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Volume II: Applications

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Fuzzy C-Means Cluster Analysis and Approximated Data Strings in Operation Prognosis for Gastric Cancer Patients

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Abstract

The chapter is composed of two parts. In the first part we aim at employing fuzzy c-means (FCM) clustering to prognosticate membership degrees pointing out possibilities for operation and none operation for a set of 25 gastric cancer patients characterized by values of decisive biological markers. The second part handles the technique of mathematical modelling of a common membership function approximating the information collected from the given set of patients. When constructing the equation of the function we are able to determine the operation and none operation diagnosis for an arbitrary gastric cancer patient.

Keywords: fuzzy c-means clustering, operation degrees for gastric cancer patients, approximation of point sets by π-functions.

1 Introduction

Nowadays we are living in an era of the rapid development of information technology. A large number of information is sent and received every day.

Therefore, finding some data processing methods to discover the partial structure in a data set and to utilize useful information to solve efficiently daily issues becomes of vital importance.

A lot of approaches in this domain have been put forward. Cluster analysis is one of them. This method involves the task of dividing data points into homogeneous classes or clusters so that items in the same class are as similar as possible and items in different classes are as dissimilar as possible [2]. Moreover, some successful applications in clustering analysis have been presented. Among clustering approaches the fuzzy c-means clustering (FCM) is regarded as well-known and efficient [1–3, 14, 16–17].

In hard clustering, data points are divided into crisp clusters, where each data point belongs to exactly one cluster [2]. In many situations, boundary data points can be difficult to be allocated. Therefore, the realistic picture of the data structure may not be correctly presented by the crisp clustering. However, fuzzy partition can make up the flaw, due to the advantage that data points are allowed to belong to more than one cluster.

Some clinical studies show that the operation is regarded as one of the most effective treatments for gastric cancer patients. Three biological markers, namely, the patient's age, the *crp*-value (the C reactive proteins) and the body weight, play a significant role in the diagnosis of operation. In this paper, the operation diagnoses of randomly selected 25 cancer patients are studied. Two approaches are applied for approximating the operation decisions.

The first method adapts the fuzzy c-means clustering analysis (FCM) to evaluate the operation possibility. We attempt to utilize the FCM algorithms in order to divide the clinical data vectors representing 25 patients into two clusters. One of them reveals the relationship between patients-vectors and "operation" possibilities via membership degrees, whereas the other samples possibility degrees being prognoses of "none operation" for each patient.

Having prepared membership degrees of "operation" in the group of 25 patients as well as the patients' characteristic values, that include all biological marker quantity codes, we will obtain a set of points being pairs (patient characteristics, membership degree for operation possibility). After inserting the points into the two-dimensional coordinate system in accordance with the ascending order of patient characteristic values we make a trial of approximating this set of points by the truncated version of the π-function [4–5, 8, 13]. This makes possible to evaluate the operation degree for any arbitrary patient by setting his/her characteristic value into the equation of the function.

The approximation of the point set and prognoses made for casual gastric cancer patients constitute the second part of the work.

2 Description of the Fuzzy C-Means Clustering Algorithm

Let us suppose that $X = \{x_1, \dots, x_n\}$ is a finite data set. Each data point $x_k = \left(x_{k_1}, \dots, x_{k_p}\right), k = 1, \dots, n,$ is a pattern vector in R^p. Fuzzy c-means algorithm tries to partition *X* in a collection of S_i subsets, $2 \leq i \leq c$, called fuzzy clusters. By running the algorithm repeatedly, a list of v_i cluster centers and a partition matrix *U* are returned.

The fuzzy c-means algorithm is based on minimizing the objective function *J* with respect to the membership values $\mu_{S_i}(x_k)$ and the distance $d(\mathrm{v}_i, x_k)$ [2], where

$$J = \sum_{k=1}^{n} \sum_{i=1}^{c} \left(\mu_{S_i}(x_k)\right)^m \cdot d(v_i, x_k). \tag{1}$$

In (1) n is the number of data points and *c* is the number of clusters. The value of $\mu_{S_i}(x_k)$ or μ_{ik} represents the value of membership degree of x_k in cluster S_i. Moreover, the sum of the membership degrees for each x_k sample in all clusters is equal to 1. The notation of $d(\mathrm{v}_i, x_k)$ indicates the Euclidean distance between the cluster center v_i and x_k. The constant $m > 1$ is called weighting exponent, which determines the fuzziness of the resulting clusters.

A linguistic description of the FCM algorithm is presented by the following steps:

1) Select the number of clusters c, initialize the value of fuzzy parameter *m* $(2 \leq m < \infty)$ and the termination tolerance ϵ.

2) Set $l = 0$.

3) Determine the initial values of membership degrees in partition matrix U^l.

4) Calculate cluster centers $\mathrm{v}_i^l, i = 1, \dots, c$, due to [2], as

$$\mathrm{v}_i^l = \frac{\sum_{k=1}^{n}\left(\left(\mu_{ik}^l\right)^m \cdot x_k\right)}{\sum_{k=1}^{n}\left(\mu_{ik}^l\right)^m}. \tag{2}$$

5) Calculate the updated partition matrix U^{l+1} by using v_i^l in formula

$$\mu_{ik}^{l+1} = \frac{\left(\frac{1}{d\left(x_k, \mathrm{v}_i^l\right)}\right)^{1/m-1}}{\sum_{j=1}^{c}\left(\frac{1}{d\left(x_k, \mathrm{v}_j^l\right)}\right)^{1/m-1}}. \tag{3}$$

6) If $\left\|U^{l+1} - U^l\right\| \geq \epsilon$, then set $l = l + 1$, and go to step 4. If $\left\|U^{l+1} - U^l\right\| \leq \epsilon$, then stop the procedure. Matrix U^{l+1} is the most optimal distribution of membership degrees of x_k in clusters S_i.

The prior determination of the membership degrees of x_k in S_i plays a crucial role in this algorithm, as their choice not only can affect the convergence speed, but also may have a direct impact on the results of the classification [2].

The initial cluster centers are just prototypes and unstable. Therefore, they need to be iteratively updated. Each iteration guarantees an improvement of the coordinates of clustering centers. The updating procedure continues until two adjacent membership matrices cease to change. Furthermore, the calculation of clustering centers depends on the values of initial membership degrees in the partition matrix. To avoid inaccuracy in final results we will discuss the own technique of calculation of membership degrees to avoid guessing at their values intuitively.

3 General Membership Functions of "the Leftmost", "in the Middle" and "the Rightmost" Expressions from the Linguistic List

The accurate evaluation of the membership of x_k in S_i can improve the iteration time and the convergence speed. In this paper, the *s*-class membership function is adopted for the further calculations due to [9–12]. We recall the formula of the s-function as

$$s(z, \alpha, \beta, \gamma) = \begin{cases} 0 & \text{for} \quad z \leq \alpha, \\ 2\left(\frac{z-\alpha}{\gamma-\alpha}\right)^2 & \text{for} \quad \alpha \leq z \leq \beta, \\ 2\left(\frac{z-\gamma}{\gamma-\alpha}\right)^2 & \text{for} \quad \beta \leq z \leq \gamma, \\ 1 & \text{for} \quad z \geq \gamma. \end{cases} \tag{4}$$

The curve, implemented as a graph of (4), starts with point $(0, \alpha)$ and ends with $(\gamma, 1)$, whereas β is the arithmetic mean value of α and γ.

By referring to the most decisive medical factors, such as the patient's age, weight and *crp*-values (C-reactive proteins), operation prognoses usually can be expressed by "operation" and "none operation". The possibilities of the decision evaluation can be described by some linguistic terms.

Let us suppose that $L = \{L_1, \dots, L_\omega\}$ is a linguistic list consisting of ω words. Each word is associated with a fuzzy set. In compliance with [11–12], ω is a positive odd integer. Furthermore, let E be the length of a common reference set *R*, designed for all restrictions characterizing the fuzzy sets from *L*, provided that $z \in R$. We now wish to divide the linguistic terms into three groups recognized as a left group, a middle group and a right group.

The membership functions assigned to the leftmost terms are generated by (5) as [11–12]

$$\mu_{L_t}(z) = \begin{cases} 1 \text{ for } z \leq \frac{E(\omega-1)}{2(\omega+1)}\delta(t), \\ 1 - 2\left(\frac{z - \frac{E(\omega-1)}{2(\omega+1)}\delta(t)}{\frac{E(\omega-1)}{\omega(\omega+1)}\delta(t)}\right)^2 \\ \text{ for } \frac{E(\omega-1)}{2(\omega+1)}\delta(t) \leq z \leq \frac{E(\omega-1)}{2\omega}\delta(t), \\ 2\left(\frac{z - \frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)}\delta(t)}{\frac{E(\omega-1)}{\omega(\omega+1)}\delta(t)}\right)^2 \\ \text{for } \frac{E(\omega-1)}{2\omega}\delta(t) \leq z \leq \frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)}\delta(t), \\ 0 \text{ for } z \geq \frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)}\delta(t), \end{cases} \quad (5)$$

where $\delta(t) = \frac{2t}{\omega-1}, t = 1, \dots, \frac{\omega-1}{2}$ is a parametric function depending on left function number *t*. When *t* is equal to 1, the formula implies the first leftmost membership function. If *t* takes the value of $\frac{\omega-1}{2}$, then we will obtain the last left membership function.

The membership function in the middle has the form of a clock. It is given by (6) in the form of [11–12]

$$\mu_{L_{\frac{\omega+1}{2}}}(z)=\begin{cases}0 & \text{for} \quad z\le\frac{E(\omega-2)}{2\omega},\\ 2\left(\frac{z-\frac{E(\omega-2)}{2\omega}}{\frac{E}{\omega}}\right)^2 & \text{for} \quad \frac{E(\omega-2)}{2\omega}\le z\le\frac{E(\omega-1)}{2\omega},\\ 1-2\left(\frac{z-\frac{E}{2}}{\frac{E}{\omega}}\right)^2 & \text{for} \quad \frac{E(\omega-1)}{2\omega}\le z\le\frac{E}{2},\\ 1-2\left(\frac{z-\frac{E}{2}}{\frac{E}{\omega}}\right)^2 & \text{for} \quad \frac{E}{2}\le z\le\frac{E(\omega+1)}{2\omega},\\ 2\left(\frac{z-\frac{E(\omega+2)}{2\omega}}{\frac{E}{\omega}}\right)^2 & \text{for} \quad \frac{E(\omega+1)}{2\omega}\le z\le\frac{E(\omega+2)}{2\omega},\\ 0 & \text{for} \quad z\ge\frac{E(\omega+2)}{2\omega}.\end{cases} \tag{6}$$

Finally, the membership functions on the right-hand side can be expressed by (7) as [11–12]

$$\mu_{L_{\frac{\omega+3}{2}+t-1}}(z)=\begin{cases}0 \text{ for } z\le E-\frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)}\cdot\varepsilon(t),\\ 1-2\left(\frac{z-\left(E-\frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)}\cdot\varepsilon(t)\right)}{\frac{E(\omega-1)}{\omega(\omega+1)}\cdot\varepsilon(t)}\right)^2\\ \text{for } E-\frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)}\cdot\varepsilon(t)\le z\le E-\frac{E(\omega-1)}{2\omega}\cdot\varepsilon(t),\\ 2\left(\frac{z-\left(E-\frac{E(\omega-1)}{2(\omega+1)}\cdot\varepsilon(t)\right)}{\frac{E(\omega-1)}{\omega(\omega+1)}\cdot\varepsilon(t)}\right)^2\\ \text{for } E-\frac{E(\omega-1)}{2\omega}\cdot\varepsilon(t)\le z\le E-\frac{E(\omega-1)}{2(\omega+1)}\cdot\varepsilon(t),\\ 1 \text{ for } z\ge E-\frac{E(\omega-1)}{2(\omega+1)}\cdot\varepsilon(t).\end{cases} \tag{7}$$

A new function $\varepsilon(t)=1-\frac{2(t-1)}{\omega-1}, t=1,\dots,\frac{\omega-1}{2}$ allows creating all rightmost functions one by one when setting *t*-values in (7).

4 Initialization of the Partition Matrix

To make a decision "operate" contra "do not operate", concerning an individual patient via high or low membership degree in the last partition matrix, we have to involve the medical experience in the decision process. To facilitate a conversation with a physician we have prepared a linguistic list named "The primary judgment concerning possibility that a physician recommends "operation" = L = $\{L_1 = \text{"none"},\ L_2 = \text{"little"},\ L_3 = \text{"medium"},\ L_4 = \text{"large"},\ L_5 = \text{"total"}\}$.

The excerpt of the data set, shown in Table 1, consists of the patients' clinical records and primary judgments of operation possibilities made by the medical expert. The total medical report contains 25 gastric cancer patients randomly selected.

Two surgery states "operation" and "none operation" assist two clusters S_1 and S_2 respectively. By selecting words from the list the experienced surgeon makes the primary graded decision about possibilities of operating or not operating on the patient.

Table 1. The data set of 25 gastric cancer patients randomly selected

Patient x_k	**Attribute-vectors and operation possibilities**		
	Attribute-vectors (Age, weight, crp)	**Operation cluster S_1**	**None Operation cluster S_2**
x_1	(71, 85, 1)	Total	Little
x_2	(81, 70, 9)	Medium	Large
x_3	(50, 67, 4)	Large	Medium
x_4	(64, 84, 13)	Large	Little
x_5	(41, 95, 4)	Large	Little
…	…	…	…
x_{25}	(54, 49, 36)	None	Large

Each verbal expression, being the term of L, is associated with a fuzzy set. L_1 and L_2 represent two left fuzzy sets, L_3 is the fuzzy set in the middle, whereas L_4 and L_5 constitute two rightmost fuzzy sets. Unfortunately, these linguistic items do not provide us with any information about membership degrees expected in matrix U^0 as primary recommendation states of "operation" or "none operation". Therefore we adopted the following technique to assign numerical substitutes to verbal expressions from the list.

By inserting E = 100 (the length of the reference set R = [0,100] – typical of density measures in medical investigations), ω = 5 and t = 1, 2, in (5), we obtain the membership functions of the first two fuzzy sets, namely, $L_1 =$ "none" given as

$$\mu_{L_1}(z) = \begin{cases} 1 & \text{for} \quad z \leq 16.7, \\ 1 - 2\left(\frac{z-16.7}{6.6}\right)^2 & \text{for} \quad 16.7 \leq z \leq 20, \\ 2\left(\frac{z-23.3}{6.6}\right)^2 & \text{for} \quad 20 \leq z \leq 23.3, \\ 0 & \text{for} \quad z \geq 23.3 \end{cases} \tag{8}$$

and $L_2 =$ "little" prepared as

$$\mu_{L_2}(z) = \begin{cases} 1 & \text{for} \quad z \leq 33.3, \\ 1 - 2\left(\frac{z-33.3}{13.4}\right)^2 & \text{for} \quad 33.3 \leq z \leq 40, \\ 2\left(\frac{z-46.7}{13.4}\right)^2 & \text{for} \quad 40 \leq z \leq 46.7, \\ 0 & \text{for} \quad z \geq 46.7\,. \end{cases} \tag{9}$$

By substituting E = 100 and ω = 5 in (6), the membership function of $L_3 =$ "medium" is given as the structure

$$\mu_{L_3}(z) = \begin{cases} 0 & \text{for} \quad z \leq 30, \\ 2\left(\frac{z-30}{10}\right)^2 & \text{for} \quad 30 \leq z \leq 40, \\ 1 - 2\left(\frac{z-50}{10}\right)^2 & \text{for} \quad 40 \leq z \leq 50, \\ 1 - 2\left(\frac{z-50}{10}\right)^2 & \text{for} \quad 50 \leq z \leq 60, \\ 2\left(\frac{z-70}{10}\right)^2 & \text{for} \quad 60 \leq z \leq 70, \\ 0 & \text{for} \quad z \geq 70. \end{cases} \tag{10}$$

Finally, for E = 100, ω = 5 and t = 1, 2, inserted in (7), we get the membership functions of $L_4 =$ "large" in the form of

$$\mu_{L_4}(z) = \begin{cases} 0 & \text{for} \quad z \leq 53.3, \\ 1 - 2\left(\frac{z-53.3}{13.4}\right)^2 & \text{for} \quad 53.3 \leq z \leq 60, \\ 2\left(\frac{z-66.7}{13.4}\right)^2 & \text{for} \quad 60 \leq z \leq 66.7, \\ 1 & \text{for} \quad z \geq 66.7 \end{cases} \tag{11}$$

and $L_5 = $ "total" as

$$\mu_{L_5}(z) = \begin{cases} 0 & \text{for} \quad z \leq 76.7, \\ 1 - 2\left(\frac{z-76.7}{6.6}\right)^2 & \text{for} \quad 76.7 \leq z \leq 80, \\ 2\left(\frac{z-83.3}{6.6}\right)^2 & \text{for} \quad 80 \leq z \leq 83.3, \\ 1 & \text{for} \quad z \geq 83.3\,. \end{cases} \tag{12}$$

When substituting $\alpha = 0, \beta = 50$ and $\gamma = 100$ in a new s-function impacted over set *R* we determine

$$\mu_R(z) = s(z, 0{,}50{,}100)$$
$$= \begin{cases} 0 & \text{for} \quad z \leq 0, \\ 2\left(\frac{z}{100}\right)^2 & \text{for} \quad 0 \leq z \leq 50, \\ 2\left(\frac{z-100}{100}\right)^2 & \text{for} \quad 50 \leq z \leq 100 \\ 1 & \text{for} \quad z \geq 100. \end{cases} \tag{13}$$

After sampling all membership functions (8)–(13) in Figure 1, we aim at evaluating the membership degrees taking place in the first partition matrix U^0.

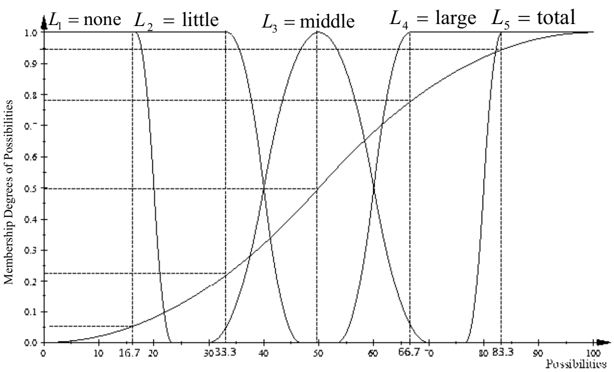

Figure. 1. The collection of membership functions of L_1–L_5 and *R*

The degrees of the intersection points between $\mu_R(z)$ and $\mu_{L_l}(z)$, *l*=1,…,5 represent “none” by 0.056, “little” by 0.22, “medium” by 0.5, “large” by 0.78 and "total” by 0.944, respectively.

After the arrangements of numerical substitutions, the words in Table 1 are replaced by numerical values put in Table 2.

Table 2. Data set with initial membership values

Patient x_k	Attribute-vectors and operation possibilities		
	Attribute-vectors (Age, weight, crp)	$\mu_{S_1}(x_k)$	$\mu_{S_2}(x_k)$
x_1	(71, 85, 1)	0.944	0.22
x_2	(81, 70, 9)	0.5	0.78
x_3	(50, 67, 4)	0.78	0.5
x_4	(64, 84, 13)	0.78	0.22
x_5	(41, 95, 4)	0.78	0.22
…	…	…	…
x_{25}	(54, 49, 36)	0.056	0.78

It is assumed that the sum of membership grades in clusters S_1 and S_2 should be equal to 1 for each x_k, $k = 1,\ldots,25$. It can happen that the distinct sums differ from 1. In such cases some adjustments need to be made; therefore the following techniques are applied.

Case 1: $\mu_{S_1}(x_k) + \mu_{S_2}(x_k) > 1$.

If the sum is greater than 1, we calculate a quotient q_1, designed as

$$q_1 = \frac{\mu_{S_1}(x_k) + \mu_{S_2}(x_k) - 1}{2}.$$

Hence, two adjusted membership degrees are given by the following formulations:

$\mu'_{S_1}(x_k) = \mu_{S_1}(x_k) - q_1$ and $\mu'_{S_2}(x_k) = \mu_{S_2}(x_k) - q_1$.

Proof:

$$\mu'_{S_1}(x_k) + \mu'_{S_2}(x_k) = \mu_{S_1}(x_k) + \mu_{S_2}(x_k) - 2q_1 =$$
$$\mu_{S_1}(x_k) + \mu_{S_2}(x_k) - 2 \cdot \frac{\mu_{S_1}(x_k) + \mu_{S_2}(x_k) - 1}{2} =$$
$$\mu_{S_1}(x_k) + \mu_{S_2}(x_k) - \mu_{S_1}(x_k) - \mu_{S_2}(x_k) + 1 = 1.$$

In contrast with Case 1, Case 2 handles the situation that the sum is less than 1.

Case 2: $\mu_{S_1}(x_k) + \mu_{S_2}(x_k) < 1$.
We now have to derive another fraction q_2, given by

$$q_2 = \frac{1 - \mu_{S_1}(x_k) - \mu_{S_2}(x_k)}{2}.$$

Membership values typical of Case 2 are verified by

$$\mu'_{S_1}(x_k) = \mu_{S_1}(x_k) + q_2 \text{ and } \mu'_{S_2}(x_k) = \mu_{S_2}(x_k) + q_2$$

Proof:

$$\mu'_{S_1}(x_k) + \mu'_{S_2}(x_k) = \mu_{S_1}(x_k) + \mu_{S_2}(x_k) + 2q_2 =$$
$$\mu_{S_1}(x_k) + \mu_{S_2}(x_k) + 2 \cdot \frac{1 - \mu_{S_1}(x_k) - \mu_{S_2}(x_k)}{2} =$$
$$\mu_{S_1}(x_k) + \mu_{S_2}(x_k) + 1 - \mu_{S_1}(x_k) - \mu_{S_2}(x_k) = 1.$$

After revising the membership degrees due to Case 1 or Case 2 we rearrange the last two columns of Table 2 to renew it as Table 3.

The entries of the initial partition matrix U^0 consist of the values coming from the last two columns in Table 3. U^0 is a 2x25 matrix given by

Table 3. Data set with verified membership degrees

Patient x_k	**Attribute-vectors and operation possibilities**		
	Attribute-vectors (Age, weight, crp)	$\mu'_{S_1}(x_k)$	$\mu'_{S_2}(x_k)$
x_1	(71, 85, 1)	0.862	0.138
x_2	(81, 70, 9)	0.36	0.64
x_3	(50, 67, 4)	0.64	0.36
x_4	(64, 84, 13)	0.78	0.22
x_5	(41, 95, 4)	0.78	0.22
…	…	…	…
x_{25}	(54, 49, 36)	0.138	0.862

$$U^0 = \begin{matrix} & x_1 & x_2 & x_3 & \cdots & x_{25} \\ S_1 & 0.862 & 0.36 & 0.64 & \cdots & 0.138 \\ S_2 & 0.138 & 0.64 & 0.36 & \ldots & 0.862 \end{matrix}_{2x25}.$$

The numerical values in the first row in matrix U^0 propose membership degrees for patients $x_k, k = 1, \ldots, 25,$ in cluster S_1, whereas the second row suggests the membership values for patients $x_k, k = 1, \dots, 25,$ in cluster S_2. The sum of membership degrees in each column is equal to one.

If we go back to the FCM algorithm and let $l = 0$, $m = 3$ and $\epsilon = 10^{-8}$ then, by using Matlab, we note that the cluster centers become stable and do not change their coordinates after 31 iterations due to $||U^{31} - U^{30}|| = 9.93692 \times 10^{-9} < 10^{-8}$. The last two partition matrices and the optimal cluster centers are listed in the patterns

$$U^{30} = \begin{matrix} & x_1 & \cdots & x_{25} \\ S_1 & 0.74366062 & \cdots & 0.39447229 \\ S_2 & 0.25633938 & \dots & 0.60552771 \end{matrix}_{2x25}$$

and

$$U^{31} = \begin{matrix} & x_1 & \cdots & x_{25} \\ S_1 & 0.74366062 & \cdots & 0.39447229 \\ S_2 & 0.25633938 & \dots & 0.60552771 \end{matrix}_{2x25},$$

as well as

$$v_1^{31} = (65.9704, 74.2257, 6.50373)$$

and

$$v_2^{31} = (70.4353, 69.735, 35.8068).$$

The final membership degrees for 25 patients, classified in S_1 and S_2, are depicted in Figure 2. In this manner the primary operation hypotheses, formulated by verbal structures, have been secondarily confirmed or denied by the strength of corresponding membership degrees in both clusters.

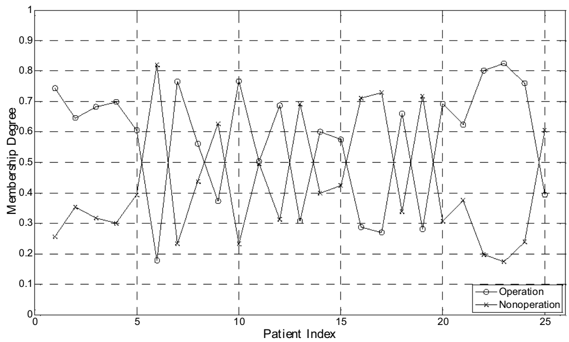

Figure 2. The final cluster membership degrees

5 The Introduction to the Point Set Approximation

In the first step of the study we have shown the practicability of fuzzy c-means clustering analysis in expressing operation possibilities via the values of membership degrees for randomly selected 25 gastric cancer patients. We note that the points (x_k, $\mu_{S_i}(x_k)), k = 1, \dots, n, i = 1,2,$ have been casually scattered along the x-axis since x_k have not been characterized by any quantity.

We wish now to assign a characteristic value to each patient x_k to be able to place x_k's characteristic values in an ascending order. The new points will possess coordinates (characteristics of x_k, membership degree of operation possibility for x_k). The membership degrees are fetched from the first row of matrix U^{31}. This allows applying an approximation of the newly-created point set by a truncated version of the π-function [4–5, 8, 13]. Having got an equation of the function we can make operation prognoses for arbitrary patients.

6 The Determination of Characteristic Values

The patients' ages, the *crp*-values and the body weights play a significant role in the operation diagnosis. Let us denote the space of ages [0, 100] by A, the space of *crp*-values [0, 85] by CRP and express the body weight by BW = [40, 120]. A characteristic function [12] denoted by $f_{x_k}(a^c\ crp^c\ bw^c) = w_1 a^c + w_2 crp^c + w_3 bw^c$, $k = 1,\dots,n$, will be constructed to determine the characteristic value for each patient. It is worthwhile to emphasize that $(a^c\ crp^c\ bw^c)$ is defined as a code vector [12], in which a^c, crp^c and bw^c symbolize assigned codes to the age $a \in A$, the *crp*-value $crp \in CRP$ and the body weight $bw \in BW$. The multipliers w_1, w_2 and w_3 are the importance weights emphasizing the decisive power of each biological parameter for the operation decision.

By returning to formulas (5–7) and using the technique expanded in [11–12] we make a partition of A, CRP and BW into five subintervals with assigned to them codes in the range 0–4.

We divide A = "age" into five subintervals $A_0 = [0, 20]$, $A_1 = [20, 40]$, $A_2 = [40, 60]$, $A_3 = [60, 80]$ and $A_4 = [80, 100]$ referred to codes 0, 1, 2, 3 and 4 respectively.

For CRP = "*crp*-value" we find subintervals $CRP_0 = [0, 17]$, $CRP_1 = [17, 34]$, $CRP_2 = [34, 51]$, $CRP_3 = [51, 68]$ and $CRP_4 = [68, 85]$ with the corresponding codes 0, 1, 2, 3 and 4 assigned to CRP_p, $p = 0,\dots,4$, one by one.

Finally, BW = "body weight" is split into $BW_0 = [40, 56]$, $BW_1 = [56, 72]$, $BW_2 = [72, 88]$, $BW_3 = [88, 104]$ and $BW_4 = [104, 120]$ with the code indices p assigned to BW_p, $p = 0,\dots,4$.

To find values of importance weights w_1, w_2 and w_3 we adopt the idea coming from the contribution [15] by Saaty.

Assuming that we have s_g, $g = 1,...,r$, objects (biological markers) we want to construct a scale, rating these objects as to their importance with respect to the decision. We ask a decision-maker for comparing the objects in paired comparison. If we compare object s_g with object s_h, g, $h = 1,\dots,r$, then we will assign the values b_{gh} and b_{hg} as follows [6–7, 15]:

(1) $b_{gh} = \frac{1}{b_{hg}}$
(2) If object s_g is more important than object s_h then b_{gh} gets assigned a number according to the following scheme:

Intensity of importance expressed by the value of b_{gh}	*Definition*
1	Equal importance of s_g and s_h
3	Weak importance of s_g over s_h
5	Strong importance of s_g over s_h
...	...

If object s_h is more important than object s_g, we assign the value of b_{hg}. Having obtained the above judgments an $r \times r$ importance matrix $B = \left(b_{gh}\right)_{g,h=1}^{r}$ is constructed. The importance weights are decided as components of this eigenvector that corresponds to the largest in magnitude eigenvalue of the matrix *B*.

According to the expert's suggestion, we place the age (*A*), the *crp*-value (*CRP*) and the body weight (*BW*) in order of importance *CRP* >*A* >*BW*. The elements of the importance matrix *B* are arranged as [12]

$$B = \begin{matrix} & A & CRP & BW \\ A \\ CRP \\ BW \end{matrix} \begin{bmatrix} 1 & 1/3 & 3 \\ 3 & 1 & 5 \\ 1/3 & 1/5 & 1 \end{bmatrix}.$$

By using Matlab we obtain the largest eigenvalue of *B* equalling to $\lambda = 3.0385$ and the corresponding eigenvector $V = (0.37, 0.92, 0.15)$. V is composed of coordinates that are interpreted as the weights w_1, w_2, w_3 sought for *A*="*age*", *CRP*="*crp-value*", *BW*="*body weight*".

This gives $f_{x_k}(a^c\ crp^c\ bw^c) = 0.37 \cdot a^c + 0.92 \cdot crp^c + 0.15 \cdot bw^c$.

Example 1
The eighty-one year old man x_k weighing 90 kg and revealing *crp* = 16 is thus given by $f_{x_k}(a^c\ crp^c\ bw^c) = 0.37 \cdot 4 + 0.92 \cdot 0 + 0.15 \cdot 3 = 1.93$.

7 The Approximation of the Sets of Points by Membership Functions

We wish to approximate the point set of pairs $(f_{x_k}(a^c\ crp^c\ bw^c), \mu_{S_1}^{U^{31}}(x_k))$, symbolically denoted by $\left(f_{x_k}, \mu(x_k)\right)$, by a membership function. We thus need to construct a coordinate system, in which the abscissa consists of the characteristic values stated in the ascending order and the ordinate represents the membership degrees for operation possibility coming from the c-means algorithm. We at first consider the construction of the membership functions for diagnosis of "operation possibility" and take then into account the case of "none operation" as the complement of "operation".

Due to references [4–5, 8, 13] we utilize the equation of the truncated π function in the process of approximation. Without discussing the details, available in [4–5, 8, 13], we only mention that we need three characteristic points to start with the approximation. Remembering that $f_{x_k}, k = 1, \ldots, n,$ are placed in the ascending order we choose the first point (f_1, μ_1), which starts the set since $f_1 = \min_{k=1,\ldots,n}(f_{x_k})$.

The point (f_3, μ_3) with $f_3 = \max_{k=1,\ldots,n}(f_{x_k})$ will end the set. The third pair (f_2, μ_2) is characterized by the second coordinate equal to $\max_{k=1,\ldots,n}(\mu(x_k))$.

Example 2
Having collected the data of 25 patients x_k, $k = 1,\ldots,25$, we rearrange them in the ascending order due to f_{x_k}. By using the final membership degrees of fuzzy c-means clustering analysis, we select (f_1, μ_1) = (0.67, 0.56) as the start point, (f_2, μ_2) = (1.26, 0.82) as the point in which the membership function has its peak and (f_3, μ_3) = (5.31, 0.37) as the end point.

In compliance with [4–5, 8, 13] we construct the left part of the π function as

$$\mu_{\text{"operation"}}(f_{x_k}) = \begin{cases} (1)\ \ 0 & \text{for } f_{x_k} < f_1, \\ (2)\ \ \mu_2\left(1 - 2\left(\frac{f_{x_k} - f_2}{f_2 - a}\right)^2\right) & \text{for } f_1 \le f_{x_k} \le f_2, \end{cases} \tag{14}$$

where $\alpha = f_2 - \frac{f_2 - f_1}{\sqrt{\frac{\mu_2 - \mu_1}{2 \cdot \mu_2}}}$ for $\mu_1 \geq \frac{\mu_2}{2}$.

To build the right slope of the π function we employ the equation [4–5, 8, 13]

$$\mu_{"operation"}(f_{x_k}) = \begin{cases} (1)\ \ \mu_2\left(1 - 2\left(\frac{f_{x_k} - f_2}{\gamma - f_2}\right)^2\right) & \text{for } f_2 \leq f_{x_k} < \beta, \\ (2)\ \ \mu_2\left(2\left(\frac{f_{x_k} - \gamma}{\gamma - f_2}\right)^2\right) & \text{for } \beta \leq f_{x_k} \leq f_3, \end{cases} \tag{15}$$

for $\gamma = \frac{f_3 - f_2\sqrt{\frac{\mu_3}{2 \cdot \mu_2}}}{1 - \sqrt{\frac{\mu_3}{2 \cdot \mu_2}}}$ when $\mu_3 < \frac{\mu_2}{2}$ and $\beta = \frac{f_2 + \gamma}{2}$.

In order to demonstrate how functions (14)–(15) work in practice, we will make an approximation of points from Ex. 2.

Example 3

We wish to create the equation of a membership function, which approximates the set of 25 points typical of the previous collection of patients from Table 1 as to the “operation” prognosis. We use the data from Ex. 2 to derive

$$\mu_{"\text{operation}"}(f_{x_k}) = \begin{cases} 0 & \text{for } f_{x_k} < 0.67, \\ 0.82\left(1 - 2\left(\frac{f_{x_k} - 1.26}{1.26 - (-0.22)}\right)^2\right) & \text{for } 0.67 \leq f_{x_k} < 1.26, \\ 0.82 & \text{for } f_{x_k} = 1.26, \\ 0.82\left(1 - 2\left(\frac{f_{x_k} - 1.26}{8.974 - 1.26}\right)^2\right) & \text{for } 1.26 \leq f_{x_k} < 5.117, \\ 0.82\left(2\left(\frac{f_{x_k} - 8.974}{8.974 - 1.26}\right)^2\right) & \text{for } 5.117 \leq f_{x_k} \leq 5.31, \\ 0 & \text{for } f_{x_k} \geq 5.31. \end{cases} \tag{16}$$

The membership function of “none operation” will be computed as a complement of (16) to one.

The membership functions for both diagnoses are depicted in Figure 3.

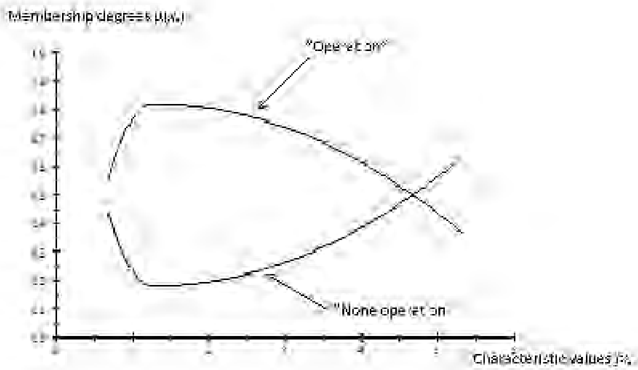

Figure 3. The membership functions for “operation” and “none operation”

Example 4
Evaluate en operation possibility for the patient from Ex. 2. The patient characteristic value has equalled 1.93. Hence, by (16), $\mu_{\text{"operation"}}(1.93) = 0.82\left(1 - 2\left(\frac{1.93-1.26}{8.974-1.26}\right)^2\right)$ = 0.807.

8 Conclusions

In the first part of this study we have adopted fuzzy 2-means clustering analysis to partition a patient data set, containing clinical records of 25 gastric cancer patients, in two fuzzy clusters. These reveal the numerical decision of states “operation” and “none operation” by the values of membership degrees due to the rule: the higher the degree is the more certain decision should be made with respect to the cluster considered.

We notice that the patients’ original clinical marker quantities lead to higher membership degrees in the initial partition matrix when comparing them to the lower values in the final matrix. This phenomenon can be explained by the fact that the decision for an individual patient has been made by the assistance of all data filling the data set. This means that the medical knowledge provided in the form of the collective information, reset numerically, could decide “softer” decisions, which have not deprive the patient of a chance for surgery. We have engaged a new form of experience performed as computerized experience constituting a database.

The obtained results converge to cautious expertise made by physicians. Therefore, fuzzy c-means cluster analysis can be seen as one of the approaches that would assist medical operation diagnosis. The method can be applied for a large number of patients.

In the second part of the study, we have constructed the characteristic function for each patient by determining the code vector of age, *crp* and weight for each individual. We have adopted another useful partition method of the marker spaces to attach to each subspace a code integer among 0 and 4.

The pairs of points with patient characteristics being the first coordinates and the membership degrees from U^{31} constituting the second coordinates create two point sets typical of states "operation" and "none operation". By applying the π truncated approximation method we have derived the equations of continuous membership functions for "operation" and "none operation" states. This allows making prognoses for any patient whose characteristic value is known.

The shape of the curve describing "operation" agrees with the medical knowledge on recommendations of surgery in the cases of gastric cancer patients. The individuals who are very young and underweighted obtain the low characteristic values and low possibility degrees for operation, which confirms the surgeons' criteria. Similarly, patients who are very old, overweighed and having the high *crp*-index are also refused to be operated. Due to Figure 3 these patients have rather low operation possibility degrees assigned. The highest operation possibility degrees belong to elderly patients whose weights are not abnormal and *crp* values do not exceed critical quantities. The result logically agrees with the physician's prediction.

Lastly, we wish to emphasize that the adaptation of membership function families to the purpose of determining the initial membership degrees in the partition matrix has been an efficient tool in the algorithm. The functions, furnished with parameters, allow constructing arbitrary linguistic lists containing many verbal judgments. The mathematical translation of words to numbers has been done systematically without predetermining any casual values. This has improved definitely the convergence speed of the algorithm.

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The papers presented in this Volume 2 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Eleventh International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2012) organized in Warsaw on October 12, 2012 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, Prof. Asen Zlatarov University, Burgas, Bulgaria, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Eleventh International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2012) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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