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DEVELOPMENT OF METHODS AND TECHNOLOGIES OF INFORMATICS FOR PROCESS MODELING AND MANAGEMENT

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This book consists of papers describing applications of informatics in process modeling and management and in environmental engineering. Problems presented in the papers concern development of methods supporting process management, development of calculation methods for process modeling and development of technologies of informatics for solving some problems of environmental engineering. In several papers results of the research projects supported by the Polish Ministry of Science and Higher Education are presented.

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CHAPTER 2

Tools of mathematical modeling; neuronal nets

PUBLIC DEBT MODELLING: APPLICATION OF THE MODEL PREDICTIVE CONTROL*

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Abstract: *In this paper we consider a national debt servicing problem. We propose the application of a model predictive control to minimizing debt servicing cost.*

Keywords: National debt, servicing cost, model predictive control.

1. Introduction

We propose a model for the optimization of the issuances of Public Debt securities that we are developing in research project. There are a number of possible fixed and floating income securities and the goal is to find the composition of the portfolio issued every month, which minimizes a "cost function".

It is clear, the lower costs of servicing the National Debt has freed up substantial Government revenue for other purposes. Sustainable management of the polish economy generally requires that the Government run a balanced current budget i.e. those Government tax revenues should pay for day-to-day running costs of Government services. A prudent fiscal policy would aim to ensure that enough revenue would be left over to pay for some, but not necessarily all, of the Government's capital investment plans.

Debt strategy is defined as the manner in which a government finances an excess of government expenditures over revenues and any maturing debt issued in previous periods. The question concerns the best way for the government to borrow these required funds. Should it, for example, use short-term debt, such as treasury bills or longer-term coupon bonds? Interestingly, an extensive academic literature on this subject does not exist. Our analysis is based on the belief that a sustainable and prudent debt structure is critical for any sovereign nation. Moreover, we take the government's fiscal policy as given and attempt to characterize the set of financing strategies that have desirable risk-cost characteristics. Indeed, our primary objective

* Research Project of the Ministry Education and Science, No 1 H02B 038 28, Application of the Artificial Intelligence Method to Public Debt Management.

is to learn more about the nature of the risk and cost trade-offs associated with different financing strategies. The practitioner literature relating to better understanding this issue is found in publications from sovereign debt managers (Bolder, 2002).

Adopting this pragmatic perspective, we prove that one can conceptualize the government's borrowing decision as an optimal-control problem in a stochastic setting.

This problem has been extensively studied in the asset-pricing setting where an investor attempts to optimally select the proportion of risky and riskless assets that maximize their expected utility subject to appropriate wealth constraints (Karatzas, Shreve, 1988). In our situation, the government is attempting to optimally select the composition of its debt portfolio to minimize expected debt costs subject to risk and liquidity constraints. Given practical complexities, however, it is not obvious how to use dynamic programming techniques to find a solution. Instead, we rely on simulation. We have also found that a simulation methodology termed dynamic financial analysis in the actuarial science literature is relevant for this task. Insurers are often faced with the problem of trying to set premiums and capital reserves, given stochastically evolving claims and investment returns. Structurally, the techniques used in dynamic financial analysis are relevant for our work in debt strategy analysis.

Because our approach to debt strategy analysis involves stochastic simulation, another objective is to present the details of this simulation framework. Moreover, it is our view that management of the government's borrowing program is an important and difficult task requiring a combination of judgment and comprehensive analytical tools.

2. Restrictions for the optimizing task

We now introduce in a more formal way the problem studied in the scientific collaboration between the Polish Ministry of Finance and the Systems Research Institute of the Polish Academy of Sciences. We basically follow the papers (Miklewski, Krawczak, 2005; Miklewski, 2005).

The Growth and Stability Pact (GSP), subscribed by the countries of the European Union (EU) in Maastricht, defines "sound and disciplined public finances" as an essential condition for strong and sustainable growth with improved employment creation. In Poland even though the general government deficit would fall to about 4½ percent of GDP, public debt would rise to 49 percent by year-end (International Monetary Fund, 2006). The Public Debt Management Division of the Polish Ministry of Finance is deeply interested in studying which securities to issue, in order to achieve an optimal debt composition (tab. 1).

Table 1. Structure of Polish Public Debt.

SDDS Data Category and Component	Unit description	Observations		
		Period of latest data	Latest data	Data for previous period
General Government or Public Sector Operations				
1. Revenue	PLN million	2004	362 757	336 490
2. Expense	"	"	387 514	366 373
3. Gross operating balance	"	"	-5 923	-12 066
4. Net operating balance	"	"	-24 757	-29 883
5. Net acquisition of nonfinancial assets	"	"	8 620	6 038
6. Net lending/borrowing	"	"	-33 377	-35 921
7. Net Acquisition of Financial Assets	"	"	5 876	2 796
7.1. Foreign			-2 713	1 722
7.2. Domestic			8 589	1 074
8. Net Incurrence of Liabilitie	"	"	39 253	38 717
8.1. Foreign			6 094	4 078
8.2. Domestic			33 159	34 639
Central Government Operations				
Total revenue and grants	PLN million	March 2006	21 843	19 936
Expenditure and lending minus repayments	"	"	29 060	29 138
Overal deficit/Surplus	"	"	-7 217	-9 202
* Financing	"	"	7 217	9 202
** Foreign	"	"	-524	3 850
** Domestic	"	"	7 741	5 352
*** Bank	"	"	3 225	2 471
*** Non-Bank	"	"	4 516	2 881
Domestic State Treasury Debt				
Domestic State Treasury debt by original maturity	PLN milion	March 2006	329 893,30	326 523,90
Short term domestic debt in treasury securities (one year and less)	"	"	20 900,00	22 800
Medium term domestic debt in treasury securities (more than one year includind five years)	"	"	165 987,30	162 201,80
Long term domestic debt in treasury securities (more than five years)	"	"	139 499.9	138 012,10
** Other domestic debt of the State Treasury	"	"	3 506,10	3 509,90
* Domestic State Treasury debt by holder	"	"	329 893,30	326 523,9
** Central bank	"	"	0	0
** Other domestic banks	"	"	74 258,90	72 275,80
** Domestic non-banking sector	"	"	182 454,70	179 841,20
** Foreign investors	"	"	73 179,70	74 406,90
* Debt guaranteed by the government of the Republic of Poland	"	Q4 2005	13 381,90	14 358,90
Foreign State Treasury Debt				
* Foreign State Treasury debt by original maturity	PLN million	March 2006	135 035,70	133 138,40
** Short term domestic debt in treasury securities (one year and less)	"	"	0	0
Medium term domestic debt in treasury securities (more than one year including five years)	"	"	11 353,50	10 945,10

** Long term domestic debt in treasury securities (more than five years)	"	"	123 682,20	122 193,20
* Foreign State Treasury debt by type of security and by holder	"	"	135 035,70	133 138,40
** Treasury Bonds	"	"	96 642,90	93 255,50
*** Foreign Bonds	"	"	92 765,90	89 464,80
*** Brady Bonds	"	"	3 876,90	3 790,70
** Loans	"	"	38 392,80	39 882,90
*** Paris Club	"	"	22 334,80	24 615,90
*** World Bank	"	"	4 818,20	4 621,70
*** Other	"	"	11 239,80	10 645,30
* Debt guaranteed by the government of the Republic of Poland	PLN million	Q4 2005	18 261,70	18 921,40
Public Debt				
Public Debt	PLN million	Q4 2005	467 806,00	461 160,50
* Central government debt	"	"	447 623,10	443 471,80
** State Treasury Debt	"	"	439 567,00	431 773,90
** Other central government debt	"	"	8 056,10	11 698,00
* Local government debt	"	"	20 182,90	17 688,70
** Local government debt entities	"	"	17 167,50	14 548,60
** Other local government debt	"	"	3 015,40	3 140,00

Source: the Public Debt Management Division of the Polish Ministry of Finance

The goal is to determine the composition of the portfolio issued every month which minimizes a predefined cost function. This can be, for instance, the width of fluctuations of deficit over a given time horizon or the interest expenses.

Mathematically speaking, this is a stochastic optimal control problem with several constraints imposed by national and supranational regulations and by market practices. Among the former, for example, the Stability and Growth Pact rules require that:

- The Budget Deficit has to be below 3% of Gross Domestic Product (GDP) (i.e. the total output of the economy).
- The Nominal Debt, that is the nominal amount of securities issued to finance the Budget Deficit, has to be less than 60% of the GDP.
- Countries should have an inflation rate within 1.5% of the three EU countries with the lowest rate.
- Long-term interest rates must be within 2% of the three lowest interest rates in EU.

Moreover, there are a number of other constraints such as the amount of money in the Treasury Cash Account. The complexity of the problem is further increased by the need for realistic solutions to take into account several side issues, like macroeconomic factors which are complicated as well, see (Demmel, 1999).

The stochastic component of the problem is represented by the evolution of interest rates and Primary Budget Surplus (PBS) (Primary Balance is defined as domestic revenue minus domestic expenditure excluding interest payments).

Once a scenario for the evolution of these variables is set-up, the portfolio optimization can be formulated as a finite dimensional Linear Programming problem, neglecting some nonlinear effects of the bond issuances (for instance, a variation of the portfolio composition might trigger, by market reaction, a change in the term structure of the interest rate).

By means of standard methods (i.e. the simplex method) we determine an optimal issuance strategy for each scenario.

The selection of the optimal strategy among the many optimal portfolios turns out to be a major problem. For example, it is likely that a combination of portfolios does not fulfill all the constraints (like the refunding of the expired securities).

Note that the Government announces the expected expenditure for the payment of interests in the yearly Financial Law that is essentially the expected balance of the State for the following year. Therefore we need to provide strong probability estimates for our optimal control problem.

We thus turned the attention to iterative control algorithms to deal with scenario realizations. In engineering literature iterative control methods, called Model Predictive Control (MPC), have been successfully used in presence of disturbances, uncertainties and strict control and state constraints. The main difference of our framework is the presence of dominant stochastic behaviors, but the same techniques can be adapted to deal with that. The use of MPC allows us to obtain reliable probability estimates for the cost function opposed to predefined strategies that appear much less reliable.

3. Mathematical model

Building the model we devised to determine the optimal issuance strategy.

For example, The National Bank issues ten different types of securities including one with outing rate. The securities differ in the maturity (or expiration date) m_k and in the rules for the payment of interests.

The first instrument (BOT - BOND1) does not have coupons. From the accounting viewpoint the issuing price p is determined with a discount factor dp = 100-d, i.e., at the maturity date the nominal value 100 is reimbursed. The second instrument (CTZ - BOND2), like BOND1, does not have coupon. The issuing price is determined in such a way that the interests are comprised in the reimbursement $p(1 + r) = 100$.

Next instruments, both the (BTP - BOND3) and (CCT - BOND4) pay cash dividends by means of coupons corresponded every 6 months. The difference among them lies in the rate of interest (i.e. the value of the coupon) that is set at issuance time for BOND3s whereas is variable for BOND4s. More precisely, the interest rate for BOND4s is determined by the interest rate for the 6–month BOND1s.

For each of these four types of bonds we make a further distinction depending on the maturity. We order the bond types with an integer k taking $K = \{1,..., 10\}$. Moreover we indicate by m_k, the maturity in months of k. The issuance dates depend on the type of bond and we indicate them by a couple (d, m) where d is the day and m the month (tab. 2).

Table 2. Example of bonds issuance

No	Bond name	Maturity	Issuance date
1	BOND1	$m_1 = 3$	(15, m), m=1,...,12
2	BOND1	$m_2 = 6$	(30, m) or (28, m), m=1,...,12
3	BOND1	$m_3 = 12$	(15, m), m=1,...,12
4	BOND2	$m_4 = 24$	(15, m), m=1,...,12
5	BOND3	$m_5 = 36$	(15, m), m=1,...,12
6	BOND3	$m_6 = 60$	(15, m), m=1,...,12
7	BOND3	$m_7 = 120$	(1, m), m=1,...,12
8	BOND3	$m_8 = 180$	(15, m), m=2, 3, 6, 7, 10, 11
9	BOND3	$m_9 = 360$	(15, m), m=1, 3, 5, 7, 9, 11
10	BOND4	$m_{10} = 84$	(1, m), m=1,...,12

Source: Own investigations on the basis Adamo et al., 2003

The bonds' portfolio is the collection of bonds issued by the National Bank that are still on the market that is bonds that have not reached their maturity.

Let $u_k(t)$ be the nominal value of all the bonds of type k issued at time t, $p_k(t)$ the unit price and $c_k(s, t)$ the coupon percentage at time s for the same bond. For each bond there is an income of $p_k(t)$ at issuance time t, a payment of the nominal value that we set as equal to 100 at maturity $t + m_k$ and possibly payments of $100c_k(s, t)$ of coupons for all times s between the issuance date and maturity. Thus for a single bond we obtain the cash flow at time s:

$$R_k(s,t) = \delta_t(s)p_k(t) - 100\left[\delta_{t+m_k}(s) + \sum_{l=1}^{m_k/6} \delta_{t+6l}(s)c_k(l,t)\right],$$

where the function $\delta_\tau(s) = 1$ if $s = \tau$ and 0 otherwise. Similarly we derive the cash flow for the whole portfolio:

$$CashFlow(s) = \sum_{k \in K} \sum_{t=s-m_k}^{s} \frac{u_k(t)}{100} R_k(s,t).$$

The cash flow of bonds' issuances and payments goes through a National Bank account Treasury Cash Account (TCA). There are some institutional positive lower bounds on the amount of money this account must have at the end of each month (for example 15 PLN billion). We indicate by *TCA(s)* the amount of money in the Treasury Cash Account at month *s*.

As to the PBS, any forecast is difficult due to many issues like seasonality and changes in the status of the economy. However, we assume that the PBS is defined every month and we indicate with *PBS(s)* the PBS at month *s*.

Institutional constraints.

A fundamental constraint is to guarantee the payment of coupons and the reimbursement of bonds at maturity:

$$TCA(s) = TCA(s-1) + CashFlows(s) + PBS(s) \geq \beta ,$$

where β = 15 PLN billion is fixed by the law as explained in a previous paragraph. Note that *PBS(s)* may be negative. The Yearly Net Issuance (YNI) measures the difference between the volume of bonds issued during the year and the volume of bonds reimbursed during the same year. There is a constraint on the YNI indicated by the Government. In formula:

$$\sum_{s=1}^{12} \sum_{k \in K} \left[p_k(t_0+s) \frac{u_k(t_0+s)}{100} - u_k(t_0+s-m_k) \right] \leq \eta ,$$

where t_0 is the first month of the year and η is fixed by the financial law. More precisely the above formula must be corrected for BOT with a 100 nominal value instead of an issuance price p_k.

The growth and stability pact constraints.

The Nominal Debt is defined as:

$$D(s) = \sum_{k \in K} \sum_{t=s-m_k+1}^{s} u_k(t)$$

and consists of all the money the Government will reimburse in the future for bonds reaching maturity. Then the GSP imposes:

$$\frac{D(s)}{GDP(s)} \leq \alpha ,$$

where α = *0.6* for the 60% constraint imposed by the Maastricht treaty that Poland is committed to reach at a satisfactory pace.

The market practice constraints.

The Treasury needs to consider also the problem of market stability. For instance, the amount of short-term bonds determines the behaviour of the corresponding market. If a significant variation of the nominal amount of a short term bill offered was proposed in a single issuance, the market would react with a major change of the issuance price.

As a consequence, there are institutional constraints on the composition of portfolio which can be classified as dynamic constraints for short term securities, namely BOT, and static constraints for the medium and long term ones, namely CTZ, BTP and CCT. Thus for $k = 1, 2$ and 3 the dynamic constraint can be modeled as:

$$\frac{u_k(t) - u_k(t - m_k)}{u_k(t - m_k)} \leq \Gamma_k ,$$

$$\frac{u_k(t - m_k) - u_k(t)}{u_k(t - m_k)} \leq \gamma_k ,$$

where the values of Γ_k, γ_k are determined by the Ministry officers relying on their experience and market knowledge. The static constraints for $k \geq 4$ are stated as:

$$\lambda_k \leq u_k(t) \leq \Lambda_k ,$$

where λ_k and Λ_k are the minimum and maximum amounts of long term bonds of each issuance.

The risk constraints

The last constraint is related to the possibility of operating changes in the issuance strategy in case of interest rates shocks. For each bond of type k issued at time t we define its Refixing Period as:

$$T_k(t,s) = m_k - (s - t) ,$$

that is the remaining time to maturity. The CCT is considered as a six month bond.

The Weighted Refixing Period (WRP) of the whole portfolio is an average time to maturity of the portfolio with weights proportional to the issued quantities:

$$WRP(s) = \frac{\sum_{k \in K} \sum_{t=s-m_k}^{s} u_k(t) T_k(t,s)}{D(s)} .$$

Since $T_k(t, s)$ is the time after which a bond has to be repaid with a (probably) different interest rate, the WRP is an estimate of the averaged time period in which the Ministry is protected against changes of interest rates.

For the zero coupon bonds (BOT and CTZ) the WRP is equivalent to the duration, whereas for BTP is the weighted average time to maturity and for CCT is the weighted average coupon refixing time.

A flexible management of the Public Debt requires that:

$$\tau_{\min} \leq WRP(s) \leq \tau_{\max},$$

for some fixed values τ_{min} and τ_{max}.

4. The cost function

A reasonable cost function is the yearly cost of the Public Debt calculated according to the European System of Accounts (Jackson, 2000) (ESA95). Roughly speaking, the ESA95 criteria consider for each bond its total cost (coupons plus the difference between nominal value and issuance price) distributed over its existence period, namely from issuance to maturity. Thus the cost over a set year is measured by the cost of bonds only for those days that fall inside the considered year. For instance, a 12-month BOT issued on July 1st 2005 counts for one half of its cost for the year 2005 according to ESA95 criteria.

In formula:

$$ESA95[t_1,t_2] = \sum_{k\in K} \sum_{t=t_1-m_k}^{t_2} \frac{u_k(t)}{100}((100-p_k(t))\frac{[t_1,t_2]\cap[t,t+m_k]}{[t,t+m_k]} + \\ + \sum_{l=1}^{m_k/6} c_k(t,l)\frac{[t_1,t_2]\cap[t+6(l-1),t+6l]}{[t+6(l-1),t+6l]})$$

is the cost for the time period $[t_1, t_2]$.

We are now ready to state our main goal:

Definition. The Optimal Issuance Strategy (OIS), is the problem of determining a strategy for the selection of Public Debt securities that minimizes, within a given probability, the expenditure for interest payment (according to the ESA95 criteria) and satisfies, at the same time, the constraints on Debt management.

A number of other possible cost functions can be chosen as an indicator of the Debt behavior. For instance, the discounted Debt which can be defined as follows.

Consider the total amount to be paid by the Treasury after some fixed time t_0, that is all the negative parts in the cash flows $R_k(s, t)$ for $k \in K$, issuance dates $t \leq t_0$ and times $s > t_0$. We denote such negative parts $Q_k(s, t)$. Let $a(t_0, s - t_0)$ be the annual interest rate of a bond with maturity $s - t_0$ (months) issued at time t_0 and $M = max_{k\in K} m_k$. In formula, the discounted Debt at time t_0 is:

$$\sum_{s=t_0+1}^{t_0+M} \sum_{k\in K} \sum_{t=s-m_k}^{s} \frac{u_k(t)}{100} Q_k(s,t) \left(\frac{1}{(1+a(t_0, s-t_0))^{\frac{s-t_0}{12}}} \right).$$

Since issuances happen at fixed dates, once per month, we use a discrete time model of evolution. For the sake of simplicity, the time step is one month. For the months in which some types of securities are not issued, the corresponding quantities are set equal to zero. We indicate by X_t the total amount of bonds that are not expired at time t. Thus X_t must contain, for every $k\in K$, one component for every $s\in\{t-m_k, ..., t-1\}$. The evolution of X_t is determined at each step by canceling bonds reaching maturity and adding the just issued ones. For example, for $k = 1$, one has to remove from X_t the quantity of 3 months BOND1 issued at time t-3 and insert that issued at time t. Clearly this can be done by shifting the components of X_t and adding the new issuances, thus we can write:

$$X_{t+1} = AX_t + BU_t, \tag{1}$$

where A is a shift matrix, $U_t = \left(\frac{u_k(t)}{100} \right)_{k\in K}$ is the vector of the new issuances and B is a sparse matrix. Hence we get a linear discrete time control system.

Note that the stochastic behavior of interest rates, or forward rates, influences the CashFlow, hence the TCA constraints, and the cost function ESA95. The latter is influenced also by the PBS.

Input and output data.

To specify completely the control problem it is necessary to set the input and output data and the optimization horizon.

The input data consist of:

- past issuances,
- issuance data,
- Gross Domestic Product and PBS forecasts.

Past issuances. If the optimization horizon starts at time t_0, then for every $k\in K$ it is necessary to know the quantities issued at all dates $t_0 - m_k, ..., t_0 - 1$.

Issuance data. The National Bank sets the dates of issuance for each type of bonds. These dates are set in advance, usually for the next two or three years, and are not part of the control problem.

GDP forecasts. This point is quite critical, since it is difficult to have reliable GDP forecasts.

The output data are represented by the number of bonds that, for each issuance, fulfill all the constraints and, at the same time, minimize the cost function.

From these data it is possible to derive:

- the Yearly Net Issuance,
- the Public Debt cost defined according to the ESA95 criteria (Jackson, 2000),
- the duration and WRP (Weighted Refixing Period) of the portfolio.

The duration of a portfolio of bonds is, from the issuer viewpoint, the weighted average of the maturity of all the outcome cash flows. The duration describes the exposure to parallel shifts in the yield curve and is a widely used indicator of the risk associated with a particular choice of a fixed income securities portfolio (Krawczak et al., 2003).

The final goal is to provide an "optimal issuance strategy". There are, at least, two possible choices: 1) define the most probable scenario for the interest rates evolution, determine the corresponding optimal strategy, estimate the consequences of applying this strategy to a set of other scenarios (this step is necessary since the forecast on the interest rates can be wrong), 2) employ an "adaptive" strategy based on the available information on interest rates at issuance date (using interest rate models) and estimate the outcoming costs on a wide set of scenarios. We call 1) Fixed (most probable) Strategy and 2) Model Predictive Control (MPC) Strategy (by similarity with engineering control problems).

For the purposes of the Ministry, a reasonable optimization horizon is 7 years (like budget of EU).

5. Optimal Control

Beside input and output data given at initial and final time respectively, there are some input and output variables evolving in the optimization horizon.

In control jargon Nominal Debt, Flow and Treasury Cash Account can be seen as output variables of the control system (1) and in formula can be indicated by:

$$Y_t = Y\left(X_t, U_t, PBS(t), p(t)\right) \tag{2}$$

In fact, all these quantities are computable since X_t, U_t and the exogenous stochastic parameters $PBS(t)$ and $p(t)$ (term structure of interest rates) are known. Finally, we get the task 1:

Task 1. The OIS (Optimal Issuance Strategy) consists of an optimal control problem for the system (1) with constraints on the outputs (2) and with a cost function defined according to the ESA95 specifications. Both constraints and cost function depend on the stochastic exogenous variables $PBS(t)$ and $p(t)$.

A wide literature for stochastic optimal control problem is available (Yong, Zhou, 1999). However, the large number of variables (some hundreds components) and the needs for strict estimate in terms of probability prevent the applicability of most techniques.

It is possible to show:

Task 2. For a fixed term structure evolution $t \mapsto p(t)$ and PBS realization $t \mapsto PBS(t)$, the optimization problem becomes a linear programming problem with linear constraints.

To solve the problem we resorted to the classic Simplex Method. In (Miklewski, 2005) we report a block diagram of the software package that we realized to manage all the phases of the optimization. The core of the optimizer is the package MATLAB (optionally MATHEMATICA and Excel) an open source linear programming solver which uses sparse matrix computations.

Once the single scenario optimization has been solved we can study the behavior of optimal controls and costs via Monte Carlo simulations. Some interesting parameters as the spread between maximum and minimum costs can be easily obtained.

6. Model Predictive Control Strategies

It is well known that interest rate models do not always provide reliable forecasts, thus we put the accent on advanced control techniques in order to reduce Debt risk. In engineering literature an iterative strategy called MPC (Model Predictive Control and/or Receding Horizon Control), is often used in industrial applications for stabilization of systems under measurement uncertainties and disturbances (Maciejowski, 2002). This approach is particularly useful in case of hard constraints, as in Stability and Growth Pact.

A typical example of such problem is portfolio optimization or public security issuances, see (Korn, 2001). Through the use of classical stochastic control methods, as dynamic programming and Hamilton-Jacobi-Bellmann equation, stochastic maximum principle, one can treat these problems providing solution methods. However, such methods are effective only for a small number of state and control variables, i.e. for a small number of assets involved in the portfolio. On the other hand, one may use Monte Carlo methods to deal with computational issue of simulations, but not easily to solve optimization problems.

MPC is a form of control in which the current control action is obtained by solving *on-line*, at each sampling instant, a finite horizon open-loop optimal control problem, using the current state of the debt as the initial state, the optimization yields an optimal control sequence and the first control in this sequence is applied to the debt.

The basic idea of MPC is the following. In a discrete time setting, at step k, obtain an estimate up to a horizon $k + H$, $H > 0$, of the system behavior. Then choose a control according to some optimization criteria, such as optimal tracking of a benchmark trajectory. Finally, apply the obtained control and repeat the operation at next step.

Let us describe more precisely our application of MPC to the OIS problem. Fix a time window, say $[t_0, t_N]$ on which the evolution of the system is considered. Our procedure consists of the following steps:

Step 1. At a given issuance time t_j, in the time window $[t_0, t_N]$, we assume to know the term structure $p(t_j, T)$ for $T = t_j + m_k$ for $k \in K$, that is the rates for all bonds. Then we use some generator (predictor) $\tilde{p}(t,T)$ for the term structures at all times up to an optimization horizon H, i.e. up to $t_j + H$.

Step 2. We solve the OIS for the considered most probable scenario according to the generator $\tilde{p}(t,T)$. Alternatively, we can use a more sophisticated selection procedure for optimal portfolio but always based on the generator. This produces optimal issuance quantities $\tilde{u}_k(t_j, s)$ for all $s \geq t_j$ in the optimization window $[t_j, t_j + H]$.

Step 3. We issue securities according to the found optimal values $\tilde{u}_k(t_j, t_j)$ and then we go back to Step 1 for the new issuance time t_{j+1}.

There are some key parameters as the optimization horizon H and the overall window $[t_0, t_N]$. However, the most interesting point is that the MPC strategy is more important than the choice of the generator $\tilde{p}$. More precisely, we show via simulations that the performance of an MPC strategy is much better, in probabilistic terms, than that of a fixed strategy for every choice of the interest rate forecast. Let us explain this in detail. Fix the model described previously: let P_{Stat} be the portfolio selected by a strategy based on the most probable scenario. For each term structure scenario p_i, we indicate by $P^i_{MPC}(\tilde{p})$ the portfolio selected by the MPC strategy in case of scenario i. Notice that obviously $P^i_{MPC}(\tilde{p})$ does depend on the scenario because the procedure measures the actual rates at issuance date. Recalling the definition of $P^i_{\min}$ min in the previous section, we evaluate the following quantities:

$$\frac{ESA95(P_{Stat}) - ESA95(P^i_{\min})}{ESA95(P^i_{\min})},$$

and

$$\frac{ESA95(P^i_{MPC}(\tilde{p})) - ESA95(P^i_{\min})}{ESA95(P^i_{\min})}.$$

Finally we consider a reasonable constant forecast for MPC strategy: at each issuance date t_j the generator simply replicates the actual term structure for all future times in the optimization horizon.

Risk estimation.

As explained at the very beginning, beside the mean value of the ESA95 cost, the Treasury must ensure the Debt performance with a very high probability.

Let us indicate by P the probability distribution over the set of scenarios for interest rates and by $\tilde{p}$ a fixed forecast.

Given a fixed probability level l we can find a ESA95 cost level $C = C(l)$ such that

$$\mathrm{P}\left\{ESA95(P^i_{\min}) \le C\right\} \ge \frac{1+l}{2}.$$

Then we find a percentage error level $\varepsilon = \varepsilon(l)$ such that

$$\mathrm{P}\left\{\frac{ESA95\left(P^i_{MPC}(\tilde{p})\right) - ESA95(P^i_{\min})}{ESA95(P^i_{\min})} \le \varepsilon\right\} \ge \frac{1+l}{2}.$$

Finally the ESA95 cost level can be set equal to $C \times (1+\varepsilon)$, that is ensured with probability greater than or equal to l. This method could, in principle, perform poorly, but for MPC strategies the error ε is extremely small, so such estimate is quite satisfactory.

7. Conclusions

Model predictive control (MPC) refers to the direct use of an explicit and separately identifiable model for controlling a debt servicing process. The core of all MPC algorithms is the moving horizon approach. The MPC designs yield control systems capable of operating without expert intervention for extended periods of time. An identified process model predicts the future response and then, the control action is determined so as to obtain the desired performance over a finite time horizon. The control problem that must be solved is an on-line optimization of the manipulated variables to satisfy multiple, changing performance criteria in the face of changing process characteristics, including hard constraints. The MPC technique is a dynamic optimization approach to control problems. The flexible constraint-handling capabilities of MPC make it most suitable for process control problems.

Neural networks can be used to determine controller parameters, because of their well-known ability to solve complex problems by learning relationships directly from data. In this decade, certain neural networks have generated a lot of interest for use in nonlinear system identification and control (Albanis et al., 1999, 2000; Krawczak et al., 2003). They have been found to approximate arbitrary nonlinear mappings and they have been effectively used for the control of complex dynamic systems. Neural networks provide a framework for deriving analytic expressions for the modeling error gradients with respect to modeling parameters. The majority of neural network control applications have centered on the use of a neural network as a controller, which is often combined with a neural network identifier. A less costly alternative is to develop approaches that would adapt conventional controller gains using neural-network algorithms, so as to successfully track the different process operating regions, and hence enhance control performance.

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DEVELOPMENT OF METHODS AND TECHNOLOGIES OF INFORMATICS FOR PROCESS MODELING AND MANAGEMENT

The purpose of this publication is to popularize application of informatics in process modeling and management and in environmental engineering. The papers published are thematically selected from the works presented during the conference '*Multi-accessible Computer Systems*' organized by the Systems Research Institute and the University of Technology and Agriculture in Bydgoszcz for several years already in Ciechocinek. Problems presented in the papers concern: development of quality and quantity methods supporting the process management, development of quantity methods for process modeling and simulation, development of technologies of informatics for solving problems of environmental engineering. In several papers results of research projects supported by the Polish Ministry of Science and Higher Education are presented.

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