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Redakcja:

Jan Studziński
Ludosław Drelichowski
Olgierd Hryniewicz

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Książka wydana dzięki dotacji KOMITETU BADAŃ NAUKOWYCH

Książka zawiera wybór artykułów poświęconych omówieniu aktualnego stanu badań w kraju, w zakresie rozwoju modeli, technik i systemów informatycznych oraz ich zastosowań w różnych dziedzinach gospodarki. Kilka artykułów omawia aplikacyjne wyniki projektów badawczych i celowych Ministerstwa Nauki i Informatyzacji.

Recenzenci artykułów:

Dr inż. Lucyna Bogdan
Prof. dr hab. inż. Ludosław Drelichowski
Prof. dr hab. inż. Olgierd Hryniewicz
Dr inż. Edward Michalewski
Dr inż. Grażyna Petriczek
Prof. dr hab. inż. Andrzej Straszak
Dr inż. Jan Studziński

Komputerowa edycja tekstu: Anna Gostyńska

Instytut Badań Systemowych PAN
ul. Newelska 6, 01-447 Warszawa

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e-mail: biblioteka@ibspan.waw.pl

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APPLICATION OF MODEL PREDICTIVE CONTROL TO DEBT SERVICING[1]

Antoni MIKLEWSKI[1], *Maciej KRAWCZAK*[2]

[1]Agricultural University in Szczecin
<miklewsk@erl.edu.pl>

[2] Systems Research Institute, Polish Academy of Sciences
[2] Warsaw School of Information Technology
<krawczak@ibspan.waw.pl>

In this paper we consider a national debt servicing problem. We propose the application of a model predictive control to minimizing debt servicing cost.

Keywords: National debt, servicing cost, model predictive control.

1. Introduction

It is clear, the lower costs of servicing the National Debt has freed up substantial Government revenue for other purposes. Sustainable management of the polish economy generally requires that the Government run a balanced current budget i.e. that Government tax revenues should pay for day-to-day running costs of Government services. A prudent fiscal policy would aim to ensure that enough revenue would be left over to pay for some, but not necessarily all, of the Government's capital investment plans.

We propose a model for the optimization of the issuances of Public Debt securities that we are developing in research project. There are a number of possible fixed and floating income securities and the goal is to find the composition of the portfolio issued every month which minimizes a "cost function".

The first instrument (BOND1) does not have coupons. From the accounting viewpoint the issuing price p is determined with a discount factor y: $p = 100-y$, i.e., at the maturity date the nominal value 100 is reimbursed. The second instrument (BOND2), like BOND1, does not have coupon. The issuing price is determined in such a way that the interests are comprised in the reimbursement $p(1 + r) = 100$.

Next instruments, both the BOND3 and BOND4 pay cash dividends by means of coupons corresponded every 6 months. The difference among them lies in the rate of interest (i.e. the value of the coupon) that is set at issuance time for

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BOND3s whereas is variable for BOND4s. More precisely, the interest rate for BOND4s is determined by the interest rate for the 6–month BOND1s.

For each of these four types of bonds we make a further distinction depending on the maturity. We order the bond types with an integer k taking $K = \{1,..., 10\}$. Moreover we indicate by m_k, the maturity in months of k. The issuance dates depend on the type of bond and we indicate them by a couple (d, m) where d is the day and m the month (tab. 1).

Mathematically speaking, this is a complex stochastic control problem with strong constraints imposed by national regulations and the Maastricht treaty. The complexity of the problem is further increased by the need for realistic solutions to take into account several side issues, like macroeconomic factors which are complicated as well. The stochastic component of the problem is represented by the evolution of interest rates. In order to generate possible "scenarios" of the interest rates' evolution it is necessary to tune the existing models according to the specific case of the market or to develop brand new models. This activity goes in parallel with the development of the optimization model and it is based on the study of specific stochastic differential equations having special properties ("mean reversion"). At this time the optimizer employs classic Linear Programming techniques (neglecting possible non-linear effects like the market reaction to changes in the Ministry's policy). However more sophisticated techniques based on Model Predictive Control (Maciejowski, 2002) strategies (an iterative approach well known in the engineering literature) are actually under development.

2. Optimization problem and Linear Programming

Since issuances happen at fixed dates, once per month, we use a discrete time model of evolution. For the sake of simplicity, the time step is one month. For the months in which some types of securities are not issued, the corresponding quantities are set equal to zero. We indicate by X_t the total amount of bonds that are not expired at time t. At the beginning of the planning period t_b, X_{tb} is given by X^{hist}. X_t must contain, for every $k \in K$, one component for every $s \in \{t-m_k,..., t-1\}$. The evolution of X_t is determined at each step by canceling bonds reaching maturity and adding the just issued ones. For example, for $k = 1$, one has to remove from X_t the quantity of 3 months BOND1 issued at time $t-3$ and insert that issued at time t. Clearly this can be done by shifting the components of X_t and adding the new issuances, thus we can write:

$$X_{t+1} = AX_t + BU_t, \tag{1}$$

where A is a shift matrix, $U_t = \left(u_k(t)\right)_{k\in K}$ is the vector of the new issuances and B is a sparse matrix. Hence we get a linear discrete time control system.

Note that the stochastic behavior of interest rates, or forward rates, influences the CashFlow, hence the Treasury Cash Account constraints (TCA), and the cost function ESA95. The latter is influenced also by the PBS (Primary Budget Surplus).

To specify completely the control problem it is necessary to set the input and output data and the optimization horizon.

The input data consist of:

- past issuances,
- issuance data,
- Gross Domestic Product and PBS forecasts.

Past issuances. If the optimization horizon starts at time t_b, then for every $k \in K$ it is necessary to know the quantities issued at all dates $t_b - m_k, \ldots, t_b - 1$.

Issuance data. The National Bank sets the dates of issuance for each type of bonds. These dates are set in advance, usually for the next two or three years, and are not part of the control problem.

GDP forecasts. This point is quite critical, since it is difficult to have reliable GDP forecasts.

The output data are represented by the number of bonds that, for each issuance, fulfill all the constraints and, at the same time, minimize the cost function.

From these data it is possible to derive:

- The Yearly Net Issuance,
- The Public Debt cost defined according to the ESA95 criteria (Jackson, 2000),
- The duration and WRP (Weighted Refixing Period) of the portfolio.

The duration of a portfolio of bonds is, from the issuer viewpoint, the weighted average of the maturity of all the outcome cash flows. The duration describes the exposure to parallel shifts in the yield curve and is a widely used indicator of the risk associated with a particular choice of a fixed income securities portfolio (Krawczak, *et al.* 2003).

The final goal is to provide an "optimal issuance strategy". There are, at least, two possible choices: 1) define the most probable scenario for the interest rates evolution, determine the corresponding optimal strategy, estimate the consequences of applying this strategy to a set of other scenarios (this step is necessary since the forecast on the interest rates can be wrong); 2) employ an "adaptive" strategy based on the available information on interest rates at issuance date (using interest rate models) and estimate the outcoming costs on a wide set of scenarios. We call 1) Fixed (most probable) Strategy and 2) Model Predictive Control (MPC) Strategy (by similarity with engineering control problems).

2.1 Optimal control

Beside input and output data given at initial and final time respectively, there are some input and output variables evolving in the optimization horizon. In control jargon Nominal Debt, Flow and Treasury Cash Account can be seen as output variables of the control system (1) and in formula can be indicated by:

$$Y_t = Y\left(X_t, U_t, PBS(t), y(t)\right) \tag{2}$$

In fact, all these quantities are computable since X_t, U_t and the exogenous stochastic parameters $PBS(t)$ and $y(t)$ (*term structure* of interest rates) are known. Finally, we get the task 1:

Task 1. The OIS (Optimal Issuance Strategy) consists of an optimal control problem for the system (1) with constraints on the outputs (2) and with a cost function defined according to the ESA95 specifications. Both constraints and cost function depend on the stochastic exogenous variables *PBS(t)* and *y(t)*.

A wide literature for stochastic optimal control problem is available (e.g., Yong, *et al.* 1999). However, the large number of variables (some hundreds components) and the needs for strict estimate in terms of probability prevent the applicability of most techniques.

2.2 Fixed scenario optimization

It is possible to show:

Task 2. For a set term structure evolution $t \mapsto y(t)$ and PBS realization $t \mapsto PBS(t)$, the optimization problem becomes a linear programming problem with linear constraints.

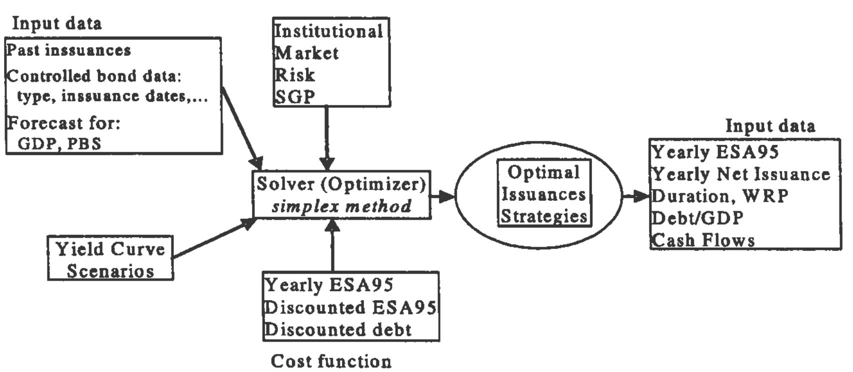

Figure 1. Optimization debt issuance.

To solve the problem we resorted to the classic Simplex Method. In fig. 1 we report a block diagram of the software package that we realized to manage all the phases of the optimization. The core of the optimizer is the package MATLAB (optionally MATHEMATICA and Excel) an open source linear programming solver which uses sparse matrix computations.

2.3 Model Predictive Control Strategies

It is well known that interest rate models do not always provide reliable forecasts, thus we put the accent on advanced control techniques in order to reduce Debt risk. In engineering literature an iterative strategy called MPC (Model Predictive Control and/or Receding Horizon Control), is often used in industrial applications for stabilization of systems under measurement uncertainties and disturbances (Maciejowski, 2002). This approach is particularly useful in case of hard constraints, as in Stability and Growth Pact (fig. 2).

A typical example of such problem is portfolio optimization or public security issuances, see (Korn, et al. 2001). Through the use of classical stochastic control methods, as dynamic programming and Hamilton-Jacobi-Bellmann equation, stochastic maximum principle, one can treat these problems providing solution methods. However, such methods are effective only for a small number of state and control variables, i.e. for a small number of assets involved in the portfolio. On the other hand, one may use Monte Carlo methods to deal with computational issue of simulations, but not easily to solve optimization problems.

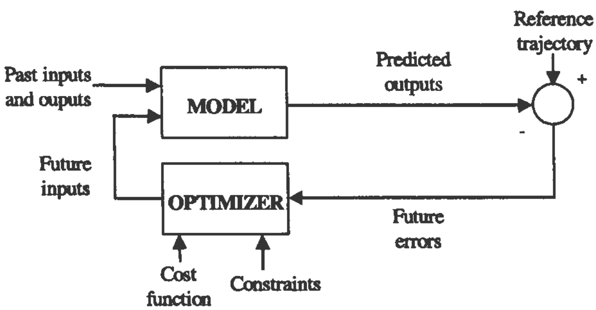

Figure 2. Basic structure of MPC

MPC is a form of control in which the current control action is obtained by solving *on-line*, at each sampling instant, a finite horizon open-loop optimal control problem, using the current state of the debt as the initial state; the optimization yields an optimal control sequence and the first control in this sequence is applied to the debt.

3. Conclusions

Model predictive control (MPC) refers to the direct use of an explicit and separately identifiable model for controlling a debt servicing process. The core of all MPC algorithms is the moving horizon approach. The MPC designs yield control systems capable of operating without expert intervention for extended periods of time. An identified process model predicts the future response and then, the control action is determined so as to obtain the desired performance over a finite time horizon. The control problem that must be solved is an on-line optimization of the manipulated variables to satisfy multiple, changing performance criteria in the face of changing process characteristics, including hard constraints. The MPC technique is a dynamic optimization approach to control problems. The flexible constraint-handling capabilities of MPC make it most suitable for process control problems.

Neural networks can be used to determine controller parameters, because of their well-known ability to solve complex problems by learning relationships directly from data. In this decade, certain neural networks have generated a lot of interest for use in nonlinear system identification and control (Albanis, et al. 1999, 2000; Krawczak *et al.*, 2003). They have been found to approximate arbitrary nonlinear mappings and they have been effectively used for the control of complex dynamic systems. Neural networks provide a framework for deriving analytic expressions for the modeling error gradients with respect to modeling parameters. The majority of neural network control applications have centered around the use of a neural network as a controller, which is often combined with a neural network identifier. A less costly alternative is to develop approaches that would adapt conventional controller gains using neural-network algorithms, so as to successfully track the different process operating regions, and hence enhance control performance.

References

Albanis G.T., Batchelor R.A. (2000) Assessing the Long-Term Credit Standing of Debt Issuers Using Dimensionality Reduction Techniques Based on Neural Networks An Alternative to Overfitting. http://www.staff.city.ac.uk/r.a.batchelor/

Albanis, G.A., Batchelor R.A. (1999) Using probabilistic neural networks and rule induction to predict bond ratings, in: M.Torres (Ed.), *Proceedings of the 5th Annual Conference on Information Systems, Analysis and Synthesis*, Orlando: IIIS (ISBN 980-07-5914-X).

Jackson D. (2000) The New National Accounts: An Introduction to the System of National Accounts 1993 and the European System of Accounts 1995, Edward Elgar Pub, Camberley UK.

Korn R., Korn E. (2001) Option pricing and portfolio optimization. *AMS Graduate Studies in Mathematics*, **31**. Providence.

Krawczak M., Jakubowski A., Konieczny P., Kulikowski R., Miklewski A., Szkatuła G., (2003) *Aktywne zarządzanie inwestycjami finansowymi – wybrane zagadnienia.* Problemy Współczesnej Nauki, Teoria I Zastosowania, Zarządzanie, Akademicka Oficyna Wydawnicza EXIT, Warszawa.

Maciejowski J.M. (2002) *Predictive Control with Constraints*. Prentice Hall, London UK.

Yong J., Zhou X.Y. (1999) *Stochastic Controls*. Springer-Verlag, New York.

Jan Studziński, Ludosław Drelichowski, Olgierd Hryniewicz
(Redakcja)

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Monografia zawiera wybór artykułów dotyczących informatyzacji procesów zarządzania, prezentując bieżący stan rozwoju informatyki stosowanej w Polsce i na świecie. Zamieszczone artykuły opisują metody, algorytmy i techniki obliczeniowe stosowane do rozwiązywania złożonych problemów zarządzania, a także omawiają konkretne zastosowania informatyki w różnych sektorach gospodarki. Kilka prac przedstawia wyniki projektów badawczych Ministerstwa Nauki i Informatyzacji, dotyczących rozwoju metod informatycznych i ich zastosowań.

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