Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

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Local entropy on IF-events

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Abstract

The notion entropy based on the probability was already described by Shannon [7] and later by Kolmogorov [3] and by Sinai [8]. We introduce the notion of local entropy introduced by Rahimi and Riazi [5] on Atanassov "intuitionistic" fuzzy (IF) sets [1] and prove some of its properties.

Keywords: entropy, local fuzzy entropy.

1 Introduction

In this section some known concepts of entropy are recalled, as there are:

Shannon's theory of entropy

Shannon [7] defined the measure of an information as follows. He has shown based on the some report that event A really occurred. Moreover, he defined the measure of information included in the given report about event A as the number

$$I = \log \frac{1}{p}.$$

Consider a random experiment with a finite number of possible results. We can model this experiment as a finite measurable space $(\Omega, \mathcal{S}, P)$, where $\Omega =$

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$\{\omega_1, ..., \omega_n\}$ is a finite set of elementary events (results of experiment) and P is a probability defined by following equations

$$P(\{\omega_i\}) = p_i, 0 \le p_i \le 1 (i = 1, ..., n), \sum_{i=1}^{n} p_i = 1.$$

Before realization of the experiment the result is not predictable. Result of the random event shows uncertainty, which is dependent on the probability measure of individual results. Here, Shannon assigned a non-negative number $H(E)$, called entropy, to the uncertainty of an experiment, defined as follows

$$H(E) = H(p_1, ..., p_n) = -\sum_{k=1}^{n} p_k \log p_k.$$

Kolomogorov-Sinai entropy theory

We shall consider classical Kolmogorov probability space $(\Omega, \mathcal{S}, P)$ and a measurable partition

$$\mathcal{A} = \{A_1, ..., A_k\},$$

i.e. a set of subsets of the set Ω such that

$$A_i \in \mathcal{S}(i = 1, ..., k), A_i \cap A_j = \emptyset (i \neq j), \bigcup_{i=1}^{k} A_i = \Omega.$$

The entropy of the measurable partition $\mathcal{A}$ is the number

$$H(\mathcal{A}) = \sum_{i=1}^{k} \varphi(P(A_i)),$$

where $\varphi(x) = -x \log x$, if $x > 0$, and $\varphi(0) = 0$.

Dynamics of a process represents a measure preserving map $T : \Omega \to \Omega$, $T^{-1}(A) \in \mathcal{S}$, and $P(T^{-1}(A)) = P(A)$, for any $A \in \mathcal{S}$. If $\mathcal{A}$ is a measurable partition, then $T^{-1}(\mathcal{A}) = \left\{T^{-1}(A_1), ..., T^{-1}(A_k)\right\}$ is a measurable partition, too. Common refinement of two measurable partitions $\mathcal{A}, \mathcal{B}$ defined by the formula

$$\mathcal{A} \vee \mathcal{B} = \{A_i \cap B_j; A_i \in \mathcal{A}, B_j \in \mathcal{B}\}.$$

generates a measurable partition. It can be proved that there exists

$$h(\mathcal{A}, T) := \frac{1}{n} \lim_{n\to\infty} H(\bigvee_{i=0}^{n-1} T^{-i}(\mathcal{A})).$$

The Kolmogorov-Sinai [3], [8] entropy of a dynamical system $(\Omega, \mathcal{S}, P, T)$ is defined by the formula

$$h(T) := \sup_{\mathcal{A}} \{h(\mathcal{A}, T)\}.$$

The Kolmogorov-Sinai theorem results from the research of entropy of a classical dynamical system. Let $(\Omega, \mathcal{S}, P, T)$ be a dynamical system and $\mathcal{A}$, the finite measurable partition, be a generator of dynamical system, i.e. $\mathcal{A}$ is a finite measurable partition of a space Ω such that

$$\sigma(\bigcup_{n=1}^{\infty} T^{-1}(\mathcal{A})) = \mathcal{S},$$

then

$$H(T) = H(\mathcal{A}, T).$$

In order to prove that two dynamical systems are not isomorphic, entropy can by used as an argument for their unisomorphism. Every dynamical system can by assigned by its entropy so, that isomorphic systems possess the same entropy. Thus, any pair of dynamical systems differing in their entropies cannot be isomorphic. Hence, the existence of unisomorphic Bernoulli schemes was identified. Since the entropy of Bernoulli's scheme given by the numbers $p_0, p_1, ..., p_{n-1}$ is $\sum_{k=0}^{n-1} p_k \log p_k$, it is no challenge to find two schemes with different entropies.

Maličký-Riečan fuzzy entropy

The notion of the entropy has been extended using the fuzzy partitions instead of partitions. A fuzzy partition is a set of non-negative measurable functions $f_1, ..., f_k, f_i : \Omega \to [0, 1]$ $(i = 1, ..., k)$ such that

$$\sum_{i=1}^{k} f_i = 1_\Omega.$$

Evidently, any partition $\mathcal{A} = \{A_1, ..., A_k\}$ can be regarded as a fuzzy partition, if we consider the characteristic functions

$$\sum_{i=1}^{k} \chi_{A_1} = 1.$$

On the set of all measurable functions $\mathcal{T}$ we define two binary operations $\oplus, \odot$ based on Lukasiewicz connectives:

$$f \oplus g(\omega) = S_L(f(\omega), g(\omega)) = \min(f(\omega) + g(\omega), 1),$$

and

$$f \odot g(\omega) = T_L(f(\omega), g(\omega)) = \max(f(\omega) + g(\omega) - 1, 0).$$

Probability $m : \mathcal{T} \to [0,1]$ is defined as any mapping satisfying the following conditions:

(i) $m(1_\Omega) = 1$,

(ii) if $f \odot g = 0$ then $m(f \oplus g) = m(f) + m(g)$,

(iii) if $f_n \nearrow f$ then $m(f_n) \nearrow m(f)$.

The dynamics of a fuzzy system represents a mapping $\mathcal{U} : \mathcal{T} \to \mathcal{T}$ such that

(i) $m(\mathcal{U}(f \oplus g) = m(\mathcal{U}(f) \oplus \mathcal{U}(g))$,

(ii) $m(\mathcal{U}(f)) = m(f)$.

If $\mathcal{A}$ is a finite measurable fuzzy partition, we define its entropy by the formula

$$H(\mathcal{A}) = -\sum_{i=1}^{k} \varphi(m(f_i)).$$

Common refinement of two partitions $\mathcal{A}$ and $\mathcal{B} = \{g_1, ..., g_l\}$ is defined using of standard product of functions

$$\mathcal{A} \vee \mathcal{B} = \{f_i.g_j; i = 1, ..., k, j = 1, ..., l\}.$$

Denote $\bigvee_{k=1}^{n-1} T^{-k}(\mathcal{A})$ fuzzy partition generated by the sets

$$f_1, ..., f_n, \mathcal{U}^1(f_1), ..., \mathcal{U}^1(f_n), ..., \mathcal{U}^{n-1}(f_1), ..., \mathcal{U}^{n-1}(f_n).$$

Maličký-Riečan [4] define their entropy by the formula

$$H(\mathcal{A}, \mathcal{U}(\mathcal{A}), ..., \mathcal{U}^{n-1}(\mathcal{A})) =$$
$$= \inf\left\{H(\mathcal{C}); \mathcal{C} \geq \mathcal{A}, \mathcal{C} \geq \mathcal{U}^{-1}(\mathcal{A}), ..., \mathcal{C} \geq \mathcal{U}^{(n-1)}(\mathcal{A})\right\}$$

and further define a number

$$h(\mathcal{A}, \mathcal{U}) = \lim_{n\to\infty} \frac{1}{n} H(\mathcal{A}, \mathcal{U}(\mathcal{A}), ..., \mathcal{U}^{n-1}(\mathcal{A})).$$

and for any set $\mathcal{G} \subset \mathcal{T}$ define the entropy of a fuzzy dynamical system $(\mathcal{T}, m, \mathcal{U})$ as the number

$$h_G(\mathcal{U}) = \sup_{\mathcal{A}} \{h(\mathcal{A}, \mathcal{U}), \mathcal{A} \subset G\}.$$

The local fuzzy entropy

Several authors have also investigated the fuzzy entropy. Dumitrescu [2] introduced the fuzzy entropy on the $\sigma-$algebra of fuzzy sets, Riečan-Markechová [6] presented an abstract model of a fuzzy entropy. Rahimi-Riazi [5] investigated the local entropy on fuzzy sets as follows.

Consider a compact metric space Ω and continuous measure preserving transformation $T : \Omega \to \Omega$. Denote by $\mathcal{F} \subset [0,1]^{\Omega}$ the $\sigma-$algebra of Borel measurable maps f:$\Omega \to [0,1]$.

For any $\omega \in \Omega$ and $f \in \mathcal{F}$ a number is defined

$$x_T(\omega, f) := \lim_{n\to\infty} \sup \frac{1}{n} \sum_{k=0}^{n-1} f \circ T^k(\omega).$$

For any partitions $\mathcal{A} = \{f_1, ..., f_k\}$ and $\mathcal{B} = \{g_1, ..., g_l\}$ and $\omega \in \Omega$ the numbers are defined

$$X_T(\omega, \mathcal{A}) := -\sum_{i=1}^{k} \varphi(x_T(\omega), f_i),$$

where $\varphi(x) = -x \log x$, if $x > 0$, and $\varphi(0) = 0$ and

$$X_T(\omega, \mathcal{A}|\mathcal{B}) := -\sum_{i,j} x_T(\omega, f_i) \log \frac{x_T(\omega, f_i.g_j)}{x_T(\omega, g_j)},$$

and finally the local fuzzy entropy of T with respect to $\mathcal{A}$ was defined as a number

$$H(T, \omega, \mathcal{A}) := \lim_{n\to\infty} \sup \frac{1}{n} X_T(\omega, \bigvee_{i=0}^{n-1} T^{-i}\mathcal{A}).$$

2 IF partition

The crucial point in the definition of the local entropy is the notion of an IF partition. We shall consider a classical dynamical system $(\Omega, \mathcal{S}, P, T)$ and the clan $\mathcal{T}$ of $\mathcal{S}-$measurable functions $f : \Omega \to [0,1]$. We assume that $f \circ T \in \mathcal{T}$ whenever $f \in \mathcal{T}$. Atanassov sets are a natural generalization of fuzzy sets. IF-set (= Atanassov set [1]) is a couple of functions

$$A = (\mu_A, \nu_A)$$

such that $\mu_A, \nu_A : \Omega \to [0,1]$ and $\mu_A + \nu_A \leq 1$. If the mappings μ_A, ν_A are $\mathcal{S}$-measurable, then the pair (μ_A, ν_A) is called IF-event. Denote by $\mathcal{F}$ the set of all IF-events.

Consider the set

$$\mathcal{K} = \{(x,y); x,y \in R, 0 \le x,y \le 1\}.$$

On the set $\mathcal{K}$ we define partial ordering as follows

$$(a_1,b_1) \le (a_2,b_2) \Leftrightarrow a_1 \le a_2, b_1 \ge b_2,$$

here $(0,1)$ is the last element. Denote by $O = (0,1), A = (a_1,b_1), B = (a_2,b_2)$. The sum $A \oplus B$ will be defined as the sum of vectors $\vec{OA} = (a_1, 1-b_1)$ and $\vec{OB} = (a_2, 1-b_2)$ hence

$$\vec{OA} \oplus \vec{OB} = (a_1,b_1) \oplus (a_2,b_2) = (a_1+a_2, 1-(1-b_1)-(1-b_2).$$

The binary operation $\oplus$ defined on the set $\mathcal{K} \times \mathcal{K}$ is commutative and associative and

$$\bigoplus_{i=1}^{n}(a_i,b_i) = (\sum_{i=1}^{n} a_i, \sum_{i=1}^{n} b_i - (n-1)).$$

Denote by $\mathcal{M}$ the set of all pairs (μ_A,ν_A) of $\mathcal{S}-$measurable functions $\mu_A,\nu_A : \Omega \to [0,1]$, i.e.

$$\mathcal{M} = \{(\mu_A,\nu_A); \mu_A,\nu_A : \Omega \to [0,1]\},$$

where μ_A,ν_A are $\mathcal{S}$-measurable. Evidently $\mathcal{F} \subset \mathcal{M}$. On the set $\mathcal{M}$ we define a partial ordering as follows

$$(\mu_A,\nu_A) \le (\mu_B,\nu_B) \Leftrightarrow \mu_A \le \mu_B, \nu_A \ge \nu_B,$$

and on the set $\mathcal{M} \times \mathcal{M}$ we define a partial binary operation $\oplus$ by the formula

$$\bigoplus_{i=1}^{n}(\mu_{A_i},\nu_{A_i}) = (\sum_{i=1}^{n} \mu_{A_i}, \sum_{i=1}^{n} \nu_{A_i} - (n-1)).$$

Definition 1 *An IF partition is any set*

$$\mathcal{A} = \{A_1,...,A_n\} = \{(\mu_{A_1},\nu_{A_1}),...,(\mu_{A_n},\nu_{A_n})\}$$

such that $A_i = (\mu_{A_i},\nu_{A_i}) \in \mathcal{M} (i = 1,...,n)$ *and there holds*

$$\bigoplus_{i=1}^{n}(\mu_{A_i},\nu_{A_i}) = (1,0).$$

Proposition 1 *If* $\mathcal{A} = \{(\mu_{A_1}, \nu_{A_1}), ..., (\mu_{A_n}, \nu_{A_n})\}$ *is an IF partition then* $\mathcal{A}^\flat = \{\mu_{A_1}, ..., \mu_{A_n}\}$ *and* $\mathcal{A}^\sharp = \{1 - \nu_{A_1}, ..., 1 - \nu_{A_n}\}$ *are the fuzzy partitions.*

Proof 1 *If* $\mathcal{A} = \{(\mu_{A_1}, \nu_{A_1}), ..., (\mu_{A_n}, \nu_{A_n})\}$ *is an IF partition then there holds:*

$$\sum_{i=1}^{n} \mu_{A_i} = 1,$$

and

$$\sum_{i=1}^{n} \nu_{A_i} - (n-1) = n - \sum_{i=1}^{n} \nu_{A_i} - 1 = n - 1 - (n-1) = 0,$$

hence

$$\sum_{i=1}^{n} (1 - \nu_{A_i}) = 1.$$

Therefore $\mathcal{A}^\flat$ *and* $\mathcal{A}^\sharp$ *are fuzzy partitions.*

We shall consider the mapping $\tau : \mathcal{M} \to \mathcal{M}$ defined for any $A = (\mu_A, \nu_A) \in \mathcal{M}$ by the formula

$$\tau(A) = (\mu_A \circ T, \nu_A \circ T).$$

Evidently for any $A, B \in \mathcal{M}$ there holds

$$\tau(A \oplus B) = (A \oplus B) \circ \tau = (A \circ \tau) \oplus (B \circ \tau) = \tau(A) \oplus \tau(B).$$

Proposition 2 *Let* $\mathcal{A} = \{(\mu_{A_1}, \nu_{A_1}), ..., (\mu_{A_n}, \nu_{A_n})\}$ *be an IF partition then* $\tau(\mathcal{A}) = \{(\mu_{A_1} \circ T, \nu_{A_1} \circ T), ..., (\mu_{A_n} \circ T, \nu_{A_n} \circ T)\}$ *is an IF partition too.*

Proof 2 *If* $\mathcal{A} = \{(\mu_{A_1}, \nu_{A_1}), ..., (\mu_{A_n}, \nu_{A_n})\}$ *is an IF partition then there holds:* $\bigoplus_{i=1}^{n} \tau(A_i) = \tau(\bigoplus_{i=1}^{n} A_i) = \tau(1, 0) = (1, 0)$.

3 Local IF entropy

Definition 2 *For every* $\omega \in \Omega$ *and any* $A = (\mu_A, \nu_A) \in \mathcal{M}$ *we define*

$$x_T(\omega, A) := (\lim_{n\to\infty} \sup \frac{1}{n} \sum_{k=0}^{n-1} (x_T(\omega, \mu_A)), \lim_{n\to\infty} \sup \frac{1}{n} \sum_{k=0}^{n-1} (x_T(\omega, 1 - \nu_A))),$$

where

$$x_T(\omega, f) = \lim_{n\to\infty} \sup \frac{1}{n} \sum_{k=1}^{n-1} f \circ T^k(\omega).$$

For any $A, B \in \mathcal{M}$, $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B)$ we define the product binary operation based on the product connectives by the formula

$$A.B := (\mu_A.\mu_B, 1 - (1 - \nu_A).(1 - \nu_B)).$$

Definition 3 *A common refinement of two IF partitions*

$$\mathcal{A}, \mathcal{B} = \{(\mu_{B_1}, \nu_{B_1}), ..., (\mu_{B_m}, \nu_{B_m})\}$$

is the collection

$$\mathcal{A} \vee \mathcal{B} = \left\{(\mu_{A_i}, \nu_{A_i}).(\mu_{B_j}, \nu_{B_j}); i = 1, ..., k, j = 1, ..., m\right\}.$$

An IF partition $\mathcal{B} = \{(\mu_{B_1}, \nu_{B_1}), ..., (\mu_{B_n}, \nu_{B_n})\}$ is a refinement of an IF partition $\mathcal{A} = \{(\mu_{A_1}, \nu_{A_1}), ..., (\mu_{A_m}, \nu_{B_m})\}$ if there exists a partition $\{I_1, ..., I_m\}$ of the set $\{1, ..., n\}$ such that

$$(\mu_{A_i}, \nu_{A_i}) = \bigoplus_{j \in I(i)} (\mu_{B_j}, \nu_{B_j}),$$

for any i=1,...,m.

Proposition 3 *If $\mathcal{A}, \mathcal{B}$ are two IF partitions, then a common refinement $\mathcal{A} \vee \mathcal{B}$ is an IF partition too.*

Proof 3 *If $\mathcal{A} = (\mu_{A_1}, \nu_{A_1}, ..., (\mu_{A_m}, \nu_{A_m})), \mathcal{B} = (\mu_{B_1}, \nu_{B_1}, ..., (\mu_{B_n}, \nu_{B_n}))$ are two IF partitions, then following equalities holds*

$$\bigoplus_{i=1}^{m} \bigoplus_{j=1}^{n} (\mu_{A_i}, \nu_{A_i}).(\mu_{B_j}, \nu_{B_j}) = \bigoplus_{i=1}^{m} \bigoplus_{j=1}^{n} (\mu_{A_i}\mu_{B_j}, \nu_{A_i} + \nu_{B_j} - \nu_{A_i}\nu_{B_j}) =$$

$$= (\sum_{i=1}^{m} \sum_{j=1}^{n} \mu_{A_i}\mu_{B_j}, \sum_{i=1}^{m} \sum_{j=1}^{n} (\nu_{A_i} + \nu_{B_j} - \nu_{A_i}\nu_{B_j}) - (mn - 1)) =$$

$$= ((\sum_{i=1}^{m} \mu_{A_i})(\sum_{j=1}^{n} \mu_{B_j}), \sum_{j=1}^{n} (\sum_{i=1}^{m} \nu_{A_i}) + \sum_{i=1}^{m} (\sum_{j=1}^{n} \nu_{B_j}) -$$

$$-(\sum_{i=1}^{m} \nu_{A_i})(\sum_{j=1}^{n} \nu_{B_j}) - (mn - 1)) =$$

$$= (1, m(n - 1) + m(n - 1) - (n - 1)(m - 1) - (mn - 1)) = (1, 0),$$

hence $\mathcal{A} \vee \mathcal{B}$ is an IF partition.

Remark 1 *There holds clearly $\mathcal{A} \vee \mathcal{B} \geq \mathcal{A}$ and $\mathcal{A} \vee \mathcal{B} \geq \mathcal{B}$.*

Denote by $\bigvee_{i=0}^{n-1} A^\flat \circ T^i$ a fuzzy partition generated by functions

$$\mu_{A_1}, ..., \mu_{A_n}, \mu_{A_1} \circ T, ..., \mu_{A_n} \circ T, ..., \mu_{A_1} \circ T^{n-1}, ..., \mu_{A_n} \circ T^{n-1}$$

and denote by $\bigvee_{i=0}^{n-1} A^\sharp \circ T^i$ a fuzzy partition generated by functions $1 - \nu_{A_1}, ..., 1 - \nu_{A_n}, (1 - \nu_{A_1}) \circ T, ..., (1 - \nu_{A_n}) \circ T, ..., (1 - \nu_{A_1}) \circ T^{n-1}, ..., (1 - \nu_{A_n}) \circ T^{n-1}$.

Definition 4 *For any IF partition $\mathcal{A}$ and $\omega \in \Omega$ we define*

$$X_T(\omega, \mathcal{A}) = (X_T(\omega, A^\flat), X_T(\omega, A^\sharp)),$$

where

$$X_T(\omega, \mathcal{A}^\flat) = -\sum_{i=1}^{n} \varphi(x_T(\omega), \mu_{A_i}),$$

and

$$X_T(\omega, \mathcal{A}^\sharp) = -\sum_{i=1}^{n} \varphi(x_T(\omega), (1 - \nu_{A_i})).$$

Definition 5 *Let $\mathcal{A}$ be an IF-partition, $\bigvee_{i=0}^{n-1} A^\flat \circ T^i$ and $\bigvee_{i=0}^{n-1} A^\sharp \circ T^i$ are the fuzzy partitions. We define the local IF entropy of a partition $\mathcal{A}$ by the formula*

$$H(\omega, T, \mathcal{A}) := (H(\omega, T, \bigvee_{i=0}^{n-1} A^\flat \circ T^i), H(\omega, T, \bigvee_{i=0}^{n-1} A^\sharp \circ T^i)),$$

where

$$H(\omega, T, \bigvee_{i=0}^{n-1} A^\flat \circ T^i) = \lim_{n \to \infty} \sup \frac{1}{n} X_T(\omega, \bigvee_{k=1}^{n-1} T^{-k} \mathcal{A}^\flat),$$

and

$$H(\omega, T, \bigvee_{i=0}^{n-1} A^\sharp \circ T^i) = \lim_{n \to \infty} \sup \frac{1}{n} X_T(\omega, \bigvee_{k=1}^{n-1} T^{-k} \mathcal{A}^\sharp).$$

Theorem 1 *Suppose that $\mathcal{A} = (\mu_{A_1}, \nu_{A_1}, ..., (\mu_{A_m}, \nu_{A_m}))$ and $\mathcal{B} = (\mu_{B_1}, \nu_{B_1}, ..., (\mu_{B_n}, \nu_{B_n}))$ are two arbitrary IF partitions. Then for any $\omega \in \Omega$ there holds:*

(i) *if* $\mathcal{B} \geq \mathcal{A}$ *then* $H(\omega, T, \mathcal{B}) \geq H(\omega, T, \mathcal{A})$*,*

(ii) $H(\omega, T, \tau(\mathcal{A})) = H(\omega, T, \mathcal{A})$*,*

(iii) *if* $k \geq 1$ *then* $H(\omega, T, \mathcal{A}) = H(\omega, T, \bigvee_{i=-k}^{k} \tau^k(\mathcal{A}))$*.*

Proof 4 *(i) If* $\mathcal{B} \geq \mathcal{A}$ *then* $\bigvee_{i=0}^{n-1} \tau^k \mathcal{B} \geq \bigvee_{i=0}^{n-1} \tau^k \mathcal{A}$ *for all* $n \geq 1$*. Further there exist a partition* $\{I_1, ..., I_m\}$ *of set* $\{1, ..., n\}$ *such that*

$$(\mu_{A_j}, \nu_{A_j}) = \bigoplus_{i \in I(j)} (\mu_{B_i}, \nu_{B_i}) = (\sum_{i \in I(j)} \mu_{B_i}, \sum_{i \in I(j)} \nu_{B_i} - (|I(j)| - 1))$$

for every j=1,...,n. Therefore

$$\mu_{A_j} = \sum_{i \in I(j)} \mu_{B_i}$$

and

$$1 - \nu_{A_j} = 1 - \sum_{i \in I(j)} \nu_{B_i} - (|I(j)| - 1) = \sum_{i \in I(j)} (1 - \nu_{B_i})$$

for every j=1,...,n.
Hence

$$\mathcal{A}^\flat \leq \mathcal{B}^\flat, \mathcal{A}^\sharp \leq \mathcal{B}^\sharp$$

therefore

$$H(\omega, T, \mathcal{A}^\flat) \leq H(\omega, T, \mathcal{B}^\flat), H(\omega, T, \mathcal{A}^\sharp) \leq H(\omega, T, \mathcal{B}^\sharp)$$

and finally

$$H(\omega, T, \mathcal{B}) \geq H(\omega, T, \mathcal{A}).$$

(ii) Since

$$X_T(\omega, \bigvee_{k=1}^{n} T^i \mathcal{A}^\flat) = X_T(\omega, \bigvee_{k=0}^{n-1} T^i \mathcal{A}^\flat)$$

and

$$X_T(\omega, \bigvee_{k=1}^{n} T^i \mathcal{A}^\sharp) = X_T(\omega, \bigvee_{k=0}^{n-1} T^i \mathcal{A}^\sharp)$$

therefore we can easily have

$$H(\omega, T, \tau(\mathcal{A})) = H(\omega, T, \mathcal{A}).$$

(iii) We have from definition

$$H(\omega,T,\bigvee_{i=0}^{k}T^{-k}\mathcal{A}^{\flat})=\lim_{n\to\infty}\sup\frac{1}{n}X_T(\omega,\bigvee_{j=0}^{n-1}T^{-j}(\bigvee_{i=0}^{k}T^{-i}\mathcal{A})=$$

$$\lim_{n\to\infty}\sup\frac{1}{n}X_T(\omega,\bigvee_{j=0}^{n-1}T^{-j}(\bigvee_{i=0}^{k}T^{-i}\mathcal{A})=$$

$$\lim_{n\to\infty}\sup(\frac{k+n}{n})\frac{1}{k+n}X_T(\omega,\bigvee_{j=0}^{n-1}T^{-j}(\bigvee_{i=0}^{k}T^{-i}\mathcal{A})=H(\omega,T,\mathcal{A}^{\flat}).$$

Analogously

$$H(\omega,T,\bigvee_{i=0}^{k}T^{-k}\mathcal{A}^{\sharp})=H(\omega,T,\mathcal{A}^{\sharp}),$$

hence

$$H(\omega,T,\mathcal{A})=H(\omega,T,\bigvee_{i=-k}^{k}\tau^{k}(\mathcal{A})).$$

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) organized in Warsaw on October 8, 2010 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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