fact should be properly reflected by MCDM models. To achieve this, we propose providing the decision maker with a decision together with a measure of its quality (cf. [3, 8]). Such an evaluation would be an indicator of the reliability of the decision for the decision maker.

By the decision quality in the context of MCDM we mean the extent to which the optimal decision x^* is incontestable. The decision is incontestable if the ranking of alternatives is unequivocal in the sense of the evaluation function E and the degree to which data quality attributes influence the position of each alternative in the ranking. For example, if alternative $x_3 \in X$ is the top ranked one, but its evaluation is based on data from an unreliable source, the quality of such a decision should be considered to be low. On the other hand, if the final ranking remains unequivocal even with allowance of all ignorance, then the top ranked alternative, i.e. the decision, may be considered a high quality one. In other words, such a decision's quality measure could be an answer to the question of the amount of ignorance which needs to be omitted to make the final ranking of alternatives an unequivocal one.

The aim of the further considerations in this paper is to define a function $q^* : X \to [0, 1]$ measuring the quality of the optimal decision $x^* \in X$.

4 Construction of a decision quality measure

4.1 Interval evaluation function as an aggregation of knowledge

For the sake of simplicity we take the weighted mean as the aggregation method used in MCDM. Thus evaluation function E is then defined as $E(x_i) = [e_i^l, e_i^u]$, where

$$e_i^l = \sum_{j=1}^{n} w_j \mu_{ij}, \tag{5}$$

$$e_i^u = 1 - \sum_{j=1}^{n} w_j \nu_{ij}. \tag{6}$$

The interval evaluation of alternatives may be ambiguous, so in order to fulfill the requirement of providing a decision, we need to utilize the function $R(e_i^l, e_i^u) = r_i$, which determines a real evaluation $r_i \in E(x_i)$ and thus the final ranking of alternatives. Figure 4.1 presents an example of such an evaluation and ambiguity connected with it.

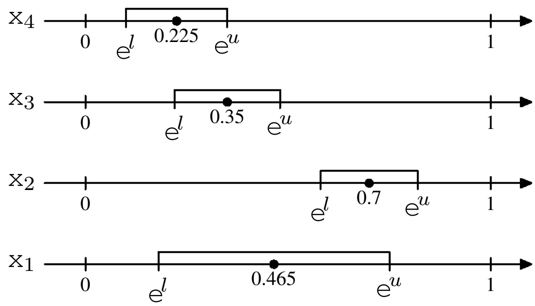

Figure 1: Ambiguous interval evaluation of alternatives in IF-Sets environment with real evaluation

Note that the use of r_i as a reliable method of constructing the final ranking is an oversimplification, since the positive and negative knowledge do not indicate directly any specified point in the interval $E(x_i)$. Thus any point from $E(x_i)$ might be proper evaluation of x_i, and nonempty intersections of $E(x_i) \cap E(x_j)$ for $x_i, x_j \in X$ only indicate higher ambiguity of the ranking and contestability of the final decision. Therefore such interval evaluations E contribute to the measure of decision quality.

4.2 Aggregation of intuitionistic fuzzy indices

In MCDM based on Atanassov's IF-Sets, the data quality attributes such as incompleteness, uncertainty or ignorance are represented by the intuitionistic fuzzy index $\pi_{ji}(x)$. We will follow the IF-Set notions in further considerations, and we will refer to all such attributes as *hesitation*.

The intuitionistic fuzzy index π_{ij}, being auxiliary to the positive and negative knowledge μ_{ji} and ν_{ji}, is not directly a part of the evaluation function E. However, it brings relevant information which, in applications, can be interpreted as a measure of hesitation.

We define the hesitation interval for each alternative $x_i \in X$ as

$$HI(x_i) = E(x_i) \cap \left[r_i - q \cdot \frac{h(x_i)}{2}, \, r_i + q \cdot \frac{h(x_i)}{2} \right], \tag{7}$$

where $h(x_i) = \sum_{j=1}^{n} w_j \pi_{ij}$ is the aggregated hesitation for x_i and some $q \in [0,1]$. q will be called the *quality coefficient*.

Figure 4.2 shows graphical interpretation of hesitation interval. Note that depending of the value of $q \in [0,1]$, it decreases ambiguity of ranking based on E and R. Thus we use it to define the decision quality measure.

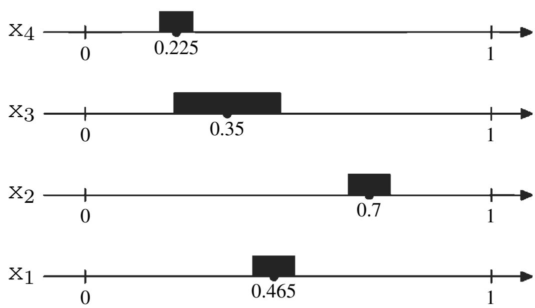

Figure 2: Hesitation Interval in IF-Sets based MCDM ($q = 1$).

4.3 Quality coefficient q^*

The quality coefficient as defined above and in [8] refers to the relations between hesitation intervals (7). For our further considerations we define decisions neighborhood as follows:

$$N(x_i) = \{x_k \in X : HI(x_i) \cap HI(x_k) \neq \varnothing\}\,. \tag{8}$$

For alternative $x_i \in X$, the set of equally evaluated alternatives will be denoted by

$$[x_i] = \{x_k \in X : r_i = r_k\}\,. \tag{9}$$

The quality of a multicriteria decision based on an IF-set is then defined as

$$q^* = \sup_{q \in [0,1]} \{q : \forall_{x_i \in X} |N(x_i)| = |[x_i]|\}. \tag{10}$$

We interpret this value as a minimum amount of hesitation which needs to be omitted to produce an unequivocal ranking of final alternatives, as presented in Figure 4.3.

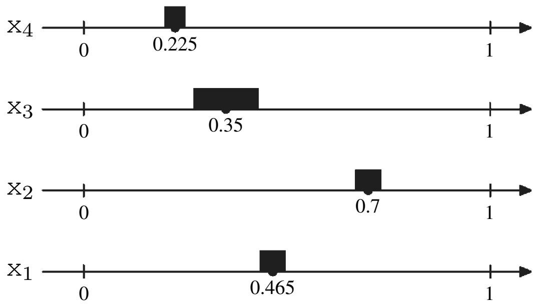

Figure 3: Hesitation Interval in IF-Sets based MCDM ($q = 0.6$).

5 Illustrative example

Let us consider a problem of choosing air-conditioning system, which has already been studied in literature (cf. [4, 7, 6, 8]). Denote the alternative set by $X = \{x_1, x_2, x_3\}$ and set of attributes by $A = \{a_1, a_2, a_3\}$. Three attributes a_1 (economical), a_2 (function), and a_3 (being operative) are taken into consideration in the process of selection. The degrees μ_{ij} of membership and the degrees ν_{ij} of non-membership for the alternative $x_j \in X$ with respect to the attribute $a_i \in A$ to the fuzzy concept „excellence" are determined in the following way:

$$((\mu_{ij},\, \nu_{ij}))_{3\times 3} = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \left(\begin{array}{ccc} x_1 & x_2 & x_3 \\ (0.75,\, 0.10) & (0.80,\, 0.15) & (0.40,\, 0.45) \\ (0.60,\, 0.25) & (0.68,\, 0.20) & (0.75,\, 0.05) \\ (0.80,\, 0.20) & (0.45,\, 0.50) & (0.60,\, 0.30) \end{array} \right). \qquad (11)$$

We define the vector of weights as

$$(w_i)_{1\times 3} = \left(\begin{array}{ccc} a_1 & a_2 & a_3 \\ 0.25 & 0.375 & 0.457 \end{array} \right). \qquad (12)$$

Set A will be treated as a set of local criteria. The evaluation function for alternatives and its aggregation are obtained using (3):

$$E(x_1) = [0.26, 0.93], \quad E(x_2) = [0.22, 0.89], \quad E(x_2) = [0.21, 0.91], \qquad (13)$$
$$r_1 = 0.56, \qquad r_2 = 0.53, \qquad r_3 = 0.54. \qquad (14)$$

The values of intuitionistic fuzzy index aggregations are then:

$$h(x_1) = 0.03, h(x_2) = 0.03, h(x_3) = 0.05. \tag{15}$$

For $q = 1$, we get the following hesitation intervals:

$$\begin{aligned} HI(x_1) &= [0.26, 0.93] \cap [0.56 - 0.015, 0.56 + 0.015] = [0.545, 0.575], \\ HI(x_2) &= [0.22, 0.89] \cap [0.53 - 0.015, 0.53 + 0.015] = [0.515, 0.545], \\ HI(x_3) &= [0.21, 0.91] \cap [0.54 - 0.025, 0.54 + 0.025] = [0.515, 0.565]. \end{aligned} \tag{16}$$

For $q = 0.5$, we obtain:

$$\begin{aligned} HI(x_1) &= [0.553, 0.568], \\ HI(x_2) &= [0.523, 0.537], \\ HI(x_3) &= [0.528, 0.552]. \end{aligned} \tag{17}$$

If $q = 0.25$, we have:

$$\begin{aligned} HI(x_1) &= [0.556, 0.564], \\ HI(x_2) &= [0.526, 0.534], \\ HI(x_3) &= [0.534, 0.546]. \end{aligned} \tag{18}$$

Only the last approximation of q^* guarantees an unambiguous final ranking. In this example the final decision is made with quality 0.25. The intuitive interpretation of this value is the answer to the question „What greatest amount of hesitation can be included into the process of decision making such that the final ranking is not ambiguous?"

6 Conclusions

In this paper we have investigated the motivations and requirements for a quality measure of multicriteria decisions in an intuitionistic fuzzy environment. We have shown the need for assessment of data quality in decision making and thus the measure of decision's quality from the perspective of decision support system design. We have presented the construction of a simple method of quality measure in such environment. The method proposed is close to the intuitive perception of decisions as being of low or high quality.

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The papers presented in this Volume 2 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) organized in Warsaw on October 8, 2010 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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