New Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics Volume II: Applications

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Modification of FCM clustering for intuitionistic fuzzy data

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Abstract

In the paper a modification of the method of fuzzy clustering basing on fuzzy intuitionistic features is presented. Objects are described by set of features with fuzzy intuitionistic numeric values. Generally, the method uses the concept of modified fuzzy c-means procedure. The author suggests different method for distance measure between cluster centers and intuitionistic data. A comparison of the results obtained for some numeric examples of clustering is presented.

Keywords: fuzzy c-means clustering, fuzzy intuitionistic data, distance measure.

1 Introduction

In fuzzy clustering the limits between clusters are fuzzy and input data can belong to different clusters partially with different levels of membership. In many practical clustering problems the input data must be treated as fuzzy sets. For example during face recognition procedure, some distances measured between face elements are rather fuzzy numbers than crisp values. Moreover, a not exceeded bounds of these distances can be done. Such situation occurs often in practice. Thus, in the paper, an approach to fuzzy clustering basing on data with intuitionistic fuzzy features is presented. Objects are described by set of features with

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intuitionistic fuzzy values [1]. Generally, the method uses modified c-means procedure applied to such data.

The applications of intuitionistic fuzzy sets in clustering problems begin for year 2004. Hung, Lee and Fuh [3], proposed the fuzzy clustering algorithm based on intuitionistic fuzzy relations. In years 2007-2008, a Greek team from the University of Piraeus, University of Athens and Technological Educational Institute of Crete was published some papers on fuzzy clustering of intuitionistic fuzzy data [5] [7] [8]. The novel variant of the FCM algorithm assumed that the features were represented by intuitionistic fuzzy values, i.e. elements of an intuitionistic fuzzy set. The elements of an intuitionistic fuzzy set were characterized by two functions representing their belongingness and non-belongingness to this set, respectively. In order to exploit this information for clustering a novel distance metric was defined especially designed to operate on intuitionistic fuzzy vectors. The distance is based on a new similarity measure between intuitionistic fuzzy sets. The paper [7] apply the method to RGB color image clustering. In the paper [5] the clustering is based on intuitionistic fuzzy intersection and is applied to computer vision problem. The paper [8] concerns application of intuitionistic fuzzy clustering to information retrieval from cultural databases. The similarity measure is applied also. In the paper [12] the clustering algorithm is based on definition of association coefficients of intuitionistic fuzzy sets. Also interval-valued intuitionistic fuzzy set is considered. The intuitionistic fuzzy hierarchical clustering algorithm was presented in [13]. There, normalized Hamming distance and the normalized Euclidean distance were applied to clustering. In [14] Xu and Wu proposed intuitionistic fuzzy C-means clustering algorithms for intuitionistic fuzzy set and interval-valued intuitionistic fuzzy set, respectively. To solve the optimization problem the Lagrange multiplier method was employed. A novel intuitionistic fuzzy c-means color clustering on human cell images is proposed by Chaira [4]. The non-membership values are calculated from Sugeno's type intuitionistic fuzzy complement. The method is applied to color space medical images. In order to incorporate intuitionistic property in conventional fuzzy clustering algorithm, the cluster centers are updated using a hesitation degree. In [11] identical similarity measure as in [7] was used as distance measure in cluster membership matrix. In [10] the concept of the α-level fuzzy relation was extended introducing the definition of (α, β)-level intuitionistic fuzzy relation. Next, the idea of intuitionistic fuzzy tolerance matrix was described and clustering algorithm based on this matrix was proposed.

In [2] the author suggested a modification of fuzzy c-means (FCM) algorithm and applied this modification to clustering of fuzzy data. The idea is developed here for intuitionistic fuzzy data.

2 Modification of FCM with Intuitionistic Data

Consider input data set $X = (x_1, ..., x_N)$ where any data x_i is described by a vector $F_i = (f_{i1}, ..., f_{iL})$ of fuzzy features f_{il}. Any feature f_{il} represents uncertain numeric values. Thus, any feature is described by set of intuitionistic fuzzy sets with membership functions μ_{lk} and non-membership ν_{lk}. In practical situations triangular or trapezoidal shapes of membership and non-membership functions are useful. Sophisticated shapes as bell, Gaussian or some other are not reasonable because of infinite support. Consider now a set $V = (V_1, ..., V_c)$ of fuzzy clusters. Let $v_1, ..., v_c$ be unknown centers of clusters. Any data x_i can belong to any cluster V_j with unknown membership u_{ij}. The goal of the robust fuzzy c-means algorithm is to find optimal number of clusters and centers of clusters to minimize objective function $J(U, V)$.

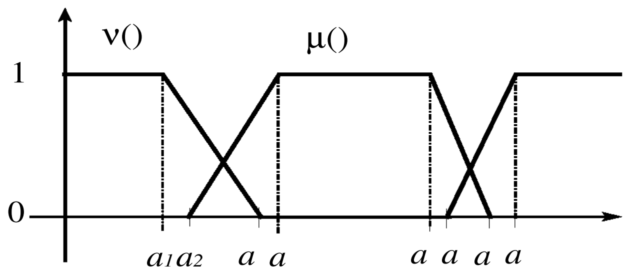

Figure 1: Membership and non-membership functions for data.

In the paper the following procedure, called IFCM, is proposed. Firstly, centers of intuitionistic fuzzy sets are found. Many methods are proposed in literature to found them: association coefficients [12], maximum and minimum values of each feature [11], tolerance value [9][10], Sugeno type intuitionistic fuzzy complement [4], etc. Here some solutions were considered but it seems that very simple procedure is the best

$$x_{il} = (a_{1il} + a_{2il} + a_{3il} + a_{4il} + a_{5il} + a_{6il} + a_{7il} + a_{8il})/8 \qquad (1)$$

where $a_{1il}..a_{8il}$ denote the characteristic values of l feature for element i. The rule can be explained as follows. Both functions - membership and non-membership have the same influence on the result. If the data are fuzzy sets (non intuitionistic) then if membership function is symmetric then rule gives geometrical center; if sides of membership trapeze are non equal then the result is sifted somewhat in the direction of longer side. It is true also for non-membership function.

Next, sufficient, surely too great number c_{max} of clusters and initial matrix of membership U are supposed. For example, without any previous knowledge u_{ij} can be equal to 0.5. Very often Huber function is applied to distance measure $\rho(x)$ between data x_i and center v_j of the cluster V_j, i.e. $\rho[d(x_i, v_j)]$. The goal of this function is to reduce the influence of outliers.

$$\rho(x) = \begin{cases} x^2/2 & \text{if } |x| \leq 1 \\ |x| - 1/2 & \text{if } |x| > 1 \end{cases} \tag{2}$$

Kersten *et al.* [6] proposed reduction of outliers using a function $\psi(x) = d\rho(x)/dx$ and weights $w(x) = \psi(x)/x$, where the weights are defined as follows

$$w(x) = \begin{cases} 1 & \text{if } |x| \leq 1 \\ 1/|x| & \text{if } |x| > 1 \end{cases} \tag{3}$$

After integration it can be obtain a formula for ρ

$$\rho(x) = \begin{cases} x^2/2 & \text{if } |x| \leq 1 \\ |x| & \text{if } |x| > 1 \end{cases} \tag{4}$$

In this paper, after many investigations, another idea are proposed. The function $\rho(x)$ has form of squares

$$\rho(x) = \begin{cases} x^2/2 & \text{if } |x| \leq 1 \\ x^2 - 1/2 & \text{if } |x| > 1 \end{cases} \tag{5}$$

The definition also reduce somewhat influence of outliers, but also fasten searching for big clusters laying far from starting point of clustering procedure. It seems reasonable.

Another measure of distance, as suggested before, the author obtained in a way somewhat similar to weighting function defined in [6] by expression with derivative $w(x) = (1/x)d\rho(x)/dx$. It is assumed for $|x| <= 1$ the weight $w(x) = 1$ and for $|x| > 1$ the weight $w(x) = 1/x^2$. Applying inversely the definition $d\rho(x)/dx = xw(x)$ to the proposed weighting function, it is possible to find appropriate $\rho(x)$ as integral. The result is as follows

$$\rho(x) = \begin{cases} x^2/2 & \text{if} \quad |x| \leq 1 \\ \ln(|x|) + 1/2 & \text{if} \quad |x| > 1 \end{cases} \tag{6}$$

Applying new definition of $\rho[d(x_i, v_j)]$ the objective function is equal

$$J(U, V) = \sum_{i=1}^{N} \sum_{j=1}^{c} u_{ij}^m \, \rho[d(x_i, v_j)/\gamma] \tag{7}$$

where γ is a scaling constant, called Huber constant. The value of γ can be found experimentally or by calculating standard deviation or median. The choice of γ was not very critical. The value $d(x_i, v_j)$ is as before a distance between data x_i and center of cluster V_j, but now both values are fuzzy. Therefore, centers of data were found firstly. The cluster centers will be considered at this moment as crisp. However, at the end of clustering procedure they will be fuzzified.

Five different procedures, replacing Huber $\rho(x)$ function, denoted here by D_L, D_S, D_P, D_H, D_K, were used in the paper for calculations using modified distance definition

$$d_m(x_i, v_j) = d(x_i, v_j)/\gamma \tag{8}$$

$$D(x_i, v_j) = \begin{cases} d_m^2(x_i, v_j)/2 & \text{if} & d_m^2(x_i, v_j) \leq 1 \quad \text{else} \\ [\ln(d_m^2(x_i, v_j) + 1)]/2 & \text{case 1} & \text{denoted as } D_L(x_i, v_j) \\ d_m^2(x_i, v_j) - 1/2 & \text{case 2} & \text{denoted as } D_S(x_i, v_j) \\ |d_m^3(x_i, v_j)| - 1/2 & \text{case 3} & \text{denoted as } D_P(x_i, v_j) \\ |d_m(x_i, v_j)| - 1/2 & \text{case 4} & \text{denoted as } D_H(x_i, v_j) \\ |d_m(x_i, v_j)| & \text{case 5} & \text{denoted as } D_K(x_i, v_j) \end{cases} \tag{9}$$

First three cases, 1, 2, 3, are suggested by the author, case 4 by Huber, and case 5 by Kersten *et al.*

Now, the matrix of membership $[u_{ij}]$ of data x_i in the cluster c_j is updated in the following way:

$$u_{ij} = \Big[\sum_{k=1}^{c} \Big(\frac{D(x_i, v_j)}{D(x_i, v_k)}\Big)^{1/(m-1)}\Big]^{-1} \tag{10}$$

New values of u_{ij} are normalized in all clusters to 1

$$u_{ij}^{'} = \frac{u_{ij}}{\sum_{j=1}^{c} u_{ij}} \tag{11}$$

In the next step the influence of outliers can be reduced using weighting function

$$w[d_m(x_i, v_j)] = \begin{cases} 1 & \text{if } 1/d_m(x_i, v_j) \leq 1, & \text{else} \\ 1/d_m(x_i, v_j) & \text{Huber weight, case 1,} & \text{denoted as } w_H \\ 1/d_m^2(x_i, v_j) & \text{new weight, case 2,} & \text{denoted as } w_S \\ 1/d_m^3(x_i, v_j) & \text{new weight, case 3,} & \text{denoted as } w_P \end{cases} \tag{12}$$

Using this definition of $D(x_i, v_j)$ and $w[d_m(x_i, v_j)]$ the results obtained by simulation were compared with other methods. For this reason, the definitions (7) (8)

(9) (10) (11) and (12) were adopted for the proposed clustering algorithm. New centers of clusters are calculated as follows:

$$v_j = \frac{\sum_{i=1}^{N} u_{ij}^m \, w[d_m(x_i, v_j)] x_i}{\sum_{i=1}^{N} u_{ij}^m \, w[d_m(x_i, v_j)]} \tag{13}$$

Now, there are two possibilities - center of cluster can be crisp or intuitionistic fuzzy. Fuzzy center is more interesting, because it may represent fuzziness of data belonging to the cluster. Because the membership function of the data have trapezoidal shape, it is reasonable to use the same type of membership for cluster center calculated as weighted mean

$$a_{2jl} = \frac{\sum_{i=1}^{N} u_{ij} \, a_{2il}}{\sum_{i=1}^{N} u_{ij}} \qquad a_{7jl} = \frac{\sum_{i=1}^{N} u_{ij} \, a_{7il}}{\sum_{i=1}^{N} u_{ij}} \tag{14}$$

The points a_{4jl}, a_{5jl}, where alpha-cut is equal to 1, are calculated in similar way. Similar procedure is used for non-membership function. It is not necessary to calculate these values during iteration. They can be found at the end of the clustering procedure.

FCM algorithm requires declaring maximal number of clusters c_{max}. During any iteration merging procedure can diminish the number of clusters if the distance between their centers is small. Several methods for merging procedure are proposed in literature. Here, merging criterion is based on concepts of variation, cardinality, and compactness. Variation σ_j of the cluster c_j is defined as weighted mean function of distance

$$\sigma_j = \sum_{i=1}^{N} u_{ij} D(x_i, v_j) \tag{15}$$

Fuzzy cardinality is a measure of the cluster size and is defined as

$$n_j = \sum_{i=1}^{N} u_{ij} \tag{16}$$

Compactness of the cluster is a ratio

$$\pi_j = \frac{\sum_{i=1}^{N} u_{ij}^m \, D(x_i, v_j)}{\sum_{i=1}^{N} u_{ij}^m} \tag{17}$$

Separation between two clusters c_j and c_k can be calculated using modified distance $d_m(x_i, v_j)$ between cluster centers v_j and v_k. Decision about merging two clusters is taken with help of validity index. Validity index is defined as ratio [2]

$$\omega_{jk} = \frac{D(v_j, v_k)}{\sqrt{\pi_j \pi_k}} \tag{18}$$

During every iteration the validity index is calculated for any pair of clusters c_j, c_k and if $\omega_{jk} < \alpha$ then merging procedure is initiated. The value $\alpha = 1$ corresponds to situation when distance between clusters is equal to geometric mean of the cluster compactness. In practice the values in the range [0.1, 0.35] work well. The center v_l of new cluster c_l is located in the weighted middle

$$v_l = \frac{v_j n_j + v_k n_k}{n_j + n_k} \tag{19}$$

Two old clusters are eliminated after merging and replaced by new cluster. After merging, the membership values are recalculated and the IFCM procedure repeats. Stop criterion is based on the change of membership values u_{ij} after each iteration. If maximal change is lower than threshold ϵ then procedure is stopped.

3 Simulation Experiments

In the paper input data have probabilistic nature. Every data x_i is two-dimensional vector of intuitionistic fuzzy trapezoidal sets (Fig. 1) $x_{il} = (a_{1il}..a_{8il})$ and $y_{il} = (b_{1il}..b_{8il})$ on the plain $(x, y) = 640 \times 480$ pixels. Probabilistic distributions for fuzzy parameters were used. As a result we obtain a fuzzy value with two-dimensional membership function in the form of pyramid with top cut off and inverse pyramid for nonmembership.

First, the values r, b were generated with uniform $[0, 1]$ distribution. The values a_{i2}, b_{i2} were generated using formula of the type:
if number of clusters $2 \leq c \leq 3$ then for $j := 1$ to c do begin
r:=Random; b:=Random;
a_2:=-300/c+j*(10+600/c)+(300/c)*r*cos(2*pi*b);
b_2:=190+100*(j mod 2)+160*r*sin(2*pi*b);
The values $sign$ and $sign1$ are equal to 1 or -1 and they were changed during generation to obtain axial symmetry of probability density. Other parameters of fuzzy numbers were obtained using formula:
$a_4 := a_2 + 4 + Random(5)$; $a_5 := a_4 + 4 + Random(5)$;
$a_7 := a_5 + 4 + Random(5)$;
$b_4 := b_2 + 3 + Random(5)$; $b_5 := b_4 + 3 + Random(5)$;
$b_7 := b_5 + 3 + Random(5)$;

$a_1 := a_2 - 4 - Random(5)$; $a_3 := a_4 - 1 - Random(5)$;
$a_6 := a_5 + 1 + Random(5)$; $a_8 := a_7 + 1 + Random(5)$;
$b_1 := b_2 - 3 - Random(5)$; $b_3 := b_4 - 1 - Random(5)$;
$b_6 := b_5 + 1 + Random(5)$; $b_8 := b_7 + 1 + Random(5)$;

If number of clusters is greater then three analogical way of data generation was used.

Every time 2% or 5% of data was generated as outliers with uniform distribution on the whole plain. Following values were used: number of data N=100, 500 or 1000, real number of clusters c=1, 2, 3, 4, maximal (initial, start) value c_{max}=4, 5 or 6, m=1.5, γ=0.1...1000, α=0.2 or 0.3, ϵ=0.005..0.01. The size of clusters was identical. An example of results is presented in Fig. 1.

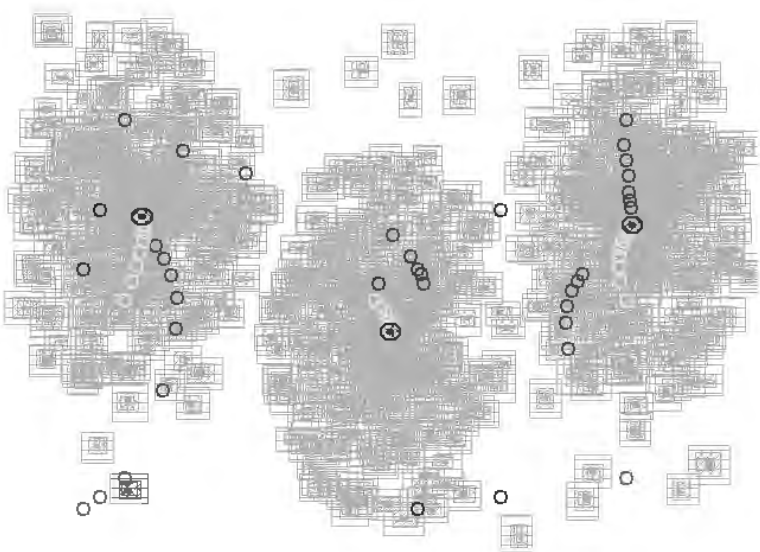

Figure 2: Clustering results where $N = 1000$. Number of initial clusters c_{max}=6, number of real clusters c=3; $m = 1.5$, $\alpha = 0.25$, $\epsilon = 0.01$, $\gamma = 100$.

Gray rectangles shaded with lines and with black centers represent fuzzy intuitionistic data. This black centers show area with membership equal to 1. Gray empty circles show actual center of clusters and displacement of the cluster's centers during actualization and merging procedure, after merging color of circles is changed (visible well only in color image). Big holes show final position of centers.

During clustering procedure the distance was calculated two times. First time, where the distance between data and cluster center is calculated (for all clusters) and second time, where objective function is calculated. Between these calculations the weight is calculated. In the paper five cases were considered for distance and three for weight. Totally, it obtains $5 \times 3 \times 5 = 75$ different cases. All these cases are compared. More of them give bad results - bad number of clusters or long time of calculations with many iterations. Only the best methods were investigated more carefully. It is not possible to look for optimal values of parameters: m, γ, α and ϵ in 75 different cases. Thus, typical "good" values are assumed for parameters: $m = 1.5$, $\alpha = 0.25$ and $\epsilon = 0.01$. Every time the clustering process, the number of iterations between merging procedures, total number of iterations, final value of objective function and time of calculations were displayed. Here, some results for best methods are shown in Table 1, where γ=100 and 2000. Other parameters were unchanged.

Table 1: Results of clustering for best methods.

Method	$\gamma = 100$		$\gamma = 2000$		Rank		
	No. of Iterat.	Error $* 10^5$	No. of Iterat.	Error $* 10^3$	Iterat.	Error	Total
$D_L\ w_S\ J_S$	18.28	1.810	10.93	7.687	5 3	5 5	V
$D_S\ w_S\ J_S$	13.78	1.293	9.81	6.065	3 2	2 2	I-II
$D_P\ w_S\ J_S$	16.27	1.260	12.07	5.909	4 5	1 1	III-IV
$D_H\ w_S\ J_S$	12.68	1.419	9.34	6.578	2 1	3 3	I-II
$D_K\ w_S\ J_S$	12.60	1.425	11.35	6.703	1 4	4 4	III-IV

In Tables following notations are used: $D_L\ D_S\ D_P\ D_H\ D_K$ - formula for distance, $w_H\ w_S\ w_P$ - formula for weight, $J_L\ J_S\ J_P\ J_H\ J_K$ - formula for distance function D used for calculation of the objective function $J(U, V)$ and merging (variation, compactness). Initial number of clusters was assumed as 6 and real number of clusters was 3. The results in Table 1 were ranked in raising way. The methods $D_S\ w_s\ J_S$ and $D_H\ w_s\ J_S$ work very good. The method $D_P\ w_s\ J_S$ works fast, but sometimes, in about 5% of cases, it stops with not correct number of clusters (4 clusters). The method $D_L\ w_s\ J_S$ works good but slow.

The influence of parameter γ is shown in Table 2, where also good methods are presented. It should be noted that initial repartition of cluster centers was random. Therefore, sometimes occurs that bad number of real clusters was found. For all cases in Table 1 and separately in Table 2 intuitionistic data were identical. For $\gamma = 100$ any method do not work good, a bad number of clusters was found very often. Simply, parameter γ is too small. The best method $D_L\ w_S\ J_H$ gives correct number of clusters only in 70% of cases. For $\gamma = 2000$ the situation is changed, the methods work good. Only for $D_S\ w_S\ J_H$ and $D_P\ w_S\ J_H$ sometimes bad number of clusters was found. The method $D_L\ w_S\ J_S$ works good in large range of γ values, unfortunately it gives greatest error. Of course, the value of ϵ can be diminish, but it entails more iterations. For $N = 1000$ and slow computer PC the clustering procedure takes 1000 ms to 2000 ms depending on data and method. In conclusion, $D_S\ w_S\ J_S$ and $D_H\ w_S\ J_S$ methods were found as the best.

Table 2: Results of clustering for different parameter γ.

Method	$\gamma = 100$		$\gamma = 500$		$\gamma = 1000$		$\gamma = 2000$	
	Iter.	Error $*10^3$	Iter.	Error $*10^3$	Iter.	Error $*10^3$	Iter.	Error $*10^2$
$D_L\ w_S\ J_H$	45.1	5.89	25.5	2.06	18.6	1.59	14.2	10.56
$D_S\ w_S\ J_H$	34.0	5.00	24.0	1.66	18.8	1.35	17.2	9.15
$D_P\ w_S\ J_H$	30.9	4.95	22.6	1.65	19.5	1.35	16.7	9.13
$D_H\ w_S\ J_H$	33.0	5.17	23.0	1.78	17.6	1.42	14.7	9.62
$D_K\ w_S\ J_H$	33.3	5.18	22.0	1.87	18.4	1.43	17.7	9.72

4 Conclusions

The main goal of the paper consists in comparison of some distance measure used for clustering problem. Only fuzzy C-means (FCM) algorithm was considered with some modifications suggested by the author. Fuzzy intuitionistic data are used. Centers of clusters are assumed also as fuzzy intuitionistic. The author thinks that in the case of many data it is not possible to use special distance measure taking in consideration all parameters of intuitionistic sets during calculations of distance between any data and fuzzy intuitionistic center of cluster. It will be sufficient to defuzzify every data at the beginning of the algorithm, curry out the FCM clusterisation and finally rebuild center of clusters as intuitionistic using mean procedure applied to members of the cluster. Interesting, but disputable, concept is suggested - apply a measures for calculation of distance between data and cluster center and similar or different measure for merging procedure, where distance between clusters is calculated. The results show that both concepts are possible, final results are correct for some different measures. Many combinations were considered, totally 75 cases. Most of them are bad, but good procedures were found. Investigation were performed for large number of data but in any case only a few trials were executed. If the method was bad investigation was interrupted.

Only good methods were tested more carefully. The data and starting points were chosen as random. The best procedures apply the measure proposed by the author and by Huber.

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The papers presented in this Volume 2 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Tenth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2011) organized in Warsaw on September 30, 2011 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

Http://www.ibspan.waw.pl/ifs2011

The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Tenth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2011) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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