POLISH ACADEMY OF SCIENCES
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PREFACE

The main difference between the work here presented and the other studies related to the same, generally speaking, domain, consists in the fact that considerations contained in this book indicate the possibility of resolving questions concerning the choice of the subject and establishment of profitability of international trade and cooperation in conditions when:

* prices on the internal market do not correspond to social costs,

* there is lack of conviction as to correctness of exchange rates,

* prices in international trade are subject to manipulations, resulting from definite interests of some countries, or they simply cannot follow the development of world production system.

As can easily be noticed these are just the conditions in which currently the international trade and cooperation system is being shaped. These particular conditions result, for instance, from governmental subsidies oriented at individual commodities or groups of commodities (e.g. food products), from existing custom tax barriers and from an extremely quick pace of technological progress in the techniques of production.

INTRODUCTION

The problem of international exchange was presented for the first time in precise mathematical terms by Wassily Leontief in his paper entitled "Factor Proportions and the Structure of American Trade", published in *Review of Economics and Statistics* (1956, vol. 38, no. 4).

The first mathematical approach to the problem presented in Poland, was of international industrial cooperation formulated in the Doctoral dissertation of Andrzej Ameljańczyk (Military Technical Academy, 1975), supervised by this author.

Earlier, a similar formulation of the problem of international trade exchange had been forwarded in the Doctoral dissertation of J.Kotyński (Main School of Planning and Statistics, Warsaw, 1968).

If we distinquish the specific problem of international economic cooperation within the broader domain of international trade exchange then the first monograph devoted entirely to economic cooperation is the book in Polish by S.Piasecki, J.Hołuniec and A.Ameljańczyk, entitled "International economic cooperation - Modelling and Optimization" (PWN, Warsaw-Łódź, 1982).

The assumption of complementarity of goods, characteristic for the problem of cooperation, was first introduced by D.Graham in 1923 in his paper "The Theory of International Values Examined" (Quarterly Journal of Economics, vol. 38, no.1).

The present publication contains the original results of studies conducted during the years 1982-1985, being a continuation of work started a dozen years before.

Models of international cooperation considered there (see Chapters 1 to 3) were much simpler than in the ones presented here. Still, they are, alas, only theoretical models, which cannot be practically applied in economic activity.

Notwithstanding this situation, the models give certain possibilities with respect to applications. I am convinced that further in-depth studies in and broadening of the theory presented here will make out of it in the future a perfect instrument for economic practice. I think that conclusions resulting from it may contribute to quicker reequilibration of the international economic system, which has been put so much off the equilibrium by the existing debts.

Against the background of existing numerous publications dealing with international trade and cooperation, as well as international specialization, the theory here presented does not require acceptance of the commonly up to date adopted assumption concerning economic equilibria within the cooperating countries, and, furthermore, this theory has much greater practical potential than the previous theories, in which it has been necessary to assume existence of economic equilibrium prices for comparing profitability of trade.

Since the theory presented in this book is independent of existence of prices, it can also be used in determination of the price structure of goods included in the trade, profitable for the partners in such an international trade deal. Thus, the structure determined ("terms of trade") guarantees stimulation of international cooperation and improvement of international specialization.

On the other hand, the theory can also be used in deciding whether the structure of prices actually existing in the international market is enhancing or, to the contrary, hindering, the development of trade, whether it does not lead to an unsound development of some of the partners at the expense of the other ones. It is not difficult to realize that the theory presented, and especially its results, concern one of the essential economic problems of present time.

The theory has, indeed, its weak points as well. A number of technical simplifying assumptions put aside (their number shall be decreasing as the theory develops), there is one fundamental assumption. It says that every participant of cooperation relation (of international trade) tries to produce the maximum of necessary goods of a given structure, entering the group considered.

When these ones are consumption goods, we are dealing with the situation, when every partner (every national economy) participating in international exchange, is geared towards maximization of the living standard of own population, given a consumption structure characteristic for this population.

When, however, these are not consumption goods, but, e.g. semi-products, then this corresponds to the situation in which every participant-producer tries to maximize own production, this production determining the structure of demand for semi-products encompassed by cooperation. From this point of view the theory presented may get applied beyond the domain of international cooperation.

Technical simplifications adopted in the book result from the wish of possibly clear and understandable presentation of the theory. Thus, wanting to show graphically the mechanism of cooperation and to illustrate the results of the theory, the present author emphasizes in the book bilateral cooperation encompassing only two kinds or groups of commodities. Analysis of the thus simplified problem is contained in first seven chapters of the book.

The eighth chapter is in a way a generalization of considerations presented in the previous chapters so as to account for the case of multilateral cooperation, involving multiple goods. This chapter may constitute a separate whole - a summary of the contents of the book.

7. THE NECESSARY CONDITION FOR EXISTENCE OF A STABLE COOPERATION

Let us note that the values determined, $\overset{*}{\alpha}{}_1^{I}, \overset{*}{\alpha}{}_1^{II}$ (and $\overset{*}{\alpha}{}_2^{I}, \overset{*}{\alpha}{}_2^{II}$) define the optimum cooperation, that is - the one which guarantees maximization of the value of $\beta_1^I+\beta_1^{II}$ (and of $\beta_2^I+\beta_2^{II}$), with the following inequality being always satisfied:

$$\overset{*}{\beta}{}_1^{I} + \overset{*}{\beta}{}_1^{II} \geq \overset{\circledcirc}{\beta}{}_1^{I} + \overset{\circledcirc}{\beta}{}_1^{II}$$

where, as before, $\overset{\circledcirc}{\beta}{}_1^{I}$ and $\overset{\circledcirc}{\beta}{}_1^{II}$ are consumption levels which can be ensured separately by each side for itself.

This, however, does not mean at all that inequalities

$$\overset{*}{\beta}{}_1^{I} \geq \overset{\circledcirc}{\beta}{}_1^{I}$$

$$\overset{*}{\beta}{}_1^{II} \geq \overset{\circledcirc}{\beta}{}_1^{II}$$

will hold. Thus, there may occur a situation when optimum cooperation guaranteeing the possibility of significant increase of the overall consumption would turn out disadvantageous for one of the participants, for it would only produce for this participant a level of consumption lower than the one that could be attained by his independent action. Obviously, the other participant of cooperation would gain the whole profit from cooperation - and even somewhat more than that.

Such a situation would of course be instable, and it could not be maintained over a longer period of time. That is why the two inequalities,

$$\overset{*}{\beta}{}_1^{I} \geq \overset{\circledcirc}{\beta}{}_1^{I}$$

$$\overset{*}{\beta}{}_1^{II} \geq \overset{\circledcirc}{\beta}{}_1^{II}$$

shall further on be referred to as the conditions of existence of stable cooperation.

Note that this condition actually concerns existence of a stable optimum cooperation. In order, therefore, to be able to call this condition the condition of existence of any stable cooperation, we would have to demonstrate that if there is no stable optimum cooperation then there is no other stable cooperation at all.

We must then, analyse in more detail the factors, upon which the division of the quantity $\beta_1^I + \beta_1^{II}$, maximized in optimum cooperation, into two components - β_1^I and β_1^{II} - depends. It is not difficult to notice that this division depends upon the value of the ratio ω_2 ("terms of trade"). And conversely - the condition for existence of stable cooperation defines the range of values of ω_2 for which cooperation has stable nature.

Let us now put our considerations in order from the point of view of determination of the ranges of values of ω_2, for which cooperative exchange shall be materially and financially balanced and shall additionally fulfill the stability condition, meaning that the division of advantages accruing from cooperation shall guarantee that individual advantages of each of the sides be greater than the ones they can ensure themselves acting in isolation.

Let us also recall that the values of $\overset{*}{\alpha}{}_1^I, \overset{*}{\alpha}{}_1^{II}$ are defined with the formulae

$$\overset{*}{\alpha}{}_1^I = \begin{cases} \max\{0, \dfrac{D - A_1^{II}B^{II}}{B^I}\} & \text{when } E \leq 0 \\ \min\{\dfrac{D}{B^I}, A_1^I\} & \text{when } E > 0 \end{cases}$$

$$\overset{*}{\alpha}{}_1^{II} = \begin{cases} \max\{0, \dfrac{D - A_1^I B^I}{B^{II}}\} & \text{when } E>0 \\ \min\{\dfrac{D}{B^{II}}, A_1^{II}\} & \text{when } E\le 0 \end{cases}$$

where

$$D = G + \frac{1}{\gamma_2}(A_2^I + A_2^{II}), \quad G = A_2^I b_{21}^I + A_2^{II} b_{21}^{II}$$

$$B^I = B_{21}^I + \frac{1}{\gamma_2} B_{12}^I, \quad B^{II} = B_{21}^{II} + \frac{1}{\gamma_2} B_{12}^{II}$$

$$B_{21}^I = 1 + b_{21}^I \cdot \frac{A_2^I}{A_1^I}, \quad B_{12}^I = b_{12}^{II} + \frac{A_2^I}{A_1^I},$$

$$B_{21}^{II} = 1 + b_{21}^{II} \cdot \frac{A_2^{II}}{A_1^{II}}, \quad B_{12}^{II} = b_{12}^{II} + \frac{A_2^{II}}{A_1^{II}}.$$

$$E = B_{21}^I - \frac{B^I}{B^{II}} B_{21}^{II} = \frac{B_{21}^I B_{12}^{II} - B_{12}^I B_{21}^{II}}{\gamma_2 B_{21}^{II} + B_{12}^{II}}$$

On the other hand the values of $\overset{\circ}{\alpha}{}_1^I, \overset{\circ}{\alpha}{}_1^{II}$ satisfy the system of equations of an autarchic economy:

$$\alpha_1^I - \alpha_2^I b_{21}^I - \beta_1^I = 0$$
$$\alpha_2^I - \alpha_2^I b_{12}^I - \gamma_2 \beta_1^I = 0$$
$$\frac{\alpha_1^I}{A_1^I} + \frac{\alpha_2^I}{A_2^I} \le 1$$

with the postulate of maximization of the value of β_1^I:

$\beta_1^I \to \max$

By solving the above problem we obtain:

$$\overset{\odot}{\alpha}{}_1^{I} = \frac{A_2^{I}}{\gamma_2 B^{I}}(1 + \gamma_2 b_{21}^{I})$$

and

$$\overset{\odot}{\beta}{}_1^{I} = \frac{A_2^{I}}{\gamma_2 B^{I}}(1 - b_{12}^{I} b_{21}^{I})$$

Similarly, we shall obtain for the second economy

$$\overset{\odot}{\alpha}{}_1^{II} = \frac{A_2^{II}}{\gamma_2 B^{II}}(1 + \gamma_2 b_{21}^{II})$$

and

$$\overset{\odot}{\beta}{}_1^{II} = \frac{A_2^{II}}{\gamma_2 B^{II}}(1 - b_{12}^{II} b_{21}^{II})$$

Recall that for the materially balanced exchange, that is - for the one which satisfies the conditions

$$\mu_1^{I} + \mu_1^{II} = 0$$

$$\mu_2^{I} + \mu_2^{II} = 0$$

but which is not necessarily financially balanced, the following equation also holds:

$$\alpha_1^{I} B^{I} + \alpha_1^{II} B^{II} = D$$

with

$$\beta_1^{I} + \beta_1^{II} = \alpha_1^{I} B_{21}^{I} + \alpha_1^{II} B_{21}^{II} - G$$

On the other hand, for the financially balanced exchange, i.e. for the one satisfying the conditions

$$\mu_1^I + \mu_2^I\omega_2 = 0$$
$$\mu_1^{II} + \mu_2^{II}\omega_2 = 0$$

but not necessarily materially balanced, we have

$$\beta_1^I = \frac{\alpha_1^I(B_{21}^I - \omega_2 B_{12}^I) + V^I}{1 + \omega_2\gamma_2}, \qquad V^I = A_2^I(\omega_2 - b_{21}^I),$$

$$\beta_1^{II} = \frac{\alpha_1^{II}(B_{21}^{II} - \omega_2 B_{12}^{II}) + V^{II}}{1 + \omega_2\gamma_2}, \qquad V^{II} = A_2^{II}(\omega_2 - b_{21}^{II}),$$

In every case, obviously, exchange guarantees an adequate structure of consumption, so that

$$\frac{\beta_2^I}{\beta_1^I} = \gamma_2 = \frac{\beta_2^{II}}{\beta_1^{II}}$$

Consequently, the materially and financially balanced exchange is defined by equations

$$\alpha_1^I B^I + \alpha_1^{II} B^{II} = D$$

$$\beta_1^I = \frac{\alpha_1^I(B_{21}^I - \omega_2 B_{12}^I) + V^I}{1 + \omega_2\gamma_2}$$

$$\beta_1^{II} = \frac{\alpha_1^{II}(B_{21}^{II} - \omega_2 B_{12}^{II}) + V^{II}}{1 + \omega_2\gamma_2}$$

with - as can be easily verified - the following equation holding in this case:

$$\beta_1^I + \beta_1^{II} = \alpha_1^I B_{21}^I + \alpha_1^{II} B_{21}^{II} - G$$

Thus, we can write down the condition of materially and financially balanced exchange in the form of the system of inequalities:

$$\alpha_1^I(B_{21}^I - \omega_2 B_{12}^I) + V^I > \overset{\odot}{\beta}{}_1^I(1 + \omega_2\gamma_2)$$

$$\alpha_1^{II}(B_{21}^{II} - \omega_2 B_{12}^{II}) + V^{II} > \overset{\odot}{\beta}{}_1^{II}(1 + \omega_2\gamma_2)$$

and the quantities α_1^I, α_1^{II} are linked by equation

$$\alpha_1^I B^I + \alpha_1^{II} B^{II} = D$$

On the other hand the overall consumption,

$$\beta_1^I + \beta_1^{II} = \alpha_1^I B_{21}^I + \alpha_1^{II} B_{21}^{II} - G$$

after substitution of

$$\alpha_1^{II} = \frac{D - \alpha_1^I B^I}{B^{II}}$$

shall be equal

$$\beta_1^I + \beta_1^{II} = \alpha_1^I E + C, \qquad C = \frac{D}{B^{II}} \cdot B_{21}^{II} - G$$

with the value of α_1^I selected from the range defined by

$$\max\{0, \frac{D - A_1^{II} B^{II}}{B^I}\} \le \alpha_1^I \le \min\{\frac{D}{B^I}, A_1^I\}$$

By solving the inequalities of the condition for stability of cooperation with regard to ω_2 we can obtain the condition on the value of ω_2. This value must namely be contained in the range defined by the pair of numbers $U^I(\alpha_1^I)$, $U^{II}(\alpha_1^{II})$, i.e.

$$U^{I}(\alpha_1^{I}) = \frac{\alpha_1^{I}B_{21}^{I} - (\overset{\circ}{\beta}{}_1^{I} + A_2^{I}b_{21}^{I})}{\alpha_1^{I}B_{12}^{I} - (A_2^{I} - \gamma_2\overset{\circ}{\beta}{}_1^{I})}$$

$$U^{II}(\alpha_1^{II}) = \frac{\alpha_1^{II}B_{21}^{II} - (\overset{\circ}{\beta}{}_1^{II} + A_2^{II}b_{21}^{II})}{\alpha_1^{II}B_{12}^{II} - (A_2^{II} - \gamma_2\overset{\circ}{\beta}{}_1^{II})}$$

This range of values is defined for the optimum cooperation in the following way:

I. If $E<0$, that is - when the inequality

$$\frac{B_{21}^{I}}{B_{12}^{I}} < \frac{B_{21}^{II}}{B_{12}^{II}}$$

is satisfied, then $U^{I}(\overset{*}{\alpha}{}_1^{I}) \le \omega_2 \le U^{II}(\overset{*}{\alpha}{}_1^{II})$, and, for $D \le A_1^{II}B^{II}$, or, otherwise, for

$$A_2^{I}(1 + \gamma_2 b_{21}^{I}) \le A_1^{II}(\gamma_2 + b_{12}^{II})$$

we have $\overset{*}{\alpha}{}_1^{I} = 0$, $\overset{*}{\alpha}{}_1^{II} = \frac{D}{B^{II}}$, while in the opposite case:

$$\overset{*}{\alpha}{}_1^{I} = \frac{D - A_1^{II}B^{II}}{B^{I}}, \qquad \overset{*}{\alpha}{}_1^{II} = A_1^{II}$$

Having introduced the above values of $\overset{*}{\alpha}{}_1^{I}$ and $\overset{*}{\alpha}{}_1^{II}$ to the inequality we obtain the condition of stability of cooperation in the form of

$$\frac{B_{21}^{I}}{B_{12}^{I}} \le \omega_2 \le \frac{B_{21}^{II}}{B_{12}^{II}}$$

independently of relations between the quantities D and $A_1^{II}B^{II}$.

II. In a similar manner, as can easily be verified, we can establish that when $E > 0$ the following inequality must hold:

$$\frac{B^{I}_{21}}{B^{I}_{12}} \le \omega_2 \le \frac{B^{II}_{21}}{B^{II}_{12}}$$

III. If $E = 0$ then, of course, we have an equality condition, that is

$$\omega_2 = \frac{B^{I}_{21}}{B^{I}_{12}} = \frac{B^{II}_{21}}{B^{II}_{12}}$$

Conclusion

The condition for existence of a stable optimum cooperation is the requirement that the value of ω_2 (the "terms of trade") belonged to the interval defined by the numbers

$$\frac{1 + b^{I}_{21}\dfrac{A^{I}_{2}}{A^{I}_{1}}}{b^{I}_{12} + \dfrac{A^{I}_{2}}{A^{I}_{1}}}, \qquad \frac{1 + b^{II}_{21}\cdot\dfrac{A^{II}_{2}}{A^{II}_{1}}}{b^{II}_{12} + \dfrac{A^{II}_{2}}{A^{II}_{1}}}$$

What still remains is to answer the following question: does there exist a materially and financially balanced cooperation, which would be advantageous for both sides in case when optimum cooperation is unprofitable?

The answer to this question can be brought by the relation between the intervals of tolerance for ω_2 in case when $\alpha^{I}_{1} = \overset{*}{\alpha}{}^{I}_{1}$, $\alpha^{II}_{1} = \overset{*}{\alpha}{}^{II}_{1}$, and when $\alpha^{I}_{1} = \bar{\alpha}^{I}_{1}$, $\alpha^{II}_{1} = \bar{\alpha}^{II}_{1}$, with $\bar{\alpha}^{I}_{1} \neq \overset{*}{\alpha}{}^{I}_{1}$, $\bar{\alpha}^{II}_{1} \neq \overset{*}{\alpha}{}^{II}_{1}$.

Assume that

$$\bar{\alpha}^{I}_{1} = \overset{*}{\alpha}{}^{I}_{1} + \Delta\alpha^{I}_{1}$$

Then, from equation

$$\alpha_1^I(B_{21}^I + \frac{1}{\gamma_2}B_{12}^I) + \alpha_1^{II}(B_{21}^{II} + \frac{1}{\gamma_2}B_{12}^{II}) = D$$

we can conclude that

$$\Delta\alpha_1^{II} = -\frac{B^I}{B^{II}}\cdot\Delta\alpha_1^I$$

and therefore

$$\bar{\alpha}_1^{II} = \overset{*}{\alpha}_1^{II} - \frac{B^I}{B^{II}}\cdot\Delta\alpha_1^I$$

I. Let first consider the case when $E \le 0$. Then, the optimum value is the lowest of all the possible ones. In connection with this in the relation

$$\bar{\alpha}_1^I = \overset{*}{\alpha}_1^I + \Delta\alpha_1^I$$

quantity $\Delta\alpha_1^I$ may only take positive values. Since the tolerance interval for the value of ω_2 is for $E \le 0$ defined by the pair of numbers

$$(U^I(\bar{\alpha}_1^I),\ U^{II}(\bar{\alpha}_1^{II}))$$

then, if we develop function U in a Taylor series around the point $\overset{*}{\alpha}_1^I$, the condition of stability of cooperation shall take on the form of

$$U^I(\overset{*}{\alpha}_1^I) + \frac{d}{d\alpha_1^I}U^I(\alpha_1^I)\Big|_{\alpha_1^I=\overset{*}{\alpha}}\cdot\Delta\alpha_1^I + \ldots \le \omega_2 \le U^{II}(\overset{*}{\alpha}_1^{II}) + \frac{d}{d\alpha_1^{II}}U^{II}(\alpha_1^{II})\Big|_{\alpha_1^{II}=\overset{*}{\alpha}_1^{II}}\cdot\Delta\alpha_1^{II} + \ldots$$

In order for the interval of tolerance for the value of ω_2 was in the case of $\alpha_1^I=\bar{\alpha}_1^I$ not greater than in the case of $\alpha_1^I=\overset{*}{\alpha}_1^I$, the following inequalities must hold (considering that $\Delta\alpha_1^I>0$ and $\Delta\alpha_1^{II}>0$):

$$\frac{d}{d\alpha_1^{II}}U^{II}(\alpha_1^{II})\Big|_{\alpha_1^{II}=\overset{*}{\alpha}_1^{II}}\cdot\Delta\alpha_1^{II} + \ldots \ge 0$$

$$\frac{d}{d\alpha_1^I} U^{II}(\alpha_1^I)\Big|_{\alpha_1^I=\overset{*}{\alpha}{}_1^I} \cdot \Delta\alpha_1^I + \ldots \geq 0$$

Consequently, in order to prove that if there does not exist a stable optimum cooperation then there does not exist any other stable cooperation, it is sufficient to demonstrate that the two inequalities given above are satisfied.

For this purpose we shall prove the following

Lemma. The series

$$\sum_{n=1}^{\infty} \frac{d^n}{d\alpha^n} U(\alpha)\cdot\Delta\alpha^n, \qquad \Delta\alpha \to 0,$$

converges and its limit is 0.

Proof.

Take

$$U(\alpha) = \frac{L(\alpha)}{M(\alpha)} = \frac{\alpha B_{21} - (\overset{\odot}{\beta} + A_2 b_{21})}{\alpha B_{12} - (A_2 - \gamma_2 \overset{\odot}{\beta})}$$

Then, for $n=1$ we have

$$\frac{d}{d\alpha} U(\alpha) = \frac{d}{d\alpha} \frac{L(\alpha)}{M(\alpha)} = \frac{W(\alpha)}{M^2(\alpha)}$$

where

$$W(\alpha) = L(\alpha) B_{12} - M(\alpha) B_{21}$$

Similarly, for $n=2$ we obtain

$$\frac{d^2}{d\alpha^2} U(\alpha) = \frac{d}{d\alpha}\left(\frac{d}{d\alpha} U(\alpha)\right) = (-1)(-2) B_{12} \frac{W(\alpha)}{M^3(\alpha)}$$

since

$$\frac{d}{d\alpha} W(\alpha) = B_{21} B_{12} - B_{12} B_{21} = 0$$

Generally, as can easily be verified, we have, for any n,

$$\frac{d^n}{d\alpha^n}U(\alpha) = (-1)^n n! \frac{W(\alpha)}{M^{n+1}(\alpha)}$$

But

$$W(\alpha)=L(\alpha)B_{12}-M(\alpha)B_{21}=[\alpha B_{12}-(A_2-\gamma_2\overset{\odot}{\beta})]\cdot B_{21}-[\alpha B_{21}-(\overset{\odot}{\beta}+A_2b_{21})]B_{12}=$$

$$=B_{12}(\overset{\odot}{\beta}+A_2b_{21})-B_{21}(A_2-\gamma_2\overset{\odot}{\beta})=$$

$$=\gamma_2\overset{\odot}{\beta}[1+b_{21}\cdot\frac{A_2}{A_1}+\frac{1}{\gamma_2}(b_{12}+\frac{A_2}{A_1})]-A_2(1-b_{12}b_{21})=$$

$$=\gamma_2B\cdot[\overset{\odot}{\beta}-\frac{A_2}{\gamma_2B_2}(1-b_{12}b_{21})] = \gamma_2B\cdot[\overset{\odot}{\beta}-\overset{\odot}{\beta}] = 0$$

so that

$$\frac{d^n}{d\alpha^n}U(\alpha) = (-1)^n n! \frac{W(\alpha)}{M^{n+1}(\alpha)} = 0$$

as well as

$$\sum_{n=1}^{\infty}\frac{d^n}{d\alpha^n}U(\alpha)\Delta\alpha^n = 0$$

QED.

Consequently, by making use of the lemma proved above we have also demonstrated that the following theorem is true:

Theorem 4.

If for a given value of ω_2 ("terms of trade") there does not exist a stable optimum cooperation, then there does not exist any other stable cooperation, materially and financially balanced.

This theorem is very important, for it allows abandoning of the search of other cooperation solutions when optimum cooperation is disadvantageous for one of the partners in view of the current value of ω_2 ("terms of trade") valid on the international market.

Since optimum cooperation scheme guarantees an increase of the overall consumption, then, in the case when the sharing of this increase, defined by the value of ω_2, is unfair - that is: one of the sides obtains less than could be secured by it when acting separately - this does not mean that optimum cooperation is wrong, but only - that the current value of ω_2 is inappropriate.

In such a situation one should, of course, adopt some other value of ω_2 through negotiations, with the new value still being contained within the tolerance interval, but irrespective of the value of ω_2 valid in the international trade, and to strike an adequate bilateral deal.

8. COOPERATION OF MULTIPLE PARTNERS IN PRODUCTION OF MULTIPLE COMMODITIES

Every participant of cooperation is defined by characteristics of its economy and society, forming a state or regional organism of high autonomy.

Let us denote by α_j the magnitude of production (e.g. in the annual period) of the commodity j. Then, if the technological coefficient b_{ij} defines the normative use of commodity j per unit of commodity i produced, we have

$$\varepsilon_j = \alpha_j - \sum_i \alpha_i b_{ij}$$

as the definition of the resulting output of commodity j in the economy. Using vector notation we may write

$$\hat{\varepsilon} = \hat{\alpha}[I - B]$$

where

$\hat{\varepsilon} = \langle \varepsilon_1, \dots, \varepsilon_i, \dots, \varepsilon_I \rangle$ is the vector of net product,

$\hat{\alpha} = \langle \alpha_1, \dots, \alpha_i, \dots, \alpha_I \rangle$ is the vector of global production $(\alpha \geq 0)$,

I - unit matrix,

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