New Trends in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics
Volume II: Applications

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Multidimensional approach to interval uncertainty calculations

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Abstract

Mathematic operations on intervals are basis of fuzzy set arithmetic and also of intuitionistic fuzzy set arithmetic. Fuzzy arithmetic is mostly realized with method of alpha-cuts where support of any cut is an interval. Thus, fuzzy arithmetic is fully based on interval arithmetic. If interval arithmetic used by fuzzy arithmetic is incorrect, then results of such fuzzy arithmetic also are incorrect. Fuzzy arithmetic is basis for Computing with Words and for other branches of Artificial Intelligence. The mostly used type of fuzzy arithmetic is Moore-arithmetic. It is known from calculation paradoxes. However, it is further on the mostly used interval arithmetic. The proposed paper will present a new (according to authors' knowledge) type of interval arithmetic that is free from paradoxes of Moore arithmetic and delivers correct results both in forward and in backward (equation solutions) interval calculations. This arithmetic called RDM interval-arithmetic is based on multidimensional approach to interval calculations. Author of the concept of RDM interval-arithmetic is Andrzej Piegat.

Keywords: interval arithmetic, RDM interval arithmetic, multidimensional interval arithmetic, interval mathematics, interval analysis, granular computing, computing with words, artificial intelligence.

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1 Introduction

Fuzzy arithmetic [6, 9, 14, 15] is strongly connected with interval arithmetic [4, 12, 14, 19] because one of main calculation methods in fuzzy arithmetic is α-cut method. Fig. 1 shows multiplication of two fuzzy numbers "about 2" and "about 4" with use of this method.

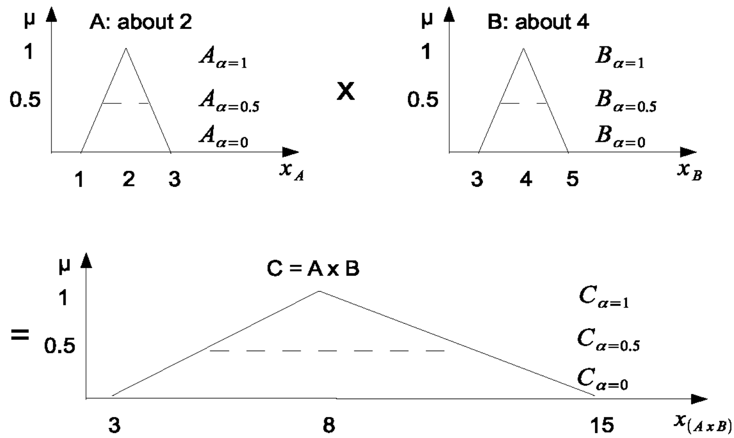

Figure 1: Example of multiplication of two fuzzy numbers A (about 2) and B (about 4) with the use of α-cut method based on interval arithmetic.

If interval arithmetic used in a fuzzy arithmetic is incorrect, then this fuzzy arithmetic also will be incorrect. Fuzzy arithmetic is of great importance because it is used in Computing with Words [1, 21], in branch of Artificial Intelligence that enables automatic thinking similar to human one and based on information granules [14]. At present there exists a number of interval arithmetic types: Moore interval arithmetic [12, 13], non-standard interval arithmetic of Markov [11], generalized interval arithmetic of Hansen [5], segment interval analysis of Sendov [16], centralized interval arithmetic of Moore [12], MV-form interval arithmetic of Caprani/Madsen [3]. However, the mostly used interval arithmetic is further on Moore-arithmetic [12, 13]. Why? The reason can be explained by quotation from [4]: "All these approaches provide good results only in specific conditions. On the other hand, in practice, the so-called "naive" form proposed by Moore [12] is proved to be the best one". It can be confirmed by many new books, as e.g. [6, 10, 13, 17] in which Moore-arithmetic is used in various new methods as e.g. in Grey Systems [10]. Moore-arithmetic has many faults that rather are good known [4]: a) "the excess width effect" problem, b) "dependency" problem, c)

”difficulties of solving even simplest equation” problem, d) ”interval equation’s right-hand side” problem, e) ”absurd solutions and request to introduce negative entropy into the system” problem. Problem a) means that uncertainty of operation results on intervals is great and it grows rapidly with uncertainty of the operation components. If e.g. two intervals are added $[1, 3] + [2, 5]$ with widths 2 and 3, then result achieved with Moore-arithmetic $[3, 8]$ has uncertainty equal to 5 that is greater than uncertainty of particular components. This phenomenon is also determined as increasing entropy principle [4]. Description of other faults of Moore-arithmetic can be found in e.g. [4] or in Wikipedia. According to Moore-arithmetic basic arithmetic operations should be realized with use of formulas (1)–(4), where $[\underline{x}, \overline{x}]$ means operation result.

$$[\underline{a}, \overline{a}] + \left[\underline{b}, \overline{b}\right] = [\underline{x}, \overline{x}] = \left[\underline{a} + \underline{b}, \overline{a} + \overline{b}\right] \tag{1}$$

$$[\underline{a}, \overline{a}] - \left[\underline{b}, \overline{b}\right] = [\underline{x}, \overline{x}] = \left[\underline{a} - \overline{b}, \overline{a} - \underline{b}\right] \tag{2}$$

$$[\underline{a}, \overline{a}] \cdot \left[\underline{b}, \overline{b}\right] = [\underline{x}, \overline{x}] = \left[\min\left\{\underline{a}\underline{b}, \underline{a}\overline{b}, \overline{a}\underline{b}, \overline{a}\overline{b}\right\}, \max\left\{\underline{a}\underline{b}, \underline{a}\overline{b}, \overline{a}\underline{b}, \overline{a}\overline{b}\right\}\right] \tag{3}$$

$$[\underline{a}, \overline{a}] \,/\, \left[\underline{b}, \overline{b}\right] = [\underline{x}, \overline{x}] = \left[\min\left\{\underline{a}/\underline{b}, \underline{a}/\overline{b}, \overline{a}/\underline{b}, \overline{a}/\overline{b}\right\}, \max\left\{\underline{a}/\underline{b}, \underline{a}/\overline{b}, \overline{a}/\underline{b}, \overline{a}/\overline{b}\right\}\right] \tag{4}$$

Further on RDM-arithmetic will be presented that is free from faults of Moore-arithmetic.

2 Addition of intervals with RDM interval arithmetic

The abridgment RDM means Relative Distance Measure. If an information piece is given that variable x has a value that is contained in interval $x \in [\underline{x}, \overline{x}]$, where $\underline{x}$ is the lower limit and $\overline{x}$ is the upper limit of the interval, then this fact can be described with formula (5).

$$x \in [\underline{x}, \overline{x}] : x = \underline{x} + \alpha_x (\overline{x} - \underline{x}), \alpha_x \in [0, 1] \tag{5}$$

Variable α_x can be interpreted as measure of relative distance. This notion is illustrated by Fig. 2 for interval $[3, 5]$.

Let us assume that two intervals $[a]$ and $[b]$ should be added, formulas (6) and (7).

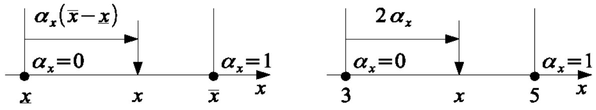

Figure 2: Illustration of notion Relative Distance Measure (RDM), $\alpha_x \in [0, 1]$.

$$[\underline{a}, \overline{a}] + \left[\underline{b}, \overline{b}\right] = [\underline{x}, \overline{x}] =? \tag{6}$$

$$[0, 2] + [1, 4] = [\underline{x}, \overline{x}] =? \tag{7}$$

Intervals $[a]$ and $[b]$ are written with use of RDM-variables α_a and α_b, formulas (8) and (9).

$$a = 0 + 2\alpha_a, \alpha_a \in [0, 1] \tag{8}$$

$$b = 1 + 3\alpha_b, \alpha_b \in [0, 1] \tag{9}$$

Sum of both intervals $[a]$ and $[b]$ is determined by formula (10).

$$[a] + [b] = 1 + 2\alpha_a + 3\alpha_b = x, \alpha_a \in [0, 1], \alpha_b \in [0, 1] \tag{10}$$

It should be noted that sum $x = a + b$ is 3-dimensional: it depends on 2 variables α_a and α_b. Table 1 shows values of x for border values of RDM-variables α_a and α_b.

Table 1: Values of the sum $[x] = [a] + [b]$ for various border values of RDM-variables $\alpha_a \in [0, 1]$ and $\alpha_b \in [0, 1]$.

α_a	0	0	1	1
α_b	0	1	0	1
x	$(\underline{a} + \underline{b})$	$\left(\underline{a} + \overline{b}\right)$	$(\overline{a} + \underline{b})$	$\left(\overline{a} + \overline{b}\right)$
x	1	4	3	6

In Fig. 3, on the addition surface $a + b = x$ contour lines of constant values of the sum are marked. Lengths of particular lines are differentiated. For example, contour line of the sum $x = 1$ has an infinitely small length because only one event $(a, b) = (0, 1)$ gives the sum $x = 1$. The longest contour lines correspond to all sum values $3 \leq x \leq 4$. There exists an infinite number of events (tuples of values of a and b) that results in the sum $x = 3$. Length of particular lines can be interpreted as non-normalized probability density of the event $a + b =$

$x = const$. Sense of this explanation can the more be shown in Fig. 4 which presents projection of 3D-granule of the problem solution $x = a + b$ on 2D-space of variables a and b.

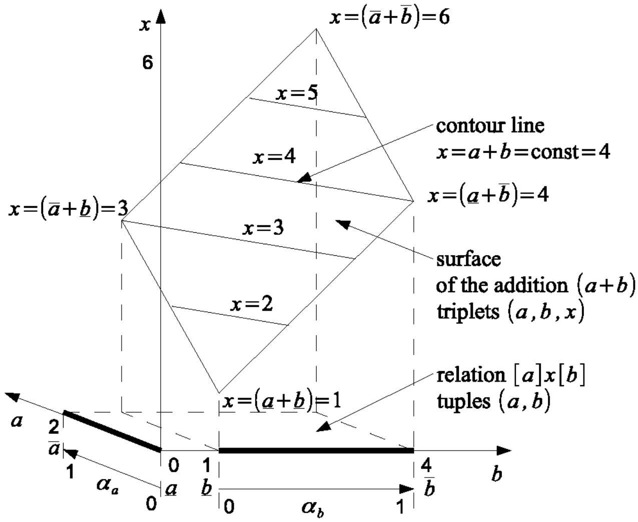

Figure 3: Illustration of interval addition $[\underline{a}, \overline{a}] = [0, 2]$ and $\left[\underline{b}, \overline{b}\right] = [1, 4]$ with use of RDM-arithmetic, where $\alpha_a \in [0, 1]$ and $\alpha_b \in [0, 1]$, in the 3D-space of the problem.

Though Fig. 4 presents the addition problem in 2D-space it gives in practice the same information about interval addition as Fig. 3 in which the addition is presented in 3D-space. Fig. 5 shows distribution $pd(x)$ of probability density of the result $[a] + [b] = [x]$ of interval addition. This distribution has meaning of a priori distribution because it was achieved at assumption of uniform distributions of variables a and b. Such assumption can be made if experimental distributions $pd(a)$ and $pd(b)$ of the variables are not known. If the experimental distributions are known, then they should be used for determining distribution $pd(x)$ of $x = a + b$.

The analyzed addition problem can also be solved with Moore arithmetic (11).

$$[\underline{a}, \overline{a}] + \left[\underline{b}, \overline{b}\right] = [\underline{x}, \overline{x}] = [0, 2] + [1, 4] = [1, 6] \tag{11}$$

The solution achieved with Moore arithmetic is visualized in Fig. 6.

Comparison of Fig. 5 and Fig. 6 shows that the maximal widths of result intervals provided by both arithmetics are identic. However, RDM-arithmetic delivers results that are more informative. It delivers 3 types of results. The first

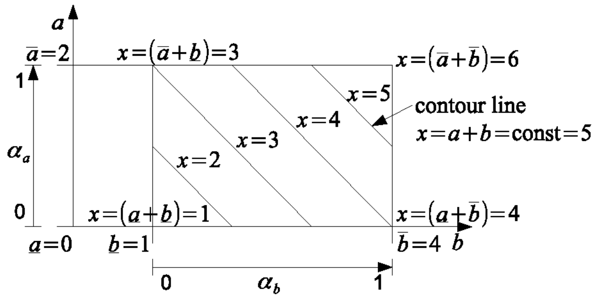

Figure 4: Interval addition $[\underline{a}, \overline{a}] + \left[\underline{b}, \overline{b}\right] = [0, 2] + [1, 4]$ with RDM-method in 2D-space, $\alpha_a \in [0, 1]$, $\alpha_b \in [0, 1]$.

result is the 3D-one (Fig. 3), the second is the 2D-version, and the third is the 1-variable version (Fig. 5) in form of probability density distribution. A very important advantage of RDM-arithmetic is that it enables equation solving whereas Moore-arithmetic is not able to realize this task.

3 Equation solving with use of RDM-arithmetic

Let us consider solving task of the equation given by formula (12).

$$\begin{array}{c}[\underline{a}, \overline{a}] + [\underline{x}, \overline{x}] = [\underline{c}, \overline{c}] \\ [0, 2] + [\underline{x}, \overline{x}] = [1, 6]\end{array} \tag{12}$$

First, the equation will be solved with use of Moore arithmetic (13).

$$\begin{array}{l}\underline{a} + \underline{x} = \underline{c}, 0 + \underline{x} = 1, \underline{x} = 1 \\ \overline{a} + \overline{x} = \overline{c}, 2 + \overline{x} = 6, \overline{x} = 4 \\ [\underline{x}, \overline{x}] = [1, 4]\end{array} \tag{13}$$

However, analysis of the obtained solution shows that it is incomplete and thus incorrect (solutions achieved with Moore-arithmetic also can be overcomplete). E.g. values $a = 1$ and $x = 0$ satisfy solution (12). But value $x = 0$ is not contained in the solution interval $[\underline{x}, \overline{x}] = [1, 4]$. Similarly, values $a = 1$ and $x = 5$ also satisfy solution (13) in spite of the fact that x=5 is not contained in interval $[1, 4]$. Now, let us solve equation (12) with use of RDM-arithmetic. Intervals $[a]$ and $[b]$ are expressed with use of variables α_a and α_c (14).

$$\begin{array}{l}[\underline{a}, \overline{a}] = [0, 2] = 0 + 2\alpha_a, \alpha_a \in [0, 1] \\ [\underline{c}, \overline{c}] = [1, 6] = 1 + 5\alpha_c, \alpha_c \in [0, 1]\end{array} \tag{14}$$

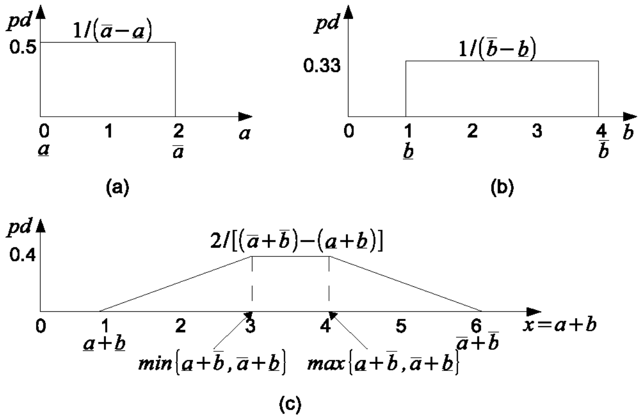

Figure 5: Distribution of a priori probability density $pd(x)$ of interval addition result achieved with use of RDM-method, subject to uniform distributions of $pd(a)$ and $pd(b)$.

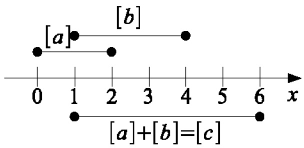

Figure 6: Visualization of interval addition realized with Moore arithmetic.

Next, the interval equation (12) is transferred in equation (15).

$$\begin{array}{l} \underline{a} + \alpha_a \left(\overline{a} - \underline{a} \right) + x = \underline{c} + \alpha_c \left(\overline{c} - \underline{c} \right) \\ 0 + 2\alpha_a + x = 1 + 5\alpha_c \\ x = 1 - 2\alpha_a + 5\alpha_c, \alpha_a \in [0,1], \alpha_c \in [0,1] \end{array} \tag{15}$$

Let us note, that value of variable x depends in (15) on two variables α_a and α_c. Thus, $x = f(\alpha_a, \alpha_c)$ and the solution problem of equation (12) became 3-dimensional, whereas Moore arithmetic treats it as 1-dimensional problem (see Fig. 6). Table 2 shows values of result variable x for border values of α_a and α_c.

The obtained solution is shown in Fig. 7.

The solution shown in Fig. 7 in 3D-space can also be presented with use of contour lines $c = a + x = const.$ in 2D-space, Fig. 8.

As can be seen in Fig. 8 solution of equation (12) is not 1-dimensional but 2-dimensional. This solution consists of set of tuples (a, x) satisfying dependence (16).

Table 2: Symbolic and numeric values of result x of equation $[\underline{a}, \overline{a}] + [\underline{x}, \overline{x}] = [\underline{c}, \overline{c}] = [0, 2] + [\underline{x}, \overline{x}] = [1, 6]$.

α_a	0	0	1	1
α_c	0	1	0	1
x	$(\underline{c} - \underline{a})$	$(\overline{c} - \underline{a})$	$(\underline{c} - \overline{a})$	$(\overline{c} - \overline{a})$
x	1	6	-1	4

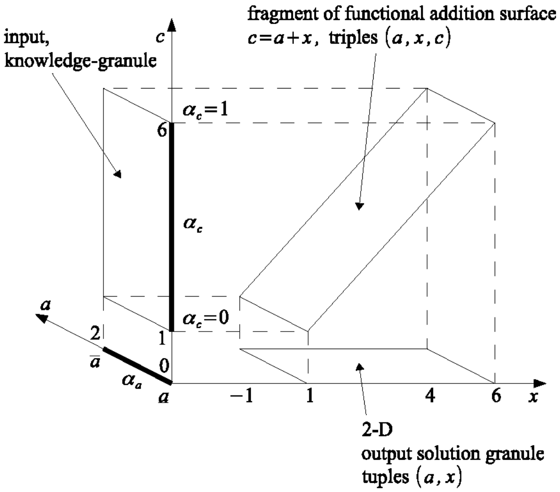

Figure 7: Input-knowledge granule and solution granule of equation $[\underline{a}, \overline{a}] + [\underline{x}, \overline{x}] = [\underline{c}, \overline{c}] = [0, 2] + [\underline{x}, \overline{x}] = [1, 6]$ in 3D-space.

$$[(a, x)] : a = 2\alpha_a, x = 1 - 2\alpha_a + 5\alpha_c, \alpha_a \in [0, 1], \alpha_c \in [0, 1] \tag{16}$$

Because solution (16) is 2-dimensional (space $A \times X$) it can not in any way be presented in 1-dimensional form $[\underline{x}, \overline{x}]$ as it is suggested by Moore arithmetic. Because of this fact, Moore arithmetic has very limited possibilities. It is able to solve only simple problems but is unable to solve such problems as relatively non-complicated equation $[a] + [x] = [c]$. However, this can be done by multidimensional RDM-arithmetic.

4 Special features of the interval addition operation

In case of interval addition (similarly as in case of other arithmetic operations) one cannot give such general addition formulas as in case of classic arithmetic of precise numbers (singletons), formulas that could be used for all problems. In

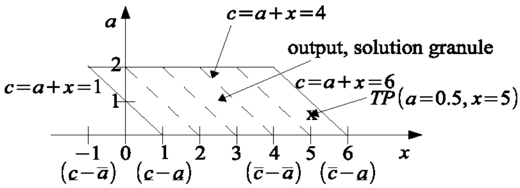

Figure 8: Solution granule of interval equation $[\underline{a},\overline{a}] + [\underline{x},\overline{x}] = [\underline{c},\overline{c}] = [0,2] + [\underline{x},\overline{x}] = [1,6]$ in 2D-space.

singleton arithmetic the sum $2+2$ is always the same. In interval arithmetic sum of two intervals e.g. $[1,3]+[1,3]$ is not always the same. The number of possible addition results increases with the number of added intervals. However, now let us limit to 2 intervals. A parable as below will be considered.

Parable about father and two sons

A father from Wild West wants to give 10g gold sand for any of his two sons. Father has at disposal a spring scales of maximal error 1g and a very precise comparative lever scales (Fig. 10) with a negligibly small error. Father decided to give to both sons possibly precisely 20g of gold sand. However, he does not want the gold weight considerably exceeds 20g because rest of the possessed gold he plans to invest in other important aims. To prevent exceeding 20g father considers 2 following methods of gold sand weighing (also other methods are possible). Method 1: he is pouring the sand on the spring scales until it shows 10g and then gives this gold to son 1. Next, he repeats the operation and gives the so weighed gold to son 2. Because the spring scales has the maximal error 1g, each son achieves gold which true weight is contained in interval $[9,11]$g, Fig. 9.

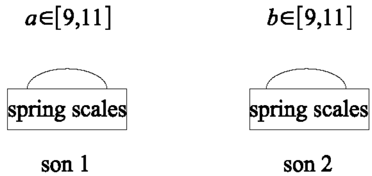

Figure 9: Weighing the gold sand according to method 1 on the spring scales with the max. error 1g.

It should be noted that after applying method 1 gold weights a and b presented to the sons will not be equal. For example: son 1 can achieve 9.5g and son 2 10.5g of gold. Both weights are contained in the scales-error interval $[9,11]$. Method 2: Father is pouring gold sand on the spring scales until it shows 10g. The so weighed gold belongs to son 1. Next, he takes this gold and puts it on one of scales of the lever scales and on the second scale he begins to pour gold sand. He is continuing the pouring until both scales show a balance. Then one can assume with sufficient accuracy that gold on both scales has the same weight a, where $a \in [9, 11]$, Fig. 10. The gold sand from the second scale belongs to son 2. The question in the problem consists in deciding which of both gold partitioning method better prevents exceeding, and especially considerable exceeding 20g of gold presented to the sons by the father?

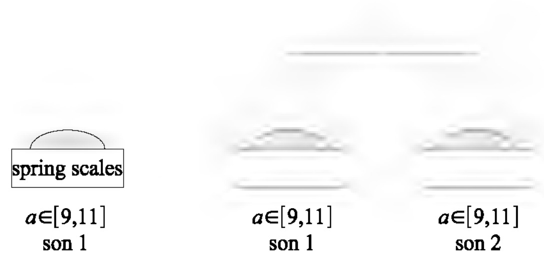

Figure 10: Illustration of method 2 of gold allowance.

First, let us try to compare both methods with use of Moore arithmetic. According to method 1, son 1 achieves amount of gold equal to $a \in [9, 11]$ and son 2 a different amount $b \in [9, 11]$. The sum $a + b = x$ can be calculated with formula (17).

$$[a] + [b] = [9, 11] + [9, 11] = [18, 22] = [\underline{x}, \overline{x}] \tag{17}$$

According to method 2 son 1 obtains an amount $a \in [9, 11]$ and son 2 the same amount $a \in [9, 11]$g. Thus, the gold sum presented to both sons $[\underline{x}, \overline{x}]$ is determined by formula (18).

$$[a] + [a] = [9, 11] + [9, 11] = [18, 22] = [\underline{x}, \overline{x}] \tag{18}$$

According to Moore-arithmetic it has no meaning which method will be applied if the criterion of not exceeding 20g of gold is considered. Now, let us apply RDM-arithmetic. In method 1 the gold amount $a \in [9, 11]$ given to son 1 can be expressed by formula (19) and the amount $b \in [9, 11]$ given to son 2 ($a \neq b$) by formula (20).

$$a = 9 + 2\alpha_a, \alpha_a \in [0, 1] \tag{19}$$

$$b = 9 + 2\alpha_b, \alpha_b \in [0, 1] \tag{20}$$

The sum $a + b = x$ presented to sons is determined by formula (21).

$$\begin{array}{l} a + b = x = (9 + 2\alpha_a) + (9 + 2\alpha_b) = 18 + 2\alpha_a + 2\alpha_b \\ \alpha_a \in [0, 1], \alpha_b \in [0, 1] \end{array} \tag{21}$$

Summing the intervals $[a] + [b] = [x]$ is visualized in Fig. 11.

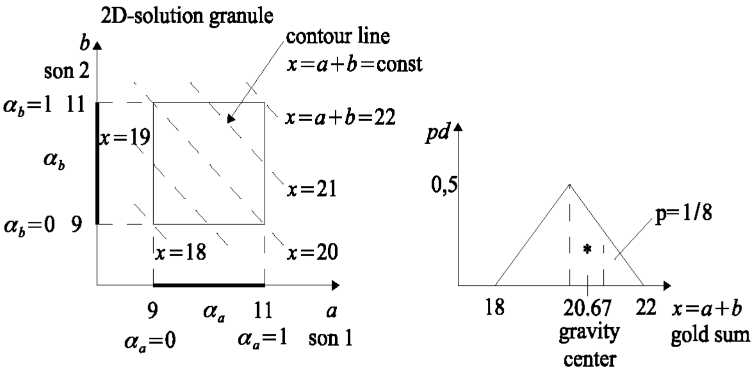

Figure 11: Illustration of method 1 of gold partitioning: solution granule of the problem in 2D-space and $pd(x)$ - a priori probability density of the sum x obtained at assumption of uniform density distributions of components $pd(a) = const = 0.5$ and $pd(b) = const = 0.5$.

In method 2 the amount a of gold weighed with the spring scales and presented to son 2 is contained in interval $a \in [9, 11]$g. This weight is known only approximately. However, owing to pouring gold sand on the lever scales son 2 will obtain the same amount $a \in [9, 11]$ of gold. If the approximately known weight a of gold is expressed with use of RDM-variables as in formula (22) then the gold sum presented to sons is given by formula (23).

$$a = 9 + 2\alpha_a, \alpha_a \in [0, 1] \tag{22}$$

$$x = a + a = 18 + 2\alpha_a, \alpha_a \in [0, 1] \tag{23}$$

Addition operation of two approximately known but equal gold amounts a+a is shown in Fig. 12.

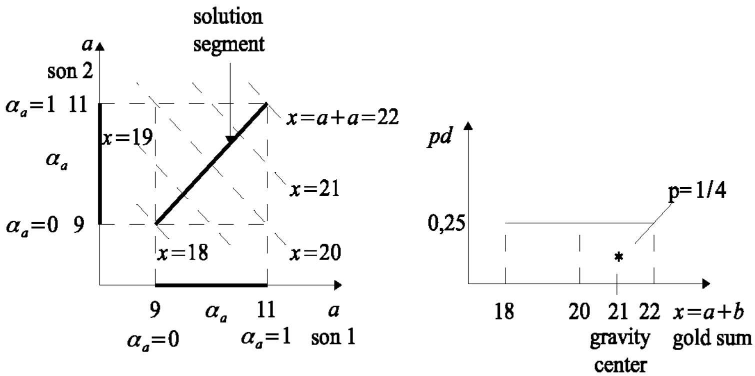

Figure 12: Illustration of method 2 of gold allowance: solution segment in 2D-space and distribution of a priori probability density $pd(x)$ achieved at assumption of uniform distribution $pd(a) = 0.5 = const.$

Now, let us compare results obtained with Moore arithmetic and RDM-arithmetic. Moore arithmetic provides us only with information that the gold sum $a + b = x$ presented to sons is contained in interval $x \in [18, 22]$, Fig. 13.

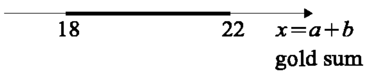

Figure 13: Visualization of the addition result $x = a + b$ achieved with Moore arithmetic (1-dimensional result) in the problem of gold allowance.

Independently whether method 1 or 2 is used, Moore arithmetic delivers the same information: the gold sum $x \in [18, 22]$. Information delivered by RDM-arithmetic is richer: we can get to know distribution of probability density of the sum $x = a+b$. In case of method 1, Fig. 11, if the gold sum x will exceed 20g (father wants to avoid it), then expected value of the surpass will be equal to $2/3$g of gold. In case of method 2 the surpass will be higher and will be equal to 1g of gold, Fig. 12. Next, in case of method 1 probability of exceeding by the gold sum 21g equals $1/8$ and in case of method 2 it equals $1/4$ and is 2 times higher. Thus, from point of view of not exceeding by the gold sum the value 20g method 1 is more advantageous. The parable about father and two sons shown that 1-dimensional Moore arithmetic does not give us possibility of determining pd-distributions and thus possibility of as precise investigation of problems as the multidimensional RDM-arithmetic. The parable also shows that in case of interval addition (also in case of remaining arithmetic operations) one can not give one general addition formula that would be correct for all real cases. Before adding one should identify whether added variable values, though known only approximately, are in the analyzed problem equal or different and then, dependently of the identification result, one (α_a) or two RDM-variables (α_a, α_b) should be used. If not 2 ($a + b$) but more, e.g. 5 approximately known variable values ($a + b + c + d + e$) are added then situation is much more complicated. However, RDM-variables cause that previously black-box interpreted intervals become brighter, they achieve their interior, what allows for better solving problems with uncertainties.

5 Conclusions

Arithmetic operations on uncertainty models such as intervals, membership functions (fuzzy arithmetic), distributions of probability density (probabilistic arithmetic [20]) and on other uncertainty models considered by Grey Systems [10] turns out to be very difficult and impossible for realization with one-dimensional arithmetics such as Moore one and some types of fuzzy arithmetics. Each uncertain parameter occurring in mathematical model of a system increases dimensionality and nonlinearity degree of the system model. Therefore solving problems with uncertainties becomes very difficult. RDM-arithmetic is an arithmetic that allows for solving problems in their full dimension and without simplifications. Thus, it provides us with credible results. Besides, these results can be tested with the testing-points. This method can be used for correctness checking of any arithmetic type.

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The papers presented in this Volume 2 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Eleventh International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2012) organized in Warsaw on October 12, 2012 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, Prof. Asen Zlatarov University, Burgas, Bulgaria, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Eleventh International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2012) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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