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POLISH ACADEMY OF SCIENCES  
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**THE INTERNATIONAL  
ECONOMIC COOPERATION**

**THEORETICAL FOUNDATIONS**

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## PREFACE

The main difference between the work here presented and the other studies related to the same, generally speaking, domain, consists in the fact that considerations contained in this book indicate the possibility of resolving questions concerning the choice of the subject and establishment of profitability of international trade and cooperation in conditions when:

- \* prices on the internal market do not correspond to social costs,

- \* there is lack of conviction as to correctness of exchange rates,

- \* prices in international trade are subject to manipulations, resulting from definite interests of some countries, or they simply cannot follow the development of world production system.

As can easily be noticed these are just the conditions in which currently the international trade and cooperation system is being shaped. These particular conditions result, for instance, from governmental subsidies oriented at individual commodities or groups of commodities (e.g. food products), from existing custom tax barriers and from an extremely quick pace of technological progress in the techniques of production.

## INTRODUCTION

The problem of international exchange was presented for the first time in precise mathematical terms by Wassily Leontief in his paper entitled "Factor Proportions and the Structure of American Trade", published in *Review of Economics and Statistics* (1956, vol. 38, no. 4).

The first mathematical approach to the problem presented in Poland, was of international industrial cooperation formulated in the Doctoral dissertation of Andrzej Ameljańczyk (Military Technical Academy, 1975), supervised by this author.

Earlier, a similar formulation of the problem of international trade exchange had been forwarded in the Doctoral dissertation of J.Kotyński (Main School of Planning and Statistics, Warsaw, 1968).

If we distinguish the specific problem of international economic cooperation within the broader domain of international trade exchange then the first monograph devoted entirely to economic cooperation is the book in Polish by S.Piasecki, J.Hołuniec and A.Ameljańczyk, entitled "International economic cooperation - Modelling and Optimization" (PWN, Warsaw-Lódź, 1982).

The assumption of complementarity of goods, characteristic for the problem of cooperation, was first introduced by D.Graham in 1923 in his paper "The Theory of International Values Examined" (*Quarterly Journal of Economics*, vol. 38, no.1).

The present publication contains the original results of studies conducted during the years 1982-1985, being a continuation of work started a dozen years before.

Models of international cooperation considered there (see Chapters 1 to 3) were much simpler than in the ones presented here. Still, they are, alas, only theoretical models, which cannot be practically applied in economic activity.

Notwithstanding this situation, the models give certain possibilities with respect to applications. I am convinced that

further in-depth studies in and broadening of the theory presented here will make out of it in the future a perfect instrument for economic practice. I think that conclusions resulting from it may contribute to quicker reequilibration of the international economic system, which has been put so much off the equilibrium by the existing debts.

Against the background of existing numerous publications dealing with international trade and cooperation, as well as international specialization, the theory here presented does not require acceptance of the commonly up to date adopted assumption concerning economic equilibria within the cooperating countries, and, furthermore, this theory has much greater practical potential than the previous theories, in which it has been necessary to assume existence of economic equilibrium prices for comparing profitability of trade.

Since the theory presented in this book is independent of existence of prices, it can also be used in determination of the price structure of goods included in the trade, profitable for the partners in such an international trade deal. Thus, the structure determined ("terms of trade") guarantees stimulation of international cooperation and improvement of international specialization.

On the other hand, the theory can also be used in deciding whether the structure of prices actually existing in the international market is enhancing or, to the contrary, hindering, the development of trade, whether it does not lead to an unsound development of some of the partners at the expense of the other ones. It is not difficult to realize that the theory presented, and especially its results, concern one of the essential economic problems of present time.

The theory has, indeed, its weak points as well. A number of technical simplifying assumptions put aside (their number shall be decreasing as the theory develops), there is one fundamental assumption. It says that every participant of cooperation relation (of international trade) tries to produce the maximum of necessary goods of a given structure, entering the group

considered.

When these ones are consumption goods, we are dealing with the situation, when every partner (every national economy) participating in international exchange, is geared towards maximization of the living standard of own population, given a consumption structure characteristic for this population.

When, however, these are not consumption goods, but, e.g. semi-products, then this corresponds to the situation in which every participant-producer tries to maximize own production, this production determining the structure of demand for semi-products encompassed by cooperation. From this point of view the theory presented may get applied beyond the domain of international cooperation.

Technical simplifications adopted in the book result from the wish of possibly clear and understandable presentation of the theory. Thus, wanting to show graphically the mechanism of cooperation and to illustrate the results of the theory, the present author emphasizes in the book bilateral cooperation encompassing only two kinds or groups of commodities. Analysis of the thus simplified problem is contained in first seven chapters of the book.

The eighth chapter is in a way a generalization of considerations presented in the previous chapters so as to account for the case of multilateral cooperation, involving multiple goods. This chapter may constitute a separate whole - a summary of the contents of the book.

where  $z_2$ , the durability of cars, is expressed in kilometers.

We can analogously determine  $b_{12}$  - "consumption" of cars per one produced machine tool, for a given output of machine tools,  $\alpha_1$ , according to the formula

$$b_{12} = \frac{z_{12}}{\alpha_1 z_2}$$

where  $z_2$  is the number of kilometers covered by the cars owned by the enterprise in connection with production of machine tools.

We may proceed in a similar way in other cases, for other commodities and other kinds of industries.

The technological structure of the portion of enterprise which is of interest for us from the point of view of production of goods "1" and "2" is shown in Fig.1.

### 3. AUTARCHIC ECONOMY

Let us try now to determine the quantities  $\alpha_1$  and  $\alpha_2$ , which maximize satisfaction of demand for commodities "1" and "2", given a predetermined structure  $\gamma_2$ , when we have no possibility of exchange. The productive potential we dispose of is characterized by the quantities  $A_1$  and  $A_2$  and by the matrix of technological normatives  $b_{ij}$  ( $i, j=1, 2$ ).

Since we assume no exchange, the structure of surplus product must correspond to the structure given,  $\gamma_2$ :

$$\frac{\epsilon_2}{\epsilon_1} : \frac{\alpha_2 - \alpha_1 b_{12}}{\alpha_1 - \alpha_2 b_{21}} = \gamma_2$$

On the other hand, we would be able to maximally satisfy the

demand if we make use of all the production capacities, that is - when the equality

$$\frac{\alpha_1}{A_1} + \frac{\alpha_2}{A_2} = 1$$

holds. Solution  $\alpha_1^0$ ,  $\alpha_2^0$  of this system of equations is illustrated by Figure 2, and the magnitudes of production guaranteeing maximum surplus product are defined by formulae:

$$\alpha_1^0 = \frac{A_2}{\frac{A_2}{A_1} + \frac{b_{12} + \gamma_2}{1 + \gamma_2 b_{21}}} = \frac{A_2(1 + \gamma_2 b_{21})}{\gamma_2(1 + b_{21} \frac{A_2}{A_1}) + b_{12} + \frac{A_2}{A_1}} = A_2 \frac{1 + \gamma_2 b_{21}}{\gamma_2 B}$$

$$\alpha_2^0 = \frac{A_1}{\frac{A_1}{A_2} + \frac{1 + \gamma_2 b_{21}}{b_{12} + \gamma_2}} = \frac{A_2(b_{12} + \gamma_2)}{\gamma_2(1 + b_{21} \frac{A_2}{A_1}) + b_{12} + \frac{A_2}{A_1}} = A_1 \frac{b_{12} + \gamma_2}{\gamma_2 B}$$

where

$$B = B_{21} + \frac{1}{\gamma_2} B_{12}$$

$$B_{21} = 1 + b_{21} \frac{A_2}{A_1}$$

$$B_{12} = b_{12} + \frac{A_2}{A_1}$$

In general, the quantity  $\beta_i$  differs from quantity  $\epsilon_i$  by the value of exports or imports  $\mu_i$  (see Fig. 1):

$$\beta_i = \epsilon_i - \mu_i$$



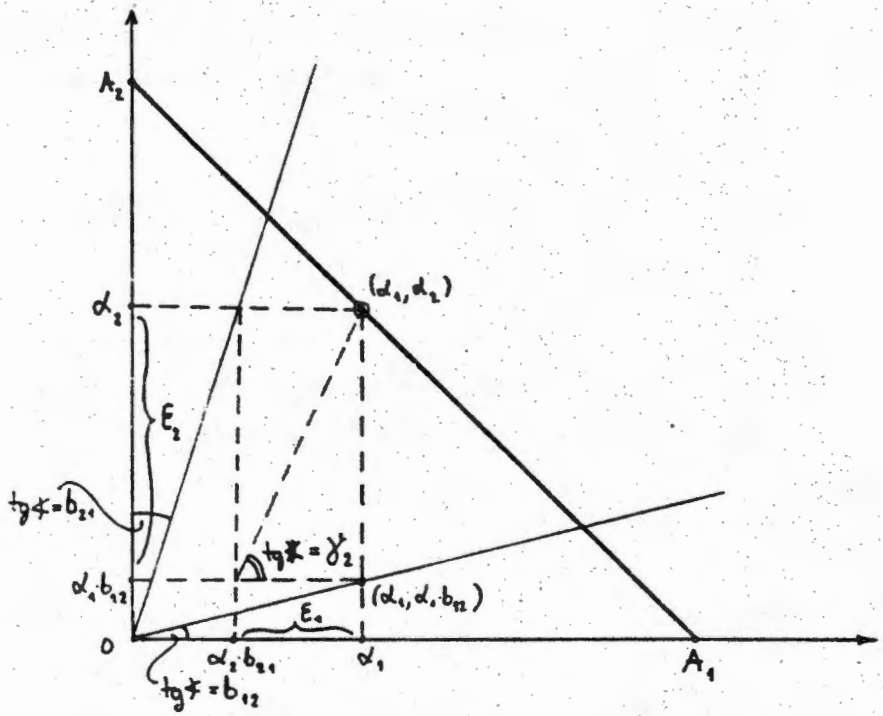


Figure 2.

When  $\mu_i$  is positive, we are dealing with net exports, when it is negative - there is net import of the good  $i$ . In our case, i.e. for an autarchic (self-sufficient, isolated) economy we have

$$\mu_i = 0$$

wherefrom equality

$$\beta_i = c_i$$

By introducing the optimum values  $\alpha_1^0, \alpha_2^0$  of net product, previously determined, to formulae defining  $c_1$  and  $c_2$  we get the maximum surplus production values,  $\xi_1^0, \xi_2^0$ , equal to maximum consumption levels,  $\beta_1, \beta_2$ , in the form of

$$\beta_1 = \xi_1^0 = A_2 \cdot \frac{1 - b_{12} b_{21}}{\gamma_2 (1 + b_{21} \frac{A_2}{A_1}) + b_{12} + \frac{A_2}{A_1}} = A_2 \cdot \frac{1 - b_{12} b_{21}}{\gamma_2 B}$$

$$\beta_2 = \xi_2^0 = \xi_1^0 \gamma_2 = A_2 \cdot \frac{1 - b_{21} b_{12}}{B}$$

In the last two formulae there appears the factor  $(1 - b_{12} b_{21})$ . Let us consider what values can product  $b_{12} b_{21}$  take in economic systems and what is its meaning.

In order to explain this question we shall look at the following example. Assume that the commodity "1" is hard coal and the commodity "2" is electricity. Quantity  $b_{12}$  corresponds then to the amount (expressed in kilowatt hours) of electricity used in coal mines for taking out one unit (ton) of coal. Thus, if production of mines is  $\alpha_1$  tons of coal per year, then consumption of electricity by mines over a year will be  $\alpha_1 b_{12}$  kilowatt hours. On the other hand, quantity  $b_{21}$  says how much coal (in tons) is used up for producing one unit (kilowatt hour) of electricity, and the product  $\alpha_2 b_{21}$  - how much coal is used up annually for electricity generation.

Note, then, that we can express the product  $b_{12}b_{21}$  in the following manner:

$$b_{12} \cdot \frac{1}{\frac{1}{b_{21}}} = \frac{b_{12}}{q_{21}}, \quad \text{where } q_{21} = \frac{1}{b_{21}}$$

with  $q_{21}$  denoting the "energy effectiveness" of coal, meaning the amount of energy (in kilowatt hours) produced out of one unit (ton) of coal (when the amount of coal used up in producing one unit of energy is  $b_{21}$ ).

The quotient  $b_{12}/q_{21}$  can be interpreted therefore, as the amount of energy used up in producing one ton of coal divided by the quantity of energy obtained from this ton of coal.

It is therefore obvious that for all the domains of economy this quotient must be less - or even much less - than 1. It is only namely then that a pair of coupled economic activities can bring a positive economic effect.

In our example this pair produces coal or energy, or - coal and energy. Thus, if the use of electric power per unit of coal produced was greater than the amount of electricity which can be obtained from this unit of coal then such a system would have to be supplied from outside with energy since the coal produced would not compensate for the energy used up.

Summing up we can assume that in the real economic systems the product  $b_{12}b_{21}$  is always less than one, so that the quantity  $1-b_{12}b_{21}$  is strictly positive.

#### 4.A MODEL OF AN OPEN ECONOMY WITH UNLIMITED EXCHANGE CAPACITIES

We shall consider now the previous case assuming, however, that there exists a possibility of buying from an external market both products - "1" and "2", with no limitations (besides the necessity of balancing exports and imports), with their prices being,

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