S. PIASECKI, J.W. OWSIŃSKI, AN INTRODUCTION TO A THEORY OF MARKET COMPETITION

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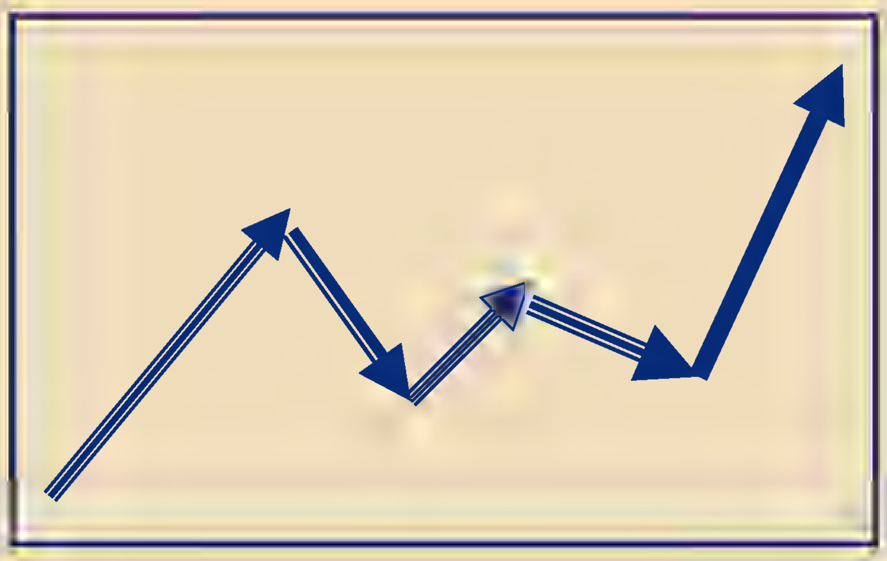
AN INTRODUCTION TO A THEORY OF MARKET COMPETITION

Volume I



Warsaw 2011

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AN INTRODUCTION TO A THEORY OF MARKET COMPETITION

Volume I

Warsaw 2011

Warsaw, January 2011

Chapter II

ACTIVITY OF A LOCAL COMPANY

1. Introduction

In the present chapter we shall consider models of activity, which do not account for the cost of transport of the products sold from the stock of the producer to the destination at the customer's.

We shall be analysing the process of activity of a local company, operating on the market having limited absorption capacity for the product considered.

For this purpose let us remind of the following notations:

μ - volume (scale) of production per unit of time, called also intensity of production, expressed in number of items or number of units of product turned out in a unit of time;

C – market price of the unit of product, expressed in monetary terms per unit of product;

Λ - market demand for the product, expressed in product units per unit of time;

$\kappa(\mu)$ – unit cost of manufacturing the product, when production intensity is μ; expressed in monetary units per unit of product;

$K(\mu)$ – unit cost of manufacturing the product, when production intensity is μ, expressed in monetary units per time unit;

$\lambda(C)$ - dependence of demand for the product from a potential customer upon the market price of the product; the value of this function is expressed in product units per unit of time. In particular, for the linear dependence, the respective function shall have the form

$$\lambda(C) = \lambda_0 \cdot q(C) = \lambda_0 \cdot \left(1 - \frac{C}{C_{mx}}\right) = a^\circ \cdot (C_{mx} - C) = \lambda_0 - a^\circ \cdot \mathrm{C}$$

$$0 \le C \le C_{mx}$$

where $\lambda_0 = \frac{1}{T}$ expresses the need of a customer for the product, also per unit of time, T being the period of use, or durability, of the product, C_{mx} is the maximum price C, for which $\lambda = \lambda_0 \cdot q(C) \to 0$, and a^o is the coefficient of proportionality, $a^o = \lambda_0/C_{mx}$;

$\Lambda(C)$ – dependence of demand (and sales – under the market equilibrium conditions) upon the price of the product, e.g. of the form,

$$\Lambda = L_{mx} \cdot \lambda(C) = L_{mx} \cdot \lambda_0 \cdot q(C) = \lambda_{mx} \cdot \left(1 - \frac{C}{C_{mx}}\right) =$$

$$= \lambda_{mx} - a \cdot C = \lambda_{mx}(C_{mx} - C)$$

where another coefficient appears, namely $a = \lambda_{mx}/C_{mx} = L_{mx}a^o$, and $\lambda_{mx} = L_{mx}\lambda_0$;

$C(\lambda)$ – dependence of the market price upon demand (in conditions of market equilibrium). In particular, in our case, for linear dependence $\Lambda(C)$, we get

$$C = \frac{\lambda_{mx} - \Lambda}{a} = \frac{\lambda_{mx} - \mu}{a} = C_{mx} \cdot \left(1 - \frac{\mu}{\lambda_{mx}}\right) = \frac{1}{a} \cdot (\lambda_{mx} - \mu),$$

where μ denotes supply, which, under market equilibrium, equals demand Λ.

$Z(C,\Lambda,\mu)$ – dependence of the company profit upon the market price, demand and production intensity. Profit is, of course, expressed in monetary terms per unit of time:

$$Z(C,\Lambda,\mu) = C\Lambda - \mu\kappa(\mu).$$

We neglect here various other costs and taxes, liabilities and outstanding payments, etc., so that the magnitude considered is the operational profit (return).

As alluded already to before, in the steady state it is reasonable to assume the equality of production and demand, $\mu = \Lambda$. Under this assumption we get

$$Z(C,\mu) = C \cdot \mu - K(\mu) = C \cdot \mu - \mu \cdot \kappa(\mu) \quad .$$

If we admit that the company aims at maximisation of profit Z, then the optimum value μ^{*}[1] is determined from the equation

$$\frac{\partial Z(C,\mu)}{\partial \mu} = 0 \; .$$

By performing differentiation, we obtain

$$C - \frac{d}{d\mu} K(\mu)\Big|_{\mu=\mu^*} = 0$$

or:

$$\frac{d}{d\mu} K(\mu) = C \; .$$

This relation is illustrated in Fig. 2.1, given further on.

2. Some elementary models

2.i. Elementary model with constant unit production cost

In this model we shall assume that the cost of turning out a unit of product does not depend upon the scale (intensity of produc-

[1] We shall use further on the asterisk (*) to denote the values of variables, corresponding to particular conditions (like maximum profit, market clearing etc.), and obtained from respective calculations.

tion), $\kappa(\mu) = b$, and demand is a linear function of C: $\Lambda = \lambda_{mx}$-aC. Then:

$Z = C\Lambda$-μb = $\Lambda(C$-$b)$, if we substitute $\mu = \Lambda$,

$\mu(C$-$b)$, if we substitute $\Lambda = \mu$.

If we now substitute, respectively, $\Lambda = \lambda_{mx}$-aC, or $C = C_{mx}$-μ/a, we obtain:

Bertrand's model:

$Z(C) = (\lambda_{mx}$-$aC)\cdot(C$-$b)$ and $dZ/dC = ab+\lambda_{mx}$-$2aC$, wherefrom

$$C^* = \frac{1}{2}(\frac{\lambda_{mx}}{a} + b),$$

and

Cournot's model:

$Z(\mu) = \mu(C_{mx}$-b-$\mu/a)$ and $dZ/d\mu = C_{mx}$-b-$2\mu/a$, wherefrom

$$\mu^* = \frac{1}{2}a(C_{mx} - b).$$

Considering the relation $aC_{mx} = \lambda_{mx}$ and maximisation of the profit we ultimately get

$$C^* = \frac{1}{2}(C_{mx} + b); \quad \mu^* = \frac{1}{2}a(C_{mx} - b);$$

$$\Lambda^* = \frac{1}{2}a(C_{mx} - b) = \frac{1}{2}aL_{mx}(C_{mx} - b); \quad Z^* = \frac{1}{4}a(C_{mx} - b)^2.$$

Two forms of the same model result from the adoption of different decision variables. In the Bertrand's model the decision variable is the sales price C, while the intensity of production, μ, is selected depending upon the demand on the market. In Carnot's model it is production intensity (or scale), μ, that constitutes the decision variable, while the price C results from the market conditions.

Market competition for the elementary model 2.i

The possibility of existence of competition regarding the sales of a given product on a market depends upon the scale of profitability of starting production (and selling the product) by the potential competitors.

We shall measure profitability of production with the rate of return on the costs borne per unit of time:

$$\varepsilon = \frac{Z}{K}.$$

In particular, when the value of the rate of return from production is lower than the interest rate on bank deposit (for the same time unit, of course), then undertaking of production activity becomes purposeless.

Let us verify the behaviour of the value of ε as a function of C or μ for the here adopted elementary model. So,

$$\varepsilon(C) = \frac{\mathrm{Z}}{\mathrm{K}} = \frac{\Lambda \cdot (C-b)}{\Lambda \cdot b} = \frac{C}{b} - 1$$

or

$$\varepsilon(\mu) = \frac{\mu \cdot (C-b)}{\mu \cdot b} = \frac{C}{b} - 1 = \frac{C_{mx} - \dfrac{\mu}{a}}{b} - 1.$$

It can be concluded from the above expression that we can increase the rate of return by raising C (beyond the value C^*), or by decreasing μ (below the value μ^*), though we then get lower profit Z. In the particular case of $C = C^*$ we then obtain

$$\varepsilon(C^*) = \varepsilon(\mu^*) = \frac{1}{2} \cdot \frac{C_{mx} - b}{b}.$$

If a new competitor wishes to enter a market, on which some company has been “residing” for long, then this newcomer must enter the market with a competitive product, sold at a lower price. After some time, let us assume, half of the customers shall be buying the cheaper products of the competitor, while the remaining half – the more expensive products of the "resident" company. If the price, quoted by the latter company, has been optimal, then the losses on profit of the "resident" company shall be lower than the losses on profit of the competitor, in relation to the maximum profit Z^{opt} that could be achieved. If, however, the "resident" company does not lower its price, then, after a sufficiently long period of time, it will get wiped away from the market. If, on the other hand, it lowers the price, this shall trigger off the price war, which shall be won by the party that can stand longer the growing losses – negative profits. In any case, under the assumptions here adopted, only one company would stay on the market. Unless, of course, they do not reach an agreement before.

2.ii. Elementary model with unit cost of production increasing in production intensity

For this model, we assume

$$\kappa(\mu) = b^0 \cdot \mu \quad ; \quad b^0 > 0 ,$$

where b^0 is the cost of manufacturing a unit of product for $\mu = 1$. For these assumptions, we have $K(\mu) = \mu\ \kappa(\mu) = b^0\mu^2$, and

$$\frac{d}{d\mu}\mathrm{K}(\mu) = \kappa(\mu) + \mu \cdot \frac{d}{d\mu}\kappa(\mu) = 2 \cdot \mu \cdot b^0$$

Since, as we have shown earlier, the general relation $\frac{d}{d\mu}K(\mu) = C$ holds, we ultimately obtain that $2\mu^* b^0 = C$ and hence $\mu^* = C/2b^0$, as well as $K(\mu^*) = \mu^*\kappa(\mu^*) = b^0(\mu^*)^2$.

Now, after substituting the expression for μ^*, we get

$$K(\mu^*) = \frac{1}{b^0}\left(\frac{C}{2}\right)^2 .$$

Next, let us note that we obtain the value of μ_{mx} (appearing in Fig. 2.1), for which profit falls down to zero, by solving the equation $C \cdot \mu_{mx} - \mathrm{K}(\mu_{mx}) = 0$. As we substitute $\mathrm{K}(\mu_{mx}) = \mu_{mx} \cdot \kappa(\mu_{mx})$, we obtain $C \cdot \mu_{mx} - \mu_{mx} \cdot \kappa(\mu_{mx}) = 0$, and hence $\kappa(\mu_{mx}) = C$.

Yet, $\kappa(\mu_{mx}) = b^0 \cdot \mu_{mx}$, so that $\mu_{mx} = \dfrac{C}{b^0}$.

The above relations are illustrated in Fig. 2.1, in which the lines *A* and *B* are parallel.

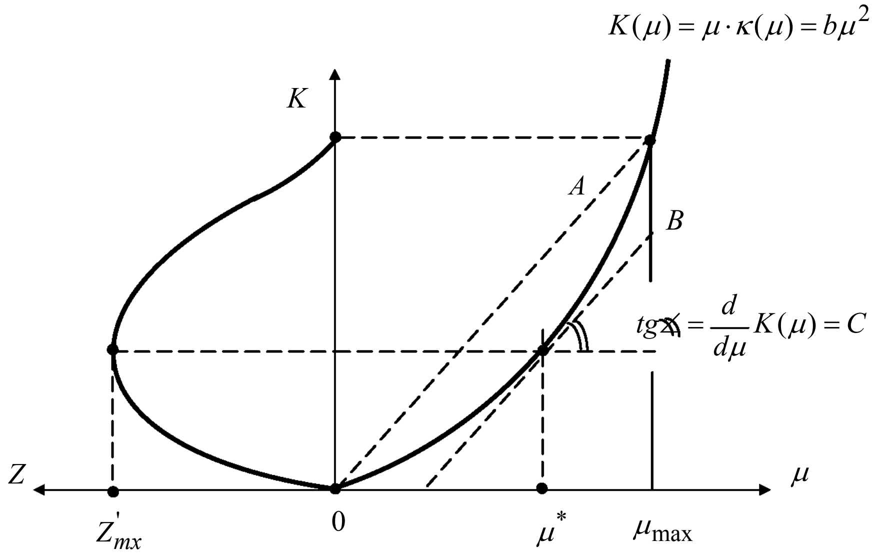

Fig. 2.1. Illustration of relations for production intensity, price and profit deduced in the text

For the assumptions adopted, let us now determine the profit of the company under optimum production intensity and given product price, with equality of the production intensity μ^* and market demand, the latter given by $\Lambda = \lambda_{mx} - aC$.

By taking into account the equality of production and demand, we get the equation

$$\frac{C}{2b} = \lambda_{mx} - a \cdot C \,.$$

As we solve this equation with respect to price, we obtain the expression for market equilibrium price:

$$C^* = \frac{2b^0 \lambda_{mx}}{1 + 2ab^0}$$

and the corresponding optimum production intensity:

$$\mu^* = \frac{C^*}{2b^0} = \frac{\lambda_{mx}}{1 + 2ab^0} \,.$$

By substituting the equilibrium price into the formula for profit, we obtain the final expressions for the maximum company profit and cost, corresponding to the optimum production intensity:

$$Z^*(\mu^*) = b^0 \left(\frac{\Lambda_{mx}}{1 + 2ab^0} \right)^2 ,$$

$$K^*(\mu^*) = \frac{1}{b^0} \left(\frac{b^0 \lambda_{mx}}{1 + 2ab^0} \right)^2 = b^0 \left(\frac{\lambda_{mx}}{1 + 2ab^0} \right)^2 = Z^*(\mu^*) \,.$$

Hence, profit attains 100% of costs!

Market competition for the elementary model 2.ii

The possibility of existence of competition on a given market depends upon the profitability of starting production and selling the product in question by the (potential) competitors.

We shall measure profitability of production with the rate of return of the outlays borne per unit time:

$$\varepsilon(\mu) = \frac{Z(\mu)}{K(\mu)}$$

Let us remind that in case the rate of return from production is lower than interest on bank deposits – for the same time unit – undertaking of production activity is purposeless.

We shall now verify the behaviour of the value of ε as a function of μ for the elementary model 2.ii:

$$\varepsilon(\mu) = \frac{C\mu - \mu^2 b^0}{\mu^2 b^0} = \frac{C - \mu \cdot b^0}{\mu \cdot b^0} = \frac{\frac{\lambda_{mx} - \mu}{a} - \mu \cdot b^0}{\mu \cdot b^0},$$

since $C = \frac{\lambda_{mx} - \mu}{a}$; $\Lambda = \mu$.

Finally, then, we get

$$\varepsilon = \frac{\lambda_{mx}}{a \cdot b^0 \cdot \mu} - \left(\frac{1}{a \cdot b^0} + 1 \right).$$

The course of this function is shown in Fig. 2.2.

When considering the course of the function, defining the value of ε for the elementary model 2.ii, we can see that every new company, starting its production with the smallest production intensity, shall have higher rate of return than a "resident" company, producing with optimum intensity μ^* for a given market price *C*. Every new company shall, therefore, be able to sell the same product for a lower price, and thus take over a part of existing demand.

Consequently, within the framework of this model, the number of producers, *n*, shall increase infinitely – in practice, though, until the rate of return from production will have fallen down to the level of interest on deposits in the banks.

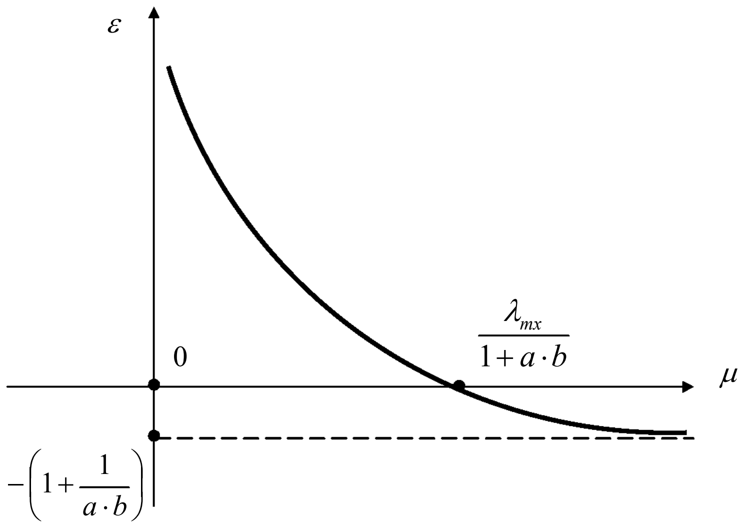

Fig. 2.2. Basic relations for the elementary model 2.ii.

Each producer, given the assumption that all of them dispose of the same technology, shall satisfy a part of demand:

$$\Lambda_n = \frac{1}{n}(\lambda_{mx} - a \cdot C)$$

and, attempting to maximise own profit, shall produce with intensity equal:

$$\mu_n = \frac{C}{2 \cdot b^0}.$$

Since, however, the equality $\mu_n = \Lambda_n$ must hold, we can determine from the equation

$$\frac{C}{2b^0} = \frac{1}{n}(\lambda_{mx} - a \cdot C)$$

The sales price, corresponding to market equilibrium (market clearing price):

$$C_n = \frac{2 \cdot b^0 \cdot \lambda_{mx}}{n + 2 \cdot a \cdot b^0}.$$

From this, ultimately, by substituting the value of C_n, we get

$$\mu_n^* = \frac{\lambda_{mx}}{n + ab^0}$$

and

$$Z_n^* = C_n \cdot \mu_n^* - b^0 \cdot (\mu_n^*)^2 = b^0 \left(\frac{\lambda_{mx}}{n + d \cdot b^0} \right)^2 .$$

If n could actually increase to infinity, then we would obtain:

$$C_n \to 0 \;\; ; \;\; Z_n^* \to 0 \;\; ; \;\; \mu_n^* \to 0 .$$

The here analysed elementary model corresponds, therefore, to the **process of ideal competition**, leading to maximum price decrease with maximum satisfaction of demand. This particular model illustrates the (limit) capacities of the mechanisms of the free market economy.

2.iii. Elementary model with constant unit costs of production and nonlinear dependence of demand upon price

We shall now assume that $\kappa(\mu) = b$ = const., and

$$\Lambda = \lambda_{\text{mx}} \frac{C_0}{C + C_0} ,$$

where C_0 denotes the level of price, for which demand falls by 50%.

If we now introduce the expression for the cost $\kappa(\mu)$ into the formula for profit, we shall get

$$Z(\Lambda, C, \mu) = \Lambda \cdot C - \mu \cdot \kappa(\mu) = \Lambda \cdot C - \mu \cdot b .$$

Next, taking into account the equality $\Lambda = \mu$, we obtain $Z(\Lambda, C) = \Lambda \cdot \left(C - b \right)$, and finally $Z(C) = \lambda_{mx} \cdot C_0 \cdot \dfrac{C - b}{C + C_0}$.

Then, as we determine the derivative of Z with respect to C, we obtain

$$\frac{dZ}{dC} = \lambda_{mx} \cdot C_0 \cdot \frac{C_0 + b}{(C_0 + C)^2}.$$

This derivative is a decreasing, always positive, function of C. Hence, profit always increases with the increase of the sales price, despite the fact that as price increases, demand and production do decrease.

Market competition for the elementary model 2.iii

By substituting the expressions for profit and cost into the formula for ε, we get:

$$\varepsilon = \frac{\Lambda \cdot (C - b)}{\Lambda \cdot b} = \frac{C}{b} - 1.$$

The above formula implies that we can obtain an increase of the rate of return by increasing product price (and decreasing production), which, at the same time, ensures increased profit.

If there are many producers of the same product on the market, then, ultimately, only one producer – of the cheapest product – shall remain on the market.

If there is a "resident" company on the market, turning out the product in question, then we can always eliminate it from the market, by introducing a product sold at lower price. If the "resident" company decides to defend itself by lowering correspondingly its price of the product, then a spiral of price decreases is set in motion, leading to minimisation of prices and profits.

At the same time, the formula implies that the rate of return shall decrease, down to the value of the interest rate on deposits. Then, production shall become (relatively) unprofitable. This is another example of the **process of ideal competition**.

2.iv. Elementary model with unit production costs increasing with production scale and nonlinear dependence of demand upon price

In this model we assume that $\kappa(\mu) = b^0\mu$, and $\Lambda = \lambda_{\text{mx}}\frac{C_0}{C + C_0}$, the latter being the demand, equal the sales of the product. As we substitute the expression for κ to the formula, defining profit, we obtain

$$Z(\Lambda,C) = C\Lambda - \mu^2 b^0 = \Lambda(C - \Lambda b^0).$$

Since $\Lambda = \mu$, then

$$Z(C) = \lambda_{mx} \cdot C_0 \cdot \frac{C^2 + C_0 \cdot C - C_0 \cdot \lambda_{mx} \cdot b}{\left(C_0 + C\right)^2},$$

so that

$$Z(C = 0) = -\lambda_{mx} \cdot b \quad ; \quad \lim_{C \to \infty} Z(C) = \lambda_{mx} \cdot C_0 .$$

Next, when we determine the derivative of Z with respect to C, we get

$$\frac{dZ}{dC} = \lambda_{mx} \cdot C_0 \cdot \frac{C + C_0(C_0 + 2\lambda_{mx} b^0)}{\left(C_0 + C\right)^3} > 0$$

so that $\frac{dZ}{dC}(C = 0) = \lambda_{mx} \cdot \left(1 + \frac{2\lambda_{mx} b}{C_0}\right) \quad ; \quad \lim_{C \to \infty} \frac{dZ}{dC} = 0 .$

Similarly as in the previous case, along with the increase of C there is an increase of profit, with simultaneous decrease of demand and production. The shape of the function $F(C) = Z(C) + Q$ is shown in Fig. 2.3.

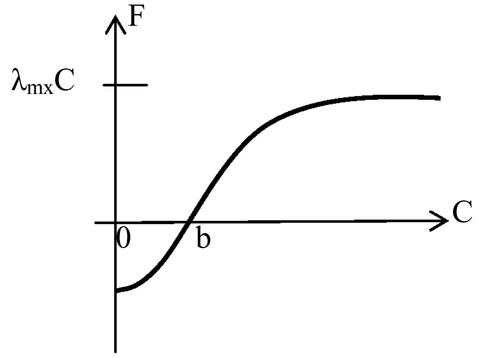

Fig. 2.3. Sketch for the shape of the function $F(C) = Z(C) + Q$.

Market competition for the elementary model 2.iv

In this case the value of the rate of return is expressed through the formula

$$\varepsilon(\Lambda,C) = \frac{\Lambda(C - \Lambda b^0)}{\Lambda^2 b^0} = \frac{C}{\Lambda b^0} - 1 .$$

When we substitute the expression for the value of demand into the above formula, then we get

$$\varepsilon(\Lambda,C) = \frac{C(C + C_0)}{\lambda_{mx} C_0 b^0} - 1 .$$

This implies that the rate of return increases with increasing price, C, and decreasing production intensity, μ. Profit, Z, also increases with the increase of price.

Conclusion is straightforward: we deal here, again, with the process of perfect competition. Entry onto a market, where another producer is active, forces us to lower the price, which, in turn, triggers off a chain process of price decreases, lasting until the decreasing rate of return becomes equal to the interest on bank deposits.

3. Basic models

3.i. Basic model with linear demand function

In reality, in the majority of cases, cost of producing a unit of product has a different form. Namely, we have

$$\kappa(\mu) = \frac{Q}{\mu} + b \ ,$$

where Q is the constant cost that has to be accounted for in production activity, and which is independent of the variable production intensity.

The value of Q depends upon the investment outlays I, associated with starting of production: purchase of machines, construction of appropriate facilities, etc., by the intermediary of amortisation costs of the fixed assets, which depend upon the admissible period of use of the company assets, T. Thus, we have, approximately – see Chapter I: $Q = I(\rho+1/T)$, where ρ is the rate of interest on credit, contracted in order to finance the production investment.

The quantity b is – as before – the **direct cost** of manufacturing a single unit of product (encompassing the cost of materials, energy, labour, use of machines, renewable turnover credit, and so on).

If we adopt such a function $\kappa(\mu)$ of unit costs, then profit shall take the form

$$Z(\Lambda,\mu,C) = \Lambda C - \mu \cdot \kappa(\mu) = \Lambda C - \mu \cdot b - Q \ .$$

Naturally, given that company conducts a rational policy, the equality $\Lambda = \mu$ must hold. We then get

$$Z(\mu,C) = \mu \cdot (C - b) - Q$$

or

$$Z(\Lambda,C) = \Lambda \cdot (C - b) - Q \ .$$

The form of the function Z implies that after introducing the respective expressions for μ and C profit shall be defined as

$$Z(C) = (\lambda_{mx} - aC)(C - b) - Q, \text{ with } Z(C{=}b) = Z(C{=}C_{mx}) = -Q,$$

or

$$Z(\mu) = \mu(C_{mx} - b - \mu/a) - Q, \text{ with } Z(\mu{=}0) = Z(\mu{=}\lambda_{mx}(1\text{-}b/C_{mx})) = -Q.$$

The shapes of the respective functions are shown in Fig. 2.4.

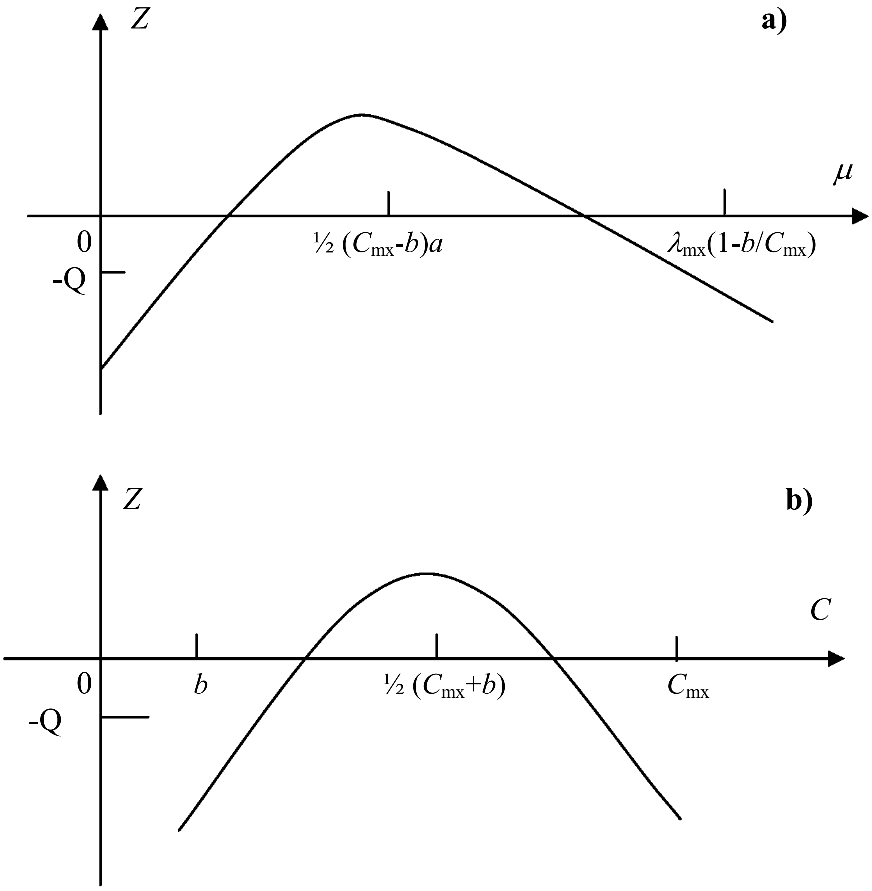

Fig. 2.4. Illustration to basic model 3.i

Let us now determine the optimum parameters of activity. We, namely, have

$$Z(C) = (\lambda_{mx} - a \cdot C) \cdot (C - b) - Q \text{ (see Fig. 2.5).}$$

The company has to regulate the intensity of production in such a manner as to achieve, under changing market price, the possibly highest profit. As we differentiate the above equation with respect to *C*, we obtain

$$\frac{dZ}{dC} = -2 \cdot a \cdot C + \lambda_{mx} + a \cdot b = 0 \, ,$$

hence

$$C^* = \frac{\lambda_{mx} + a \cdot b}{2a}$$

and

$$\Lambda^* = \lambda_{mx} - a \cdot C^* = \frac{\lambda_{mx} - a \cdot b}{2} \, .$$

If we now make use of dependence of profit on the scale of production, then we get

$$\mu^* = \frac{\lambda_{mx} - a \cdot b}{2} \, ,$$

which guarantees the highest profit. Of course, production activity shall be profitable, when the following inequality holds:

$$\mu^* > \mu_o \, .$$

Profit shall then be equal

$$Z^* = \left(\frac{\lambda_{mx} - a \cdot b}{2} \right)^2 \cdot \frac{1}{a} - Q \, .$$

The course of the function of profit, depending upon the market price, in conditions of market equilibrium, is shown in Fig. 2.6.

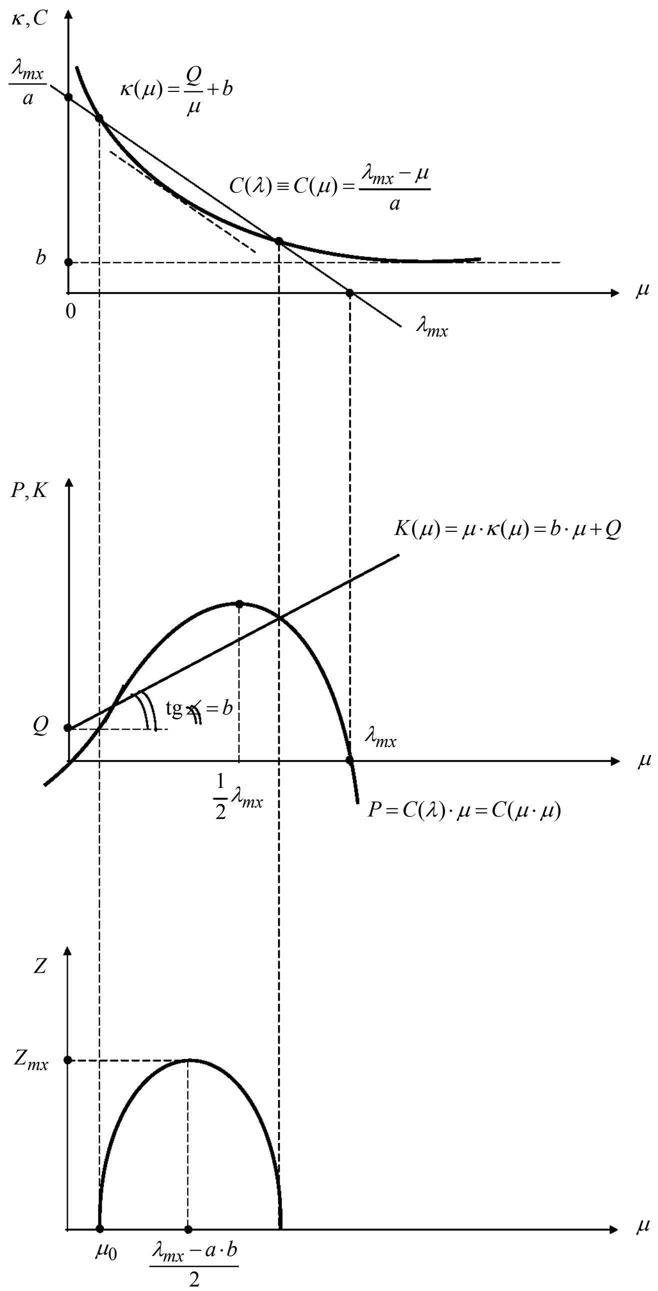

Fig. 2.5. Determination of optimum production parameters

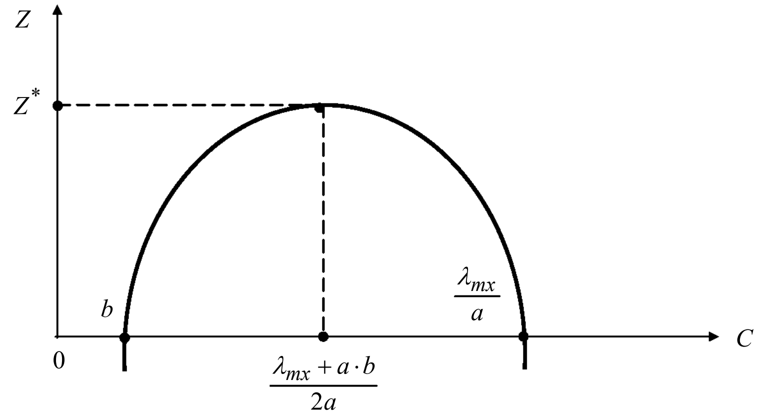

Fig. 2.6. Dependence of profit upon price in conditions of market equilibrium

Market competition for the basic model 2.i

Let us compare two production technologies, featuring maximum yields μ_{mx}^1 and μ_{mx}^2, with $\mu_{mx}^1 < \mu_{mx}^2$. They are, of course, characterised by the respective values of parameters Q_1, Q_2 and b_1, b_2, with $Q_1 < Q_2$, which is an obvious inequality, as we assume that competitors apply only rational technologies. Similarly, the following inequality should hold:

$$b_1 > b_2$$

which is no longer such an obvious necessity.

Assume that the latter inequality is not true, and that $b_1 < b_2$, so that we have, for a given $\mu < \mu_{mx}^1, \mu_{mx}^2$, the following inequality for the unit costs:

$$b_1 + \frac{Q_1}{\mu} < b_2 + \frac{Q_2}{\mu}.$$

If this inequality actually held, the second of the technologies considered would be entirely eliminated. Namely, for example, in the case of $\mu = 2\mu_{mx}^1$, one should use in production two production lines with the first technology in pace of one line with the second technology.

For the rational technologies, the inequality

$$\mu_{mx}^1 < \mu_{mx}^2$$

must entail the inequalities

$$Q_1 < Q_2 \text{ and } b_1 \geq b_2,$$

and, moreover, the inequality

$$b_1 + \frac{Q_1}{\mu} > b_2 + \frac{Q_2}{\mu}.$$

Consequently, if there is on the market a producer manufacturing a given product with optimum intensity (scale),

$$\mu^* = \frac{\lambda_{mx} - a \cdot b}{2}$$

with the sales price

$$C^* = \frac{\lambda_{mx} + a \cdot b}{2a}$$

then the condition for eliminating this producer from the market is that the competitor(s) apply a technology with bigger production scale, μ_1, such, for which the value of b_1 is smaller than the b for the current "resident" producer.

Thus, in order to push out from the market the "resident" producer, the competitor has to apply the technology featuring intensity $\mu_1 > \mu^*$, and $\kappa_1 < \kappa^* = b + \frac{Q}{\mu^*}$. Then, price C_1 can be set lower than the price to date, C^*. This will lead to bigger sales, $\Lambda_1 > \Lambda^*$, but the resulting profit shall be smaller.

In view of a similar shape of the profit function, Z, as in the elementary model 2.i, conclusions concerning the process of competition will be identical: the race shall be won by that one of the competitors, whose financial reserves are ampler. Ultimately, the market shall be entirely appropriated by the new producer, unless the “resident” company undertakes a defence.

Let us now investigate the behaviour of the rate of return from production for the here considered cost function. We now have, namely,

$$\varepsilon = \frac{Z(\mu)}{\mathrm{K}(\mu)} = \frac{C \cdot \mu - b \cdot \mu - Q}{b \cdot \mu + Q} = \frac{\dfrac{\lambda_{mx} - \mu}{a}}{b + \dfrac{Q}{\mu}} - 1$$

and the course of this function is shown in Fig. 2.7.

If, then, the “resident” company satisfies the demand given the price

$$C^* = \frac{\Lambda_{mx} + a \cdot b}{2a}$$

the new company, entering the market, must sell its product for a lower price, which can be done only with bigger production and the associated lowering of unit costs. Then, the competitor shall be able to enter the market. Consequently, the “resident” company, in order to defend its market share, shall be forced to also lower its price.

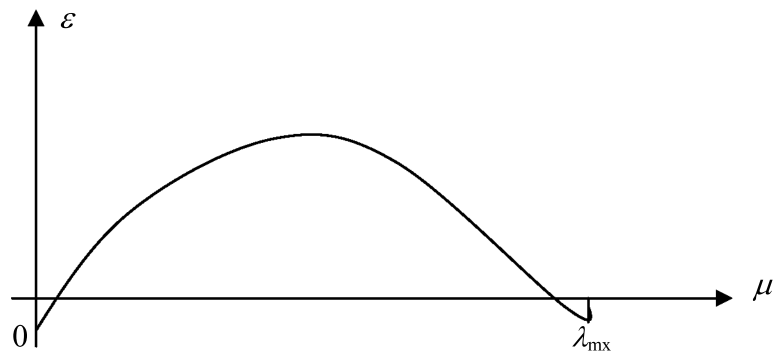

Fig. 2.7. Rate of return for the basic model 2.i

This means triggering off a price war. In any case, then, there will remain on the market only one producer-monopolist, either the "resident" one or the newly entering one.

The new producer shall be able to push away the optimal production, provided this new company is capable of lowering the value of $b+Q/\mu$. If this is impossible, the new company might secure for itself market entry by applying "dumping" prices. On the other hand, a producer behaving non-optimally (selling products for the price $C > C^*$) can be easily pushed away from the market by using the same technology in an optimal manner, i.e. with parameters C^* and μ^*.

3.ii. Basic model with quadratic dependence of demand upon price

We shall assume that demand is described with the function

$$\Lambda = \lambda_{mx}\left(1 - \frac{C}{C_{mx}}\right)^2$$

and the unit by the function

$$\kappa = \frac{Q}{\mu} + b$$

Then, profit is defined by the expression

$$Z(C) = \lambda_{mx}\left(1 - \frac{C}{C_{mx}}\right)^2 \cdot (C - b) - Q$$

The course of the function $F(C) = Z(C) + Q$ is shown in Fig. 2.8, where $C^* = \frac{1}{3}(C_{mx} + 2b)$.

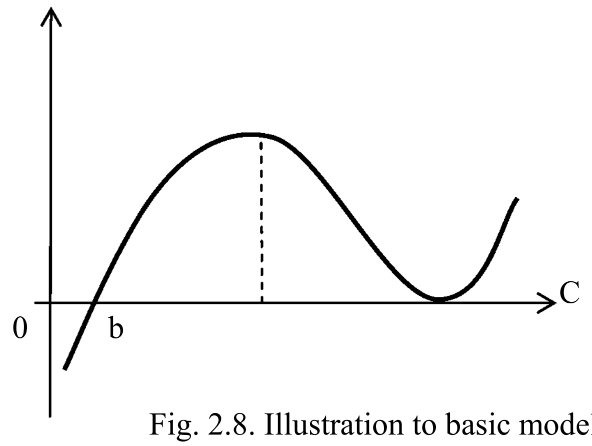

Fig. 2.8. Illustration to basic model 3.ii

Market competition for the basic model 3.ii

In view of the similar course of the function $Z(C)$ as in model 2.iv, all the conclusions pertinent to the latter model apply here.

In this model the value of the profitability (return) is defined with the expression

$$\varepsilon = \frac{(C_{mx} - C)^2 \cdot C}{(C_{mx} - C)^2 \cdot b + Q \cdot \frac{C_{mx}^2}{\lambda_{mx}}} - 1$$

The coefficient ε attains the maximum value close to the value C^*, while at the extremes of the interval (0, C_{mx}), where the function $\varepsilon(C)$ is defined, it assumes the value of -1.

Functions $Z(C)$ and $\varepsilon(C)$ have a similar course in the case, when the demand function has the form

$$\Lambda = \lambda_{mx} \frac{1 - (C / C_{mx})^2}{1 + (C / C_{mx})^2}$$

3.iii. Basic model with nonlinear dependence of demand upon price

We now assume that the cost of manufacturing a unit of product, as a function of sales price, is given by

$$\kappa(\mu) = b + \frac{Q}{\mu},$$

while demand is given by $\Lambda = \lambda_{mx}C_0/(C_0+C)$, where C_0 is the price, for which demand falls to half of its potential magnitude.

As we substitute the unit cost of production to the formula for profit, we obtain, under the condition of $\mu = \Lambda$,

$$Z(\Lambda, C, \mu) = \Lambda \cdot C - \mu \cdot \left(b + \frac{Q}{\mu} \right) = \Lambda \cdot (C - b) - Q .$$

Then, as we substitute the expression for Λ, we get

$$Z(C) = \lambda_{mx} \cdot C_0 \cdot \frac{C - b}{C + C_0} - Q$$

Note now that

$$\frac{dZ}{dC} = \lambda_{mx} \cdot C_0 \cdot \frac{C_0 + b}{(C_0 + C)^2} > 0$$

i.e. profit is an increasing function of price, with the value of the derivative tending to zero with the increase of product price. Consequently, the highest value of profit shall be given by the expression

$$\lim_{C \to \infty} Z(C) = \lambda_{mx} \cdot C_0 - Q \, .$$

We shall assume that the limit value is positive. This is insofar justified as there is no sense of conducting activity that brings losses. Since, at the same time, profit for $C = 0$ is negative, appoint must exist, at which the profit curve crosses the value of zero. This point is defined by

$$C^{\circ} = C_0 \cdot \frac{Q - \lambda_{mx} \cdot b}{\Lambda_{mx} \cdot C_0 - Q}$$

It can be easily noted that this formula is true when the following inequality holds

$$b < \frac{Q}{\lambda_{mx}} < C_0$$

confirming the previously adopted condition of $\Lambda_{mx} \cdot C_0 - Q > 0$. We shall be assuming further on that the above inequality holds.

It can be concluded from the properties of the function *Z*, listed above, that the entrepreneur shall be increasing the price of products in order to raise the profit, while consenting to the decrease of demand, as given by the formula

$$\Lambda = \lambda_{mx} \frac{C_0}{C_0 + C}.$$

Market competition for model 3.iii

As we determine the value of ε for this model, we get

$$\varepsilon = \frac{C_0 \cdot \lambda_{mx} \cdot C}{C_0 \cdot \Lambda_{mx} \cdot b + Q \cdot (C_0 + C)} - 1.$$

We note then that

$$\frac{d\varepsilon}{dC} = C_0^2 \cdot \frac{\lambda_{mx} \cdot b + Q}{\left[C_0 \cdot \Lambda \cdot b + Q(C_0 + C)\right]^2} > 0$$

which means that the function $\varepsilon(C)$ increases with *C*. Since for $C = 0$ its value is negative (-1), and

$$\lim_{C \to \infty} \varepsilon(C) = \frac{C_0 \cdot \lambda_{mx}}{Q} - 1.$$

The latter value is positive, for the assumptions adopted. It can now be easily calculated that the function $\varepsilon(C)$ crosses the zero value in the point

$$C^{\times} = C_0 \cdot \frac{\lambda_{mx} \cdot b + Q}{\lambda_{mx} \cdot C_0 - Q}.$$

For a rational design of activity, the value of C should be selected above the C^{x}.

If a newcomer company wanted to enter the market, which is ruled by a "resident" company, then it would have to start selling equivalent products at a lower price, attaining thereby lower profits. Consequently, the newcomer company cannot count on credit from the bank, whose customer is the "resident" company, enjoying higher profitability indicator. The newcomer company may count only on its own funds. As noted already, the reaction of the "resident" company shall consist in the lowering of prices of own products. A spiral of price decreases shall ensue.

This kind of situation does not encourage new firms to enter the market.

3.iv. Additional basic model with hyperbolic dependence of demand upon price

Assume that the unit cost of production is given in the form $\kappa(\mu) = Q/\mu + b$, with $b > 0$. In this model we adopt an extreme, limit dependence of demand upon price in the form $\Lambda = \alpha_0/C$, where α_0 is the annual sales value.

By substituting Λ into the formula for profit, we get

$$Z(\Lambda,\mu,C) = \Lambda\cdot C - \mu\cdot(\frac{Q}{\mu}+b) = \mu\cdot(C-b) - Q$$

If we now assume, as usual, that the market clearing condition, $\Lambda = \mu$, holds, then, after the respective substitution, we get

$$Z(\Lambda,C) = \Lambda\cdot\left(C-b\right) - Q\,.$$

Then, after substituting $\Lambda = \alpha_0/C$, we obtain

$$Z(C) = \alpha_0 - Q\cdot\frac{b\cdot\alpha_0}{C}\,.$$

As we can easily see, profit increases with the rising value of C. It can attain at most the value

$$\lim_{C \to \infty} Z(C) = \lim_{C \to \infty} \left\{ \alpha_0 - Q - \frac{b \cdot \alpha_0}{C} \right\} = \alpha_0 - Q .$$

We shall assume that the value of the limit is positive, $Q < \alpha_0$. We can then determine the minimum value of price, C_0, beyond which the value of profit becomes positive:

$$Z(C_0) = \alpha_0 - Q - \frac{b \cdot \alpha_0}{C_0} = 0$$

By solving the above equation we obtain

$$C_0 = b \cdot \frac{1}{1 - \dfrac{Q}{\alpha_0}}$$

Dependence of profit upon the value of price, C, is shown in Fig. 2.9.

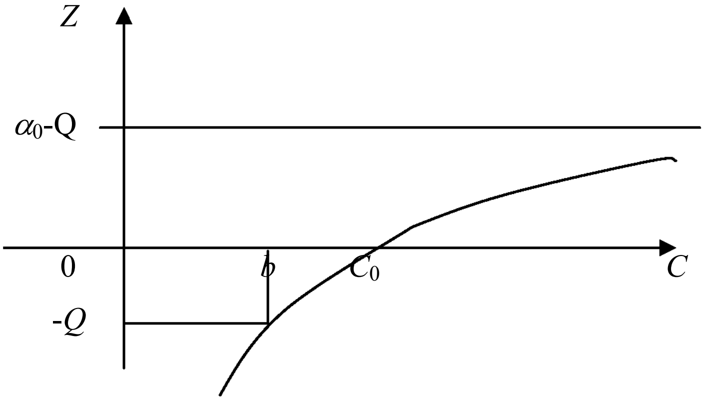

Fig. 2.9. Dependence of profit upon price for basic model 3.iv

Market competition for the additional model 3.iv

The return rate for this model is given by

$$\varepsilon = \frac{\Lambda \cdot (C-b) - Q}{\Lambda \cdot b + Q} = \frac{1}{\frac{b}{C} + \frac{Q}{\alpha_0}} - 1 .$$

The form of this expression implies that the value of the return rate increases with the increase of production intensity, and

$$\lim_{C \to \infty} \varepsilon(C) = \frac{\alpha_0}{Q} - 1 \quad \textit{and} \quad \varepsilon(C_0) = 0.$$

Hence, we see that price C ought to increase in order for the rate of return on production costs to increase, as well.

Ultimately, the system: price (C) – intensity of production (μ) shall be unstable. After the entry onto the market of a new producer, with a lower price, there would follow a spiralling price decrease, taking place in conditions of perfect competition.

3.v. Basic model with polynomial dependence of demand upon price

We assume now that dependence of demand, λ, on product price, C, is polynomial, as described in Chapter I, i.e.

$$\Lambda = \frac{\lambda_{\max}}{C_{\max}^3}\left(C_{\max}^3 - 3 \cdot C^2 \cdot C_{\max} + 2 \cdot C^3\right) =$$

$$= \frac{\lambda_{\max}}{C_{\max}^3}\left(C_{\max} - C\right)^2 \cdot \left(C_{\max} + 2C\right)$$

Then, let the cost of production have the form

$$\kappa = \frac{Q}{\mu} + b$$

We are interested in such a value of $\mu^* = \Lambda^*$, for which profit

$$Z = \Lambda \cdot C - \mu \cdot \kappa = \Lambda(C-b) - Q$$

attains the highest value.

By using the linear dependence of demand upon price (see Chapter I) we can write down

$$Z = \frac{\lambda_{\max}}{C_{\max}^3}\left(C_{\max}^3 - 3 \cdot C^2 \cdot C_{\max} + 2 \cdot C^3\right)(C-b) - Q .$$

In order to determine the value of Z^* we first determine C^*, the price, for which profit Z attains its maximum value. For this purpose we assume the derivative equals zero:

$$\frac{dZ}{dC} = \frac{d\Lambda}{dC} \cdot (C-b) + \Lambda = 0$$

By carrying out differentiation, we obtain

$$\frac{d\Lambda}{dC} = -6 \cdot \frac{\lambda_{\max}}{C_{\max}^3} \cdot (C_{\max} - C) \cdot C$$

As we substitute the latter formula to the derivative of profit, we get

$$\frac{dZ}{dC} = -6 \cdot \frac{\lambda_{\max}}{C_{\max}^3} \cdot (C_{\max} - C) \cdot C \cdot (C-b) +$$

$$+ \frac{\lambda_{\max}}{C_{\max}^3} \cdot (C_{\max} - C)^2 (C_{\max} - C)^2 (C_{\max} + 2C) =$$

$$= \frac{\lambda_{\max}}{C_{\max}^3} \cdot (C_{\max} - C) \left[C_{\max}^2 + C \cdot (C_{\max} + 6b) - 8C^2 \right]$$

We now equate the second factor of the product with zero, i.e.

$$C_{\max}^2 + C \cdot (C_{\max} + 6b) - 8C^2 = 0 .$$

The above equation has two roots,

$$C_{1,2} = \frac{-(C_{\max} + 6 \cdot b) + \sqrt{\Delta}}{-16},$$

where $\Delta = 33 C_{\max}^2 - 12 C_{\max} b + 36 b^2$.

In particular, as $b \to 0$, we get $C_1 = 0.2965 \cdot C_{mx}$ and $C_2 = 0.4215 \cdot C_{mx}$.

Consequently, we can represent the expression for the first derivative of the profit function in the form

$$\frac{dZ}{dC} = \frac{\lambda_{\max}}{C_{\max}^3} \cdot (C_{\max} - C) \cdot (C - C_1) \cdot (C - C_2)$$

The diagrams, showing the functions $Z(C)$ and $\frac{dZ}{dC}$ are shown in Fig. 2.10.

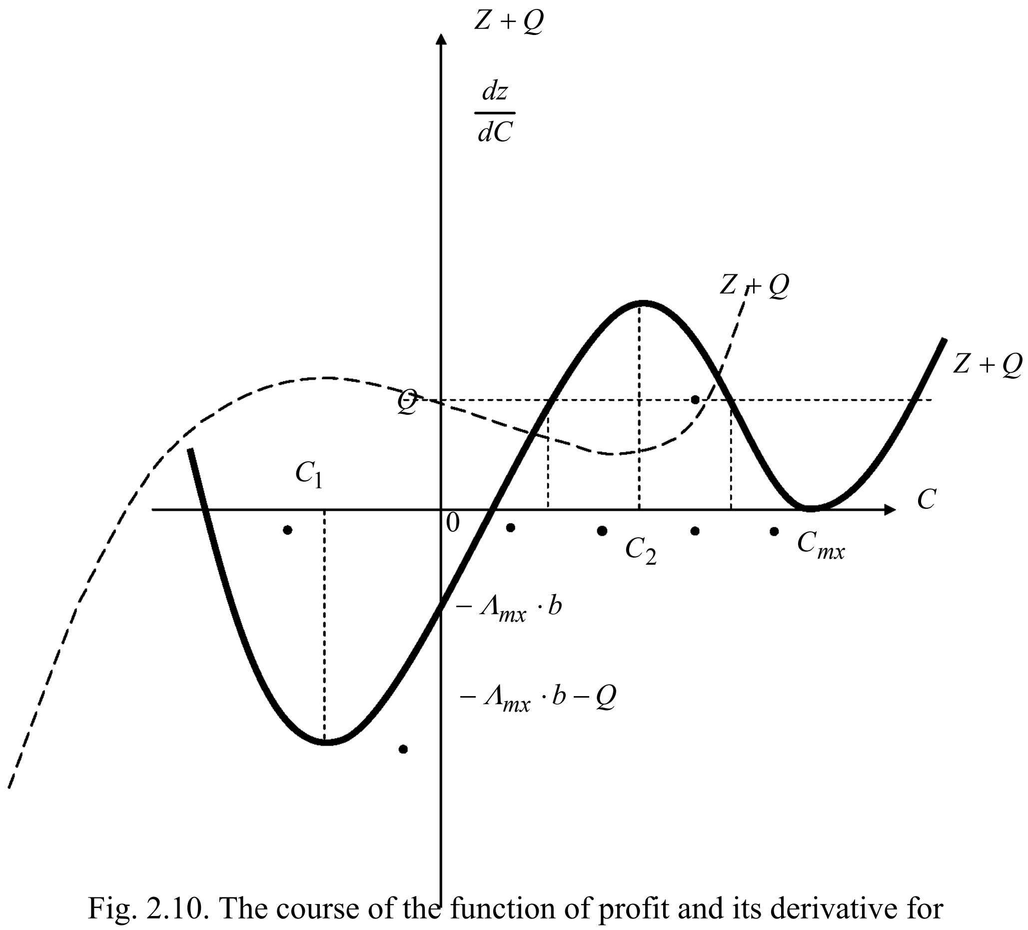

Fig. 2.10. The course of the function of profit and its derivative for model 3.v

Fig. 2.10 shows the optimum sales price value, which is here $C^* = C_2$. This value depends not only upon C_{mx}, which is a characteristic of the market, but also upon the value of b – the lower this value, the lower can be the optimum sales price C^*.

Knowing C^*, it is easy to determine the sought optimum scale of production, μ^*.

Namely,

$$\overset{*}{\mu} = \overset{*}{\Lambda} = \frac{\lambda_{mx}}{C_{mx}^3}(C_{mx} - \overset{*}{C})^2 (C_{mx} + 2\overset{*}{C})$$

Market competition for the model 3.v

Taking the formula for the return on production, we obtain

$$\varepsilon(\mu) = \frac{Z(\mu)}{K(\mu)} = \frac{\mu(C-b) - Q}{Q + \mu b} = \frac{\mu C}{Q + \mu b} - 1$$

It can be easily noticed that the respective function is increasing:

$$\varepsilon(\mu) = \frac{C}{\frac{Q}{\mu} + b} - 1$$

and has a similar shape as in Fig. 2.8.

The conclusions are identical. If the "resident" company produces with intensity μ^* under the market price C^*, then another company, wishing to enter the market with the same product, must lower its price to some $C' < C^*$, which shall, at the same time, lower its profitability of production, below the one for the "resident" company.

In such a situation it is possible to succeed only by applying the "holding out longer" strategy. One lowers the price so much and holds out so long, with this price, that the other company goes bankrupt due to the drop of sales of its product. If, however, the other company also lowers the price, the price decrease chain-reaction is triggered off, which shall be survived by the company, disposing of ampler financial reserves.

When, though, the "resident" company sells its product for an excessive price C_0, higher than the optimal one, C^*, then the newcomer firm, by selling its product for the optimum price, will be able to push out of the market the "resident" company, since, despite the lower sales price, the profits of the newcomer firm shall be higher (see Fig. 2.10, showing the functions $Z(C)$ and $\frac{dZ}{dC}$) than those of the "resident" company.

The issue of the strategy of market entry shall be the subject of further analyses in volume two of this work.

This book presents a complete exposition of a coherent and far-reaching theory of market competition. It is based on simple precepts, does not require deep knowledge of either economics or mathematics, and is therefore aimed primarily at undergraduate students and all those trying to put in order their vision of how the essential market mechanisms might work. Volume II, now in preparation, shall bring the theory to further problems and results.

The logic of the presentation is straightforward; it associates the microeconomic elements to arrive at both more general conclusions and at concrete formulae defining the way the market mechanisms work under definite assumed conditions.

Some may consider this exposition too simplistic. In fact, it is deliberately kept very simple, for heuristic purposes, as well as in order to make the conclusions more clear. Adding a lot of details that make theory more realistic – these details, indeed, changing from country to country, and from sector to sector – is mainly left to the Reader, who is supposed to be able to design the more accurate image on the basis of the foundations, provided in the book.

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