POLISH ACADEMY OF SCIENCES
SYSTEMS RESEARCH INSTITUTE

THE INTERNATIONAL ECONOMIC COOPERATION

THEORETICAL FOUNDATIONS

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PREFACE

The main difference between the work here presented and the other studies related to the same, generally speaking, domain, consists in the fact that considerations contained in this book indicate the possibility of resolving questions concerning the choice of the subject and establishment of profitability of international trade and cooperation in conditions when:

* prices on the internal market do not correspond to social costs,

* there is lack of conviction as to correctness of exchange rates,

* prices in international trade are subject to manipulations, resulting from definite interests of some countries, or they simply cannot follow the development of world production system.

As can easily be noticed these are just the conditions in which currently the international trade and cooperation system is being shaped. These particular conditions result, for instance, from governmental subsidies oriented at individual commodities or groups of commodities (e.g. food products), from existing custom tax barriers and from an extremely quick pace of technological progress in the techniques of production.

INTRODUCTION

The problem of international exchange was presented for the first time in precise mathematical terms by Wassily Leontief in his paper entitled "Factor Proportions and the Structure of American Trade", published in *Review of Economics and Statistics* (1956, vol. 38, no. 4).

The first mathematical approach to the problem presented in Poland, was of international industrial cooperation formulated in the Doctoral dissertation of Andrzej Ameljańczyk (Military Technical Academy, 1975), supervised by this author.

Earlier, a similar formulation of the problem of international trade exchange had been forwarded in the Doctoral dissertation of J.Kotyński (Main School of Plannimg and Statistics, Warsaw, 1968).

If we distinquish the specific problem of international economic cooperation within the broader domain of international trade exchange then the first monograph devoted entirely to economic cooperation is the book in Polish by S.Piasecki, J.Hołuniec and A.Ameljańczyk, entitled "International economic cooperation - Modelling and Optimization" (PWN, Warsaw-Łódź, 1982).

The assumption of complementarity of goods, characteristic for the problem of cooperation, was first introduced by D.Graham in 1923 in his paper "The Theory of International Values Examined" (Quarterly Journal of Economics, vol. 38, no.1).

The present publication contains the original results of studies conducted during the years 1982-1985, being a continuation of work started a dozen years before.

Models of international cooperation considered there (see Chapters 1 to 3) were much simpler than in the ones presented here. Still, they are, alas, only theoretical models, which cannot be practically applied in economic activity.

Notwithstanding this situation, the models give certain possibilities with respect to applications. I am convinced that further in-depth studies in and broadening of the theory presented here will make out of it in the future a perfect instrument for economic practice. I think that conclusions resulting from it may contribute to quicker reequilibration of the international economic system, which has been put so much off the equilibrium by the existing debts.

Against the background of existing numerous publications dealing with international trade and cooperation, as well as international specialization, the theory here presented does not require acceptance of the commonly up to date adopted assumption concerning economic equilibria within the cooperating countries, and, furthermore, this theory has much greater practical potential than the previous theories, in which it has been necessary to assume existence of economic equilibrium prices for comparing profitability of trade.

Since the theory presented in this book is independent of existence of prices, it can also be used in determination of the price structure of goods included in the trade, profitable for the partners in such an international trade deal. Thus, the structure determined ("terms of trade") guarantees stimulation of international cooperation and improvement of international specialization.

On the other hand, the theory can also be used in deciding whether the structure of prices actually existing in the international market is enhancing or, to the contrary, hindering, the development of trade, whether it does not lead to an unsound development of some of the partners at the expense of the other ones. It is not difficult to realize that the theory presented, and especially its results, concern one of the essential economic problems of present time.

The theory has, indeed, its weak points as well. A number of technical simplifying assumptions put aside (their number shall be decreasing as the theory develops), there is one fundamental assumption. It says that every participant of cooperation relation (of international trade) tries to produce the maximum of necessary goods of a given structure, entering the group considered.

When these ones are consumption goods, we are dealing with the situation, when every partner (every national economy) participating in international exchange, is geared towards maximization of the living standard of own population, given a consumption structure characteristic for this population.

When, however, these are not consumption goods, but, e.g. semi-products, then this corresponds to the situation in which every participant-producer tries to maximize own production, this production determining the structure of demand for semi-products encompassed by cooperation. From this point of view the theory presented may get applied beyond the domain of international cooperation.

Technical simplifications adopted in the book result from the wish of possibly clear and understandable presentation of the theory. Thus, wanting to show graphically the mechanism of cooperation and to illustrate the results of the theory, the present author emphasizes in the book bilateral cooperation encompassing only two kinds or groups of commodities. Analysis of the thus simplified problem is contained in first seven chapters of the book.

The eighth chapter is in a way a generalization of considerations presented in the previous chapters so as to account for the case of multilateral cooperation, involving multiple goods. This chapter may constitute a separate whole - a summary of the contents of the book.

disadvantageous for one of the partners in view of the current value of ω_2 ("terms of trade") valid on the international market.

Since optimum cooperation scheme guarantees an increase of the overall consumption, then, in the case when the sharing of this increase, defined by the value of ω_2, is unfair - that is: one of the sides obtains less than could be secured by it when acting separately - this does not mean that optimum cooperation is wrong, but only - that the current value of ω_2 is inappropriate.

In such a situation one should, of course, adopt some other value of ω_2 through negotiations, with the new value still being contained within the tolerance interval, but irrespective of the value of ω_2 valid in the international trade, and to strike an adequate bilateral deal.

8. COOPERATION OF MULTIPLE PARTNERS IN PRODUCTION OF MULTIPLE COMMODITIES

Every participant of cooperation is defined by characteristics of its economy and society, forming a state or regional organism of high autonomy.

Let us denote by α_j the magnitude of production (e.g. in the annual period) of the commodity j. Then, if the technological coefficient b_{ij} defines the normative use of commodity j per unit of commodity i produced, we have

$$\varepsilon_j = \alpha_j - \sum_i \alpha_i b_{ij}$$

as the definition of the resulting output of commodity j in the economy. Using vector notation we may write

$$\hat{\varepsilon} = \hat{\alpha}[I - B]$$

where

$\hat{\varepsilon} = \langle\varepsilon_1,\ldots,\varepsilon_i,\ldots,\varepsilon_I\rangle$ is the vector of net product,

$\hat{\alpha} = \langle\alpha_1,\ldots,\alpha_i,\ldots,\alpha_I\rangle$ is the vector of global production ($\alpha\geq0$),

I - unit matrix,

B - matrix of technological coefficients, b_{ij},
I - number of goods considered.

The form of dependence of the magnitude of product use, $\hat{\lambda}$, upon the production intensity magnitude, $\hat{\alpha}$, adopted here, i.e. the linear form given by $\hat{\lambda} = \hat{\alpha}B$ is of course an approximation.

Note that for the closed economies functioning in complete isolation the following condition would have to be satisfied:

$$\varepsilon_j = \alpha_j - \sum_i \alpha_i b_{ij} \geq 0$$

for every $j=1,2,\ldots,I$, excepting the commodity $j=j_*$, called (human) labor. This good is not "produced" by the economy, it is only used up by it, with the demand for labour being equal

$$\lambda_{j_*} = \sum_i \alpha_i b_{ij_*}$$

where b_{ij_*} are the labour intensity coefficients proper for production of commodities $i=1,2,\ldots,I$, $i \neq j_*$ (for $i=j$, i.e. $b_{ii}=0$ for all i). Thus, we have altogether for $j=j_*$ that

$$\varepsilon_{j_*} = -\sum_i \alpha_i b_{ij_*} \leq 0$$

since $\alpha_{j_*}=0$ (on the basis of definition of commodity j_*).

Irrespective of the limitation set by the relation constraining the capacity of free choice of the value of $\hat{\alpha}$ (in view of material limitations) this choice is additionally, independently, limited by the capacities of the fixed assets (production means) at the disposal of an economy.

Because of the limited amounts of machines and auxiliary equipment owned, the production intensity of a good $i \neq j_*$ cannot exceed a certain maximum value.

Denote by A_j the maximum intensity of production α_j that we could attain if there were no material limitations (given certain capacity of production means) and if we produced only commodity j. Then, the limitation on the production possibilities with regard to the current production capacities will have the form of

$$\sum_j \frac{\alpha_j}{A_j} \leq 1$$

This also is a linear approximation of the true form of this constraint.

Now, if we denote by $\hat{a}$ the column vector whose components are equal $a_j = \frac{1}{A_j}$, $A_j \neq 0$, then this inequality can be written down in the form of $\hat{\alpha}\hat{a} \leq 1$.

Thus, quantities $\hat{a}$ and B characterize in our considerations an economy which can be represented as the system shown in the diagram of Fig. 8.

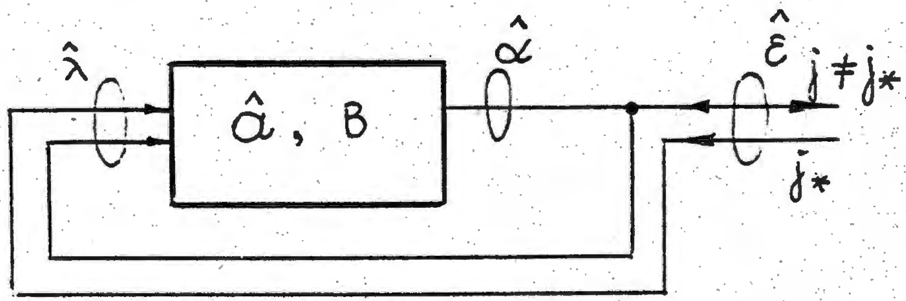

Figure 8.

In the above model the matrix B characterizes the technological perfection of the economy, while the vector $\hat{a}$ characterizes the magnitude (capacities) of the economy. Matrix B is also influenced by the accessibility of natural resources, while vector $\hat{a}$ - by the efficiency of these resources. Naturally, both the magnitudes of B and $\hat{a}$ are functions of time: the value of B depends upon the technological progress and upon the intensity of modernization of production technologies, and the value of $\hat{a}$ - upon the intensity of investment outlays.

As we mentioned already before, every participant of cooperation is, besides this, defined by the characteristics of the society. These characteristics are first of all the population number L and the consumption characteristic $\hat{\gamma}$. If we denote by b_j the statistical norm of consumption of the j-th commodity per capita then the overall consumption is equal

$$\hat{\beta} = \langle \beta_1, \dots, \beta_j, \dots, \beta_I \rangle$$

where $\beta_j = b_j L.$

The thus defined linear (again approximate!) dependence of the consumption level is characterized by the structure

$$\hat{\gamma} = \langle \gamma_1, \dots, \gamma_j, \dots, \gamma_J \rangle$$

where

$$\gamma_j = \frac{\beta_j}{\beta_{j_0}} = \frac{b_j}{b_{j_0}}$$

with j_0 denoting the commodity which was chosen as the comparative one, such that its consumption level might be the measure of the standard of living.

Now, if we adopt notation

$$\beta_{j_0} \equiv \beta$$

$$b_{j_0} \equiv b$$

then the quantity

$$b \equiv \frac{\beta}{L}$$

is the proper measure of the living standard of a statistical member of a given society.

Note that the value of

$$\eta = \frac{\beta}{\beta_{j_*}} = \frac{b}{b_{j_*}}$$

is the measure of effectiveness (in fact: efficiency) of an economy, expressing the quantity of the commodity j_0 obtained per unit of labour time (e.g. per hour of labour time), with the consumption level of this commodity expressing the standard of living of a given society. By making use of the quantities here considered we can obtain

$$\hat{\beta} = \beta\hat{\gamma} = b\gamma\hat{L}$$

In the above expression quantity L is of course an obvious function of time. Note, simultaneously, that the quantity $\hat{\beta}$ has to satisfy the condition $\beta_j \geq 0$ with exception of $j=j_*$, meaning the good being human labour. If $\hat{\beta}$ represents the use then, of course, $\beta_{j_*}<0$ (and $b_{j_*}<0$). In this, β_{j_*} has interpretation of the overall labour input into economy, and b_{j_*} expresses the norms of labour of a statistical member of a given society (labour hours per capita annually for the economy).

Naturally, the total labour output is increased by the labour connected with direct service to society's needs (the so called "service for population").

Consequently, the society, in terms of considerations here contained, can be presented in the form of the system shown in the diagram of Figure 9.

When we couple the two blocks - the one characterizing the eco nomy and the other one, representing the society, into one whole, we obtain the system presented in Figure 10, in which $\hat{\mu} = \hat{\varepsilon} - \hat{\beta}$.

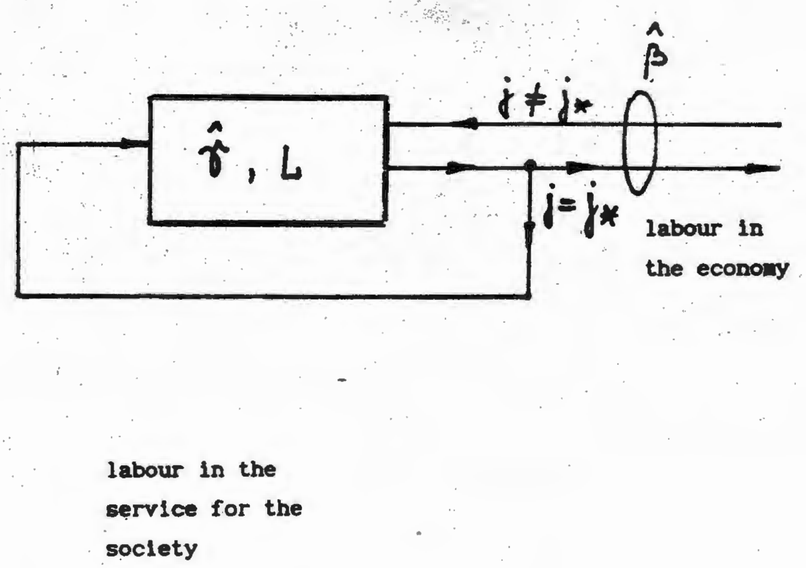

Figure 9.

In Figure 9 we have $\hat{\beta}=b\hat{\gamma}L$.

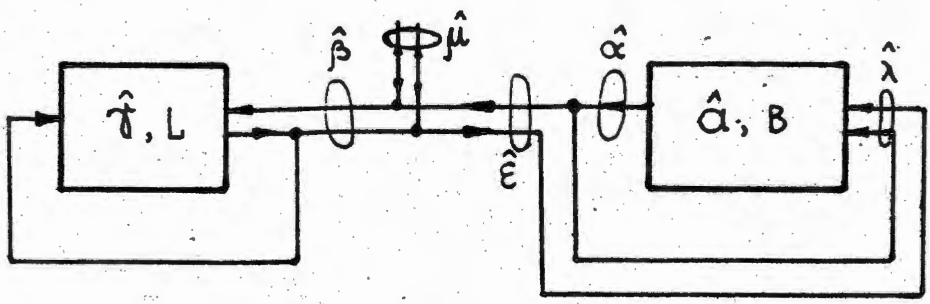

Figure 10.

Obviously, for an isolated system we have $\hat{\mu}=0$ and $\hat{\varepsilon}=\hat{\beta}$.

If we distinguish the good indexed j_* - human labour - from the general set of commodities considered, together with the balancing equation

$$\mu_{j_*} = h_{j_*} L - \sum_i \alpha_i b_{ij_*}$$

then the vector $\hat{\beta}$ fulfills the condition $\hat{\beta} \geq 0$ just as is the case with the vector $\hat{\alpha}$. In this manner we obtain a comfortable uniformity of expressions, although we omit in this the labour balance, which is sometimes significant. We shall assume in further considerations that the set of distinguished commodities does not contain labour.

The system of economy shall in this case get importantly simplified and its form shall be as presented in Figure 11.

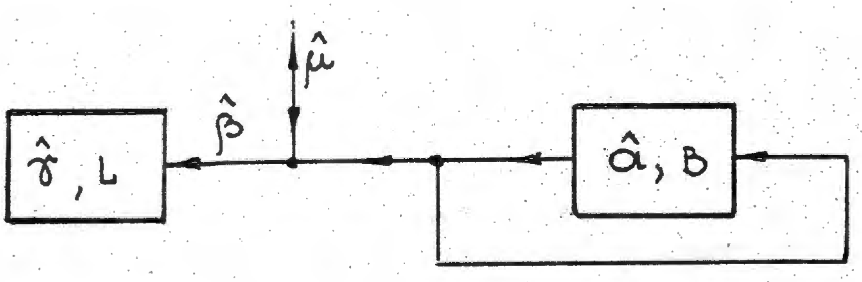

Figure 11.

If the column vector $\hat{c}$ is the vector of prices existing in the international market, then every one of the participants of cooperation tries to select $\hat{\alpha}$ in such a way as to satisfy the constraint

$$\hat{\mu}\hat{c} \geq 0$$

Consequently, optimization of the production program consists in such a selection of values of $\hat{\alpha}$ as to make the value of β or b attain its maximum

$$\beta \Rightarrow \max \quad (\text{or } b \Rightarrow \max)$$

subject to constraints

$$\left.\begin{aligned} &\hat{\alpha} \geq 0 \\ &\hat{\alpha}\hat{a} \leq 1 \\ &\hat{\mu}\hat{c} \geq 0 \end{aligned}\right\} \quad (1)$$

where

$$\hat{\mu} = \hat{\alpha}[I - B] - \beta\hat{\gamma}$$

Multilateral optimal cooperation

Consider a group of participants of a multilateral cooperation on the example of international cooperation. Let the participants be indexed with the variable k and assume that for each of them the following values are known:

L^k - population number,

$\hat{\gamma}^k$ - consumption structure,

B^k - technological characteristics of the economy,

$\hat{A}^k$ - the magnitude of the industrial potential, just as they have been introduced before. Each of the countries involved tries to select the structure and magnitude of production, $\hat{\alpha}^k$, in such a way as to maximize the consumption level $\beta^k = \beta^k_{j_0}$ of a chosen good $j = j_0$, equal for all the participants.

Obviously, for every participant, the value of $\hat{\alpha}^k$ satisfies the conditions

$$\hat{\alpha}^k \ge 0, \quad \hat{\alpha}^k \hat{a}^k \le 1$$

The magnitude of exchange is for every country defined by the vector

$$\hat{\mu}^k = \hat{\alpha}^k [I - B^k] - \beta^k \hat{\gamma}^k$$

We shall assume at the start that the values of prices on the international markets are unknown, so that values $\hat{\mu}^k$ may be arbitrary. Figure 12 illustrates the system of three cooperating economies of which every one has the form shown in Figure 11.

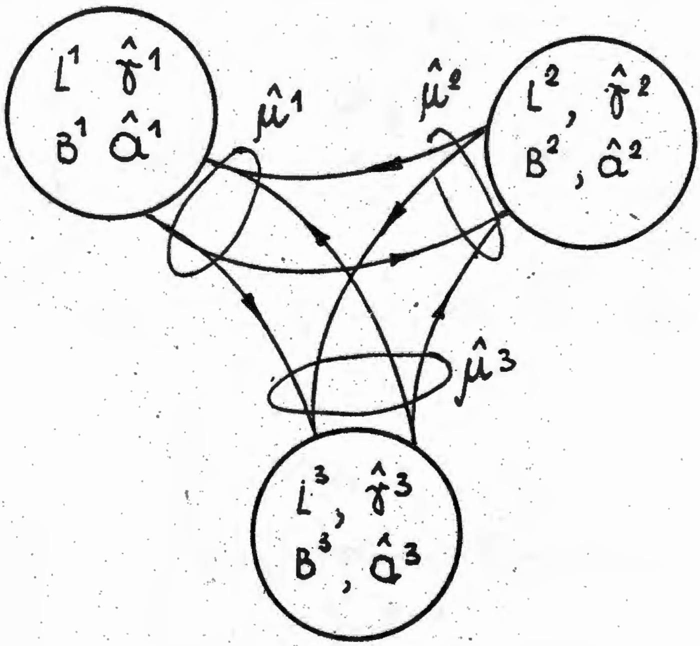

Figure 12.

Let us introduce the notion of the structure of net product, $\hat{\varepsilon}^k$, in a similar way as we introduced the notion of the consumption structure, $\hat{\gamma}^k$. We shall namely refer to the vector

$$\hat{\phi}^k = \langle \phi_1^k, \ldots, \phi_i^k, \ldots, \phi_I^k \rangle$$

in which

$$\phi_i^k = \frac{\varepsilon_i^k}{\varepsilon_{j_0}^k}$$

as to the vector of net product structure. If we put $\varepsilon_{j_0}^k = \varepsilon^k$ then we obtain

$$\hat{\varepsilon}^k = \varepsilon^k \hat{\phi}^k$$

Quantity ε^k has interpretation of the magnitude of net product of the k-th participant of cooperation, given the definite, $\hat{\phi}^k$, production structure.

Then, let us determine the structure of the resulting net product for all the countries involved:

$$\hat{\phi} = \sum_k \hat{\phi}^k \nu^k \tag{13}$$

where ν^k is the number characterizing the magnitude of the "input" of net product of the k-th participant into the overall net product ε:

$$\nu^k = \frac{\varepsilon^k}{\sum_{k=1}^{K} \varepsilon^k} = \frac{\varepsilon^k}{\varepsilon}, \tag{14}$$

with

$$\varepsilon = \sum_k \varepsilon^k$$

We shall similarly define the resulting structure of consumption

$$\hat{\gamma} = \sum_k \hat{\gamma}^k \delta^k \tag{15}$$

where δ^k is the number characterizing the magnitude of the "share" of the participant no. k in the overall consumption:

$$\delta^k = \frac{\beta^k}{\sum_k \beta^k} = \frac{\beta^k}{\beta} \tag{16}$$

with

$$\beta = \sum_k \beta^k$$

We shall say that the cooperating economic (and social) systems COMPLEMENT each other if the following equality holds:

$$\hat{\phi} = \hat{\gamma} \tag{17}$$

Theorem 5.
If the structure of consumption of every participant is identical:

$$\hat{\gamma}^k = \hat{\gamma}$$

and the net product values

$$\hat{\varepsilon}^k = \varepsilon^k \hat{\gamma} + \hat{c}^k$$

satisfy the condition

$$\sum_k \hat{c}^k = 0$$

then economic systems are COMPLEMENTARY.

Proof.
By making use of formulae (13), (14), (15) and (16) we can represent equation in the form of

$$\sum_k [\beta\hat{\varepsilon}^k - \varepsilon\hat{\beta}^k] = 0$$

Now, applying the assumption of the equality of consumption structures we can represent the latter relation as

$$\sum_k \hat{\varepsilon}^k - \varepsilon\hat{\gamma} = 0$$

But, since

$$\hat{\varepsilon}^k = \varepsilon^k\hat{\gamma} + \hat{c}^k$$

then we have

$$\sum_k \hat{\varepsilon}^k - \varepsilon\hat{\gamma} = \sum_k (\varepsilon^k\hat{\gamma} + \hat{c}^k) - \varepsilon\hat{\gamma} = \sum_k \hat{c}^k$$

so that, in fact, equality (17) shall hold if and only if

$$\sum_k \hat{c}^k = 0$$

which terminates the proof - QED.

We shall refer to exchange as CLOSED within a given group of participants if condition

$$\sum_k \hat{\mu}^k = 0$$

holds. If the values of components of $\hat{\mu}^k \neq 0$ are for some participant of one sign, i.e. we have

$$\hat{\mu}^k \geq 0 \quad \text{or} \quad \hat{\mu}^k \leq 0$$

then, in the first case, we are dealing with a (cooperation) subsidiating participant, while in the second case - with a subsidiated participant.

Theorem 6.

If all the participants of a closed exchange have identical production and consumption structures, that is, equalities

$$\hat{\phi}^k = \hat{\phi}, \qquad \hat{\gamma}^k = \hat{\gamma}$$

hold, then the group of participants, for whom $\hat{\mu}^k \neq 0$, is divided into two subgroups - of subsidiating and subsidized participants.

<u>Proof.</u>

Since, in accordance with assumptions,

$$\hat{\varepsilon}^k = \varepsilon^k \hat{\phi}, \qquad \hat{\beta}^k = \beta^k \hat{\gamma},$$

then

$$\hat{\mu}^k = \varepsilon^k \hat{\phi} - \beta^k \hat{\gamma}$$

But

$$\sum_k \hat{\mu}^k = \sum_k (\varepsilon^k \hat{\phi} - \beta^k \hat{\gamma}) = \varepsilon\hat{\phi} - \beta\hat{\gamma}$$

If, then, in accordance with assumptions, there is

$$\sum_k \hat{\mu}^k = 0$$

this implies that

$$\hat{\phi} = \hat{\gamma}$$

and

$$\varepsilon = \beta$$

The magnitude of exchange, $\hat{\mu}^k$, of every participant of cooperation can therefore be represented in the form

$$\hat{\mu}^k = \varepsilon^k \hat{\phi} - \beta^k \hat{\gamma} = (\varepsilon^k - \beta^k)\hat{\phi} = (\varepsilon^k - \beta^k)\hat{\gamma}$$

Consequently, if $\varepsilon^k \neq \beta^k$

$$\hat{\mu}^k > 0 \text{ or } \hat{\mu}^k < 0$$

so that in fact, the active participants of exchange are classified into the subsidized and the subsidizing ones.

Theorem 7.
The necessary and sufficient condition for the exchange to be CLOSED under identical consumption structure is mutual COMPLEMENTATION of the participants.

Proof.
The proof is immediate. If $\hat{\phi} = \hat{\gamma}$ then we can always select such a value of consumption, $\beta = \varepsilon$ so that the following equation be satisfied:

$$\varepsilon\hat{\phi} - \beta\hat{\gamma} = 0$$

This equation can also be written down in the form of

$$\sum_k (\varepsilon^k\hat{\phi}^k - \beta^k\hat{\gamma}) = \sum_k \hat{\mu}^k$$

since

$$\sum_k \beta^k = \beta$$

$$\sum_k \varepsilon^k\hat{\phi}^k = \varepsilon \sum_k \frac{\varepsilon^k}{\varepsilon}\cdot\hat{\phi}^k = \varepsilon\hat{\phi}$$

so that in fact, if $\hat{\phi} = \hat{\gamma}$ then $\sum_k \hat{\mu}^k = 0$

If, on the other hand, $\hat{\phi} \neq \hat{\gamma}$ then there does not exist a number β which could serve to bring about the equality

$$\sum_k \hat{\mu}^k = 0$$

We can conclude, therefore, that

$$\sum_k \hat{\mu}^k = \sum_k (\varepsilon^k \hat{\phi}^k - \beta^k \hat{\gamma}) = \varepsilon \sum_k \frac{\varepsilon^k}{\varepsilon} \hat{\phi}^k - \beta \hat{\gamma} = \varepsilon \hat{\phi} - \beta \hat{\gamma}$$

which terminates the proof - QED.

We shall refer to a closed exchange maximizing consumption of its participants as to a multilateral optimum exchange.

Theorem 8.
Optimum cooperation is - for identical consumption structures of consumption of participants - defined by the vectors

$$\hat{\mu}_0^k = \hat{\alpha}_0^k \cdot [I - B^k] - \beta^k \hat{\gamma}, \quad k=1,\dots,K$$

where quantities β^k satisfy equation

$$\sum_k \beta^k = \sum_k \hat{\alpha}_0^k \cdot [I - B^k]$$

and quantities $\hat{\alpha}^k = \hat{\alpha}_0^k$ are solutions to the (linear) problem

$$\sum_k \hat{\alpha}^k \cdot [I - B^k] \rightarrow \max$$

subject to constraints

(1) $\hat{\alpha}^k \geq 0,$ $\quad k = 1,2,\dots,K$

(2) $\hat{\alpha}^k \hat{a}^k \leq 1,$ $\quad k = 1,2,\dots,K$

(3) $\sum_k \hat{c}^k$ 0,

(4) $\hat{\alpha}^k [I - B^k] = \varepsilon^k \hat{\gamma} + \hat{c}^k,$ $\quad k = 1,2,\dots,K$

Proof.

Conditions (3) and (4) guarantee that the thus defined values of $\hat{\alpha}_0^k$ ensure complementarity of economies, which is the sufficient condition for existence of closed exchange (see Theorem 7).

Conditions (1) and (2) guarantee that the solution found be feasible, and the criterion function maximize the quantity

$$\hat{\varepsilon} = \sum_k \hat{\varepsilon}^k = \sum_k \hat{\alpha}^k[I - B^k]$$

expressing net product. Since for closed exchange we have

$$\hat{\varepsilon} = \hat{\beta}$$

or

$$\varepsilon\hat{\phi} = \beta\hat{\gamma}$$

with

$$\hat{\phi} = \hat{\gamma}, \quad \varepsilon = \beta$$

then maximization of $\hat{\varepsilon}$ is equivalent to maximization of $\hat{\beta} = \beta\hat{\gamma}$ i.e. of the magnitude of consumption. Thus, it is true that solution $\hat{\alpha}_0^k$ is an optimal exchange maximizing consumption magnitude, that is - the value of number β, QED.

Stimulation of cooperation and international prices

Let us introduce the notion of balanced exchange. We shall namely call balanced exchange the one for which the following condition is satisfied:

$$\hat{\mu}^k\hat{c} \geq 0$$

for every $k = 1,2,,\dots,K$.

Theorem 9.

If economies are mutually complementary and consumption structures are identical,

$$\hat{\gamma}^k = \hat{\gamma}$$

then exchange is balanced when consumption $\hat{\beta}^k$ of every participant of cooperation can be represented in the form of

$$\hat{\beta}^k = \hat{\varepsilon}^k - \hat{G}^k + \hat{\gamma}p^k$$

where

$$p^k = \frac{\hat{G}^k\hat{\omega}}{\hat{\gamma}\hat{\omega}}$$

$\hat{\omega} = \langle\omega_1,\ldots,\omega_i,\ldots,\omega_I\rangle$ - price structure ("terms of trade"),

with $\omega_i = \dfrac{c_i}{c_{j_0}}$

<u>Proof.</u>

Since

$$\hat{\mu}^k = \hat{\varepsilon}^k - \hat{\beta}^k = \hat{G}^k - \hat{\gamma}p^k$$

for

$$\hat{\beta}^k = \hat{\varepsilon}^k - \hat{G}^k + \hat{\gamma}p^k$$

so that

$$\hat{\mu}^k\hat{c} = \hat{G}^k\hat{c} - p^k\hat{\gamma}\hat{c}$$

But

$$p^k = \frac{\hat{G}^k\hat{\omega}}{\hat{\gamma}\hat{\omega}} = \frac{\hat{G}^k\hat{c}}{\hat{\gamma}\hat{c}}$$

according to assumptions, for $\hat{\omega} = \dfrac{1}{c_{j_0}}\cdot\hat{c}$

and hence $\hat{\mu}^k\hat{c} = \hat{G}^k\hat{c} - \dfrac{\hat{G}^k\hat{c}}{\hat{\gamma}\hat{c}}\hat{\gamma}\hat{c} \equiv 0$

Thus, in fact, if

$$\hat{\beta}^k = \hat{\varepsilon}^k - \hat{c}^k + \hat{\gamma}p^k$$

then exchange is balanced, QED.

Let us now denote with β_*^k the maximum consumption level that can be secured by the k-th participant of cooperation for himself without any true cooperation with anybody.

The value of β^k_* is the solution (with regard to β^k and $\hat{\alpha}^k$) of the problem

$$\beta^k \Rightarrow \max$$

$$\beta^k \hat{\gamma} = \hat{\alpha}^k [I - B] \qquad \text{(II)}$$

$$\hat{\alpha}^k \hat{a}^k \leq 1$$

$$\hat{\alpha}^k \geq 0$$

Theorem 10.

If there exists a labour specialization among participants, and this specialization can be represented in the form of

$$\hat{\varepsilon}^k = \varepsilon^k \hat{\gamma} + \hat{G}^k$$

where $\sum_k \hat{G}^k = 0$, satisfying for a given price structure $\hat{\omega}$ the inequality

$$[\hat{G}^k + (\varepsilon^k - \beta^k_*)\hat{\gamma}]\hat{\omega} > 0$$

then this price structure ensures the stability of cooperation relations within the set of those participants for whom this inequality holds. Such a price structure shall be called cooperative.

<u>Proof.</u>

From the condition of balancing of exchange we obtain that the level of consumption of every participant is equal

$$\hat{\beta}^k = \hat{\varepsilon}^k - \hat{G}^k + \hat{\gamma} p^k$$

or, after having substituted $\hat{\varepsilon} = \varepsilon^k\hat{\gamma} + \hat{G}^k$:

$$\hat{\beta}^k = (\varepsilon^k + p^k)\hat{\gamma}$$

where

$$p^k = \frac{\hat{G}^k\hat{\omega}}{\hat{\gamma}\hat{\omega}}$$

In order for cooperation to constitute a stable relation, the level of consumption

$$\beta^k = \varepsilon^k + p^k$$

should in conditions of cooperation be higher than the level β^k_* which every participant of potential cooperation could secure for himself. Thus, the following inequality should be satisfied:

$$\varepsilon^k + p^k > \varepsilon^k_*$$

or

$$\varepsilon^k + \frac{\hat{G}^k\hat{\omega}}{\hat{\gamma}\hat{\omega}}$$

Hence, in fact, the condition obtained after transformations,

$$[\ \hat{G}^k + (\varepsilon^k - \beta^k_*)\hat{\gamma}\ ]\hat{\omega} > 0$$

is the necessary condition for existence of stable cooperation

$$\hat{\mu}^k = \varepsilon^k\hat{\gamma} + \hat{G}^k - (\varepsilon^k + p^k)\hat{\gamma} = \hat{G}^k - p^k\hat{\gamma}$$

with

$$\sum_k \hat{G}^k = 0$$

QED.

An obvious conclusion from the last theorem is the observation that only these price structures $\hat{\omega}_0$ which satisfy inequality

$$[\hat{G}^k + (\varepsilon_0^k - \beta_*^k)\hat{\gamma}]\hat{\omega}_0 > 0$$

allow for the optimum cooperative relations being the solution to problem I (see last theorem), defined by the quantities

$$\hat{\varepsilon}_0^k = \varepsilon_0^k\hat{\gamma} + \hat{G}_0^k$$

with

$$\sum_k \hat{G}_0^k = 0$$

Other price structures do not make it possible to establish optimum relations.

Cooperative economic game

It is obvious that every element of the set of price structures,

$$\Omega_0 = \{\hat{\omega}_0 : [\hat{G}_0^k + (\varepsilon_0^k - \beta_*^k)\hat{\gamma}]\hat{\omega}_0 > 0\}$$

has a different value for various participants of cooperation. This has the following consequence: with every element of the set Ω_0 a different division β^k of the overall production ε among the participants of cooperation is connected with, naturally, the condition of

$$\sum_k \beta^k = \varepsilon$$

being satisfied. This results directly from the formula

$$\beta^k = \varepsilon^k + \frac{\hat{G}_0^k\hat{\omega}}{\hat{\gamma}\hat{\omega}}$$

which implies dependence of the level of consumption β^k of a participant upon the price structure $\hat{\omega}$.

Thus, consequently, the problem arises of the manner in which one should establish the "fair" price structure $\hat{\omega}$ or the "fair" division of the net product obtained, ε, into the parts β^k. Solution to this problem is described in ***.

9. FINAL REMARKS

The present publication shows the methods of determination of the most advantageous specialization in the domain of international exchange and production in such a way as to possibly increase the consumption level ("standard of living") of every partner of exchange.

The specialization mentioned is being determined irrespective of current internal prices of economies participating in exchange and of the international market prices, basing merely upon the characteristics of the industrial potential at hand.

We are therefore obtaining the capacity of establishing a stable international economic cooperation notwithstanding the speculative price fluctuations. The structure of the thus determined cooperation shall be undergoing modifications along with the changes of productive potentials of participants.

The cooperation established in this manner may happen to be identical with the one determined on the basis of analysis of international prices (just as it is being done presently) under the condition that these will be the prices of economic equilibrium, which are not the subject of speculation, and that the changes of these prices, brought about by the changes of production potentials would not be delayed with regard to the industrial development of the participants.

We can deduce from the theory presented a conclusion going in the opposite direction as well. Thus, if we know how to determine the optimum cooperation for participants having various internal currencies, then we can choose the international prices (expressed

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