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SYSTEMS RESEARCH INSTITUTE  
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INTEGRATED RURAL/SPATIAL DEVELOPMENT:  
ELEMENTS OF SYSTEMS ANALYTIC APPROACH

Edited by

Andrzej Straszak and Jan W. Owsinski



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Integrated Rural/Spatial Development: Elements of Systems Analytic Approach,  
edited by Andrzej Straszak and Jan W. Owsinski

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#### IV. RISK ANALYSIS

##### AN INTRODUCTION

Jan W. Owsinski

Rural economic, and therefore also social, life is subject to important risk mainly through its dependence upon agriculture. Agricultural risk appears in all three essential time horizons: in the immediate or short-term one through such dangers as flood, fire, pest or intra-seasonal drought; in the annual or seasonal time horizon through inter-seasonal variations in weather conditions; and finally in the long term through secular climatic changes and possibility of fertility decline. Besides these, typically agricultural, sources of risk, there are also other, which generally can be attributed to almost any economic activity, namely: appearance of competitive producers, price decline etc.

It is this high riskiness of farming that importantly contributed to large rural → urban migrations, in addition to other social and economic factors, indeed. Urban job is seen as less risky, with one essential risk source "attached" to it, i.e. getting and staying on the job.

Risks run by farmers also often may give rise to their attachment to traditional crops and technologies, as well as their general change aversive attitude.

In general, programming techniques reported in the previous chapter, when used in planning and policy making, tend to propose decisions which are optimal, i.e. best, but at the same time sensitive to changes in values of parameters describing resource situation, fertility etc. Hence, such solutions tend to be risky. A way of dealing with this riskiness within the framework of programming formulations mentioned is to analyze a whole range of solutions obtained for various sets of parameters. From among the various solu-

tions those can be chosen which, being quite near to the optimum in terms of evaluation criterion /objective function/ adopted, retain this small distance under various conditions. This approach, leading to definition of "robust", less sensitive plans and policies, can be reformulated into explicit optimization task in which risk, described e.g. as statistical variance of "optimal" criterion values enters objective function.

In practice, "robust" real-life solutions often imply adequate diversification of activities. This may even lead to treatment of farming as a part-time job, performed together with another.

Analysis of "robust" solutions may give information on methods of off setting high risk in economic activities and, indirectly, on ways of improving the social rural processes by introduction of more stability and self-reliance.

FARM LOCATION AND RISK STRATEGIES: OFF-FARM EMPLOYMENT  
AND CROP DIVERSIFICATION

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**Abstract:** A model of the demand for and costs of off-farm employment as a means of protecting against agricultural risk is developed. The demand for off-farm employment increases with distance from markets, and the cost appears to first decline, then increase, with distance from markets. As an insurance device, crop diversification should have a cost advantage relative to off-farm employment nearer markets, a prediction which is in accord with some suggestive evidence from India.

**Keywords:** agriculture, location, risk, employment, landuse.

Agriculture is a particularly risky economic activity. It is sufficiently pervaded with uncertainties that a substantial proportion of farmers' behavior has been designed to deal with risk. Farm policies in developed countries since the 1930's have striven to stabilize, as well as increase on average, farm incomes, thus providing some protection from price fluctuations and from adverse natural events. For less developed countries, a body of literature on the "moral economy of the peasant" attributes much peasant and tribal organization and livelihood behavior as efforts to avoid agricultural risk. Despite what I consider to be overstatement of responses to risks relative to profit opportunities in this literature, risk is great enough in these agricultural communities to make some of those initial statements not totally unreasonable.

The major sources of risk in agriculture, in both developed and less developed countries, are price fluctuations, yield fluctuations, input coordination, and policy change. Societies have dealt with these risks with several tools appropriate for use at the individual level and several at the community, or public, level. For example, individual farmers can diversify their crops, or more generally, their lines of on-farm activity, and decide to take off-farm employment if it is available. Two or more farmers can agree to vary the conditions under which they receive payments

for their separate services. This involves making choices of operating one's own farm or leasing under one or more types of tenure arrangement, specifically, cash or share rental. Farmers in a locale can agree to scatter their holdings among several plots to diversify very localized risks, although this may involve longer term implicit contracts which do not appear to vary substantially in the short run and, observationally, is mixed with effects of inheritance systems. Much larger groups of farmers, preferably over a very large area, can organize farm income insurance policies. Government programs can be instituted for farmers in general, or even more likely, for specific groups of farmers.

Agriculture is also an especially space using activity, and failure to take account of this characteristic can mislead students of agricultural behavior. Transportation costs and spatially systematic variations in land prices will induce different behavior at different locations without having any implications regarding economic efficiency or cultural differences. Agricultural risk has purely spatial (i.e., distance-from-market) components. The risk related activities of farmers involve costs and benefits that vary spatially, so the conduct of risk related activities in agriculture will differ systematically among locations. As an example, studies have found the rent on sharecropped land to be occasionally greater than that on cash rented farms (sometimes compared with owner farmed land), sometimes less and sometimes roughly the same / Johnson, 1950; Rao, 1971 /. These findings could imply the inefficiency of sharecropping as an incentive system, although that system has been shown theoretically to be efficient / Stiglitz, 1974; Reid, 1976 /. But I have shown theoretically, using the Thünen model of land use and pricing, and adapting a model of the demand for, and supply of, farm tenancy developed by Richard Dowell / 1977 /, that the proportion of sharecropped to cash rented farms should increase with distance from markets. Empirically, rents on sharecropped land would be expected to be somewhat lower, on average, than those on cash rented land simply because of locational center of gravity differences in tenure choices. Evidence from the United States from 1880 failed to refute the predictions about locational characteristics of the proportion of sharecropped to cash rented farms / Jones, 1979; 1980a; 1980b /.

In those papers I identified the nexus of farm tenure diversification, crop diversification (a special case of general on-farm activity diversification), and employment diversification as the major set of instruments with which individual farmers could diversify their income sources so as to protect themselves against risk. I developed formally the increase in price and yield risk over distance from markets and the spatial dimensions in the demand and supply of tenancy. In the present paper I focus more on the spatial changes in costs and benefits of off-farm employment by farmers or farm workers. I discuss relationships between off-farm employment and crop diversification as alternative means of farm income insurance, and I present some corroborating evidence from contemporary India on spatial patterns of crop diversification.

## 1. The demand for off-farm employment

### 1.1 The spatial setting

The situation envisaged is the usual one of the Thünen model: the homogeneous plain with the market in the center. Additionally it is presumed that agricultural production is risky, as is production at the market, but agricultural income is riskier than urban income and the returns are less than perfectly correlated. Agriculture is small relative to the urban sector of this region so that the wage facing labor in agriculture is determined by the demand for and supply of labor in the urban sector, and the rent of land in agriculture is also determined by prices set in the larger market. The rent at any location  $t$ ,  $f(t)$ , is composed of several elements. First is a reservation price that can be earned in a use alternative to agriculture,  $S(t^*)$ , below which the price of land in agricultural use cannot fall; if the alternative use brings a zero price, the location at which this is the total rent to land,  $t^*$ , is the edge of all land use, and the location at which the value of land is fully exhausted by the cost of transporting the output produced on it. At locations  $t$  closer to the market than  $t^*$ , the difference in transportation costs on the output are incorporated into rents. In addition to the increments to rents at locations nearer the market accruing from lesser transport costs, factor substitution increases the ratio of other inputs to land as the market is approached: reduced transport costs on land increase the relative price of land to other



inputs closer to the market, and the higher ratio of those other inputs to land increases the marginal product of land. Figure 1 shows these components of land rent, and expression (1) presents the components analytically.

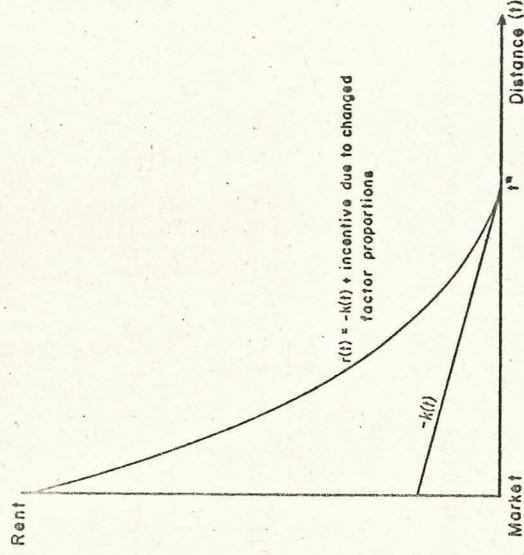


Figure 1: Components of land rents with  $S(t^*)=0$ .

$$r(t) = S(t^*) + \int_{t^*}^t k(t) dt + \int_{t^*}^t \frac{\partial r}{\partial n} \frac{\partial n}{\partial k} \frac{\partial k}{\partial t} dt, \quad (1)$$

where  $n \equiv N(t)/L(t)$ ,  $t^* > t$ , and  $\partial r / \partial n > 0$ ,  $\partial n / \partial k > 0$ ,  $\partial k / \partial t > 0$ . The first expression on the right hand side of (1),  $S(t^*)$ , is the reservation price of land at the edge of cultivation; it may be zero. The second expression is the change in the farmgate value of output due to transportation costs from the edge of cultivation to location  $t$ . The third term is the factor substitution which produces the curved line  $r(t)$  in Figure 1; as  $t$  is moved inward from  $t^*$ , transport costs fall, inducing an increase in the labor (or "outer inputs")/land ratio via a relative price mechanism, and eventually inducing an increase in the rent via the law of variable proportions as the labor/land ratio increases. Thus is the change in the quantity of output per unit of land changed as location changes. This is the familiar incorporation of transportation cost savings into land values when substitution among inputs is possible.

Two expressions for the wage rate must be developed. The demand for labor at location  $t$  will yield a wage rate  $w(t)$  which is the marginal physical product of labor at location  $t$ ,  $f_N(t)$ , valued at the farmgate price of output,  $P(t) = P_0 - \int_0^t k(t) dt$ , where  $P_0$  is the price of the output at the market:

$$w(t) = f_N(t)P(t) = \left[ f_N(0) + \int_0^t \frac{\partial f_N}{\partial n} \frac{\partial n}{\partial k} dt \right] \left[ P_0 - \int_0^t k(t) dt \right]$$

or

$$w(t) = P_0 f_N(0) - f_N(0) \int_0^t k(t) dt + P_0 \int_0^t \frac{\partial f_N}{\partial n} \frac{\partial n}{\partial k} dt - \int_0^t k(t) dt \int_0^t \frac{\partial f_N}{\partial n} \frac{\partial n}{\partial k} dt, \quad (2)$$

where  $\partial f_N / \partial n < 0$ .

The first right hand term of (2) is the wage at the market,  $w_0$ ; from it is subtracted the marginal physical product of labor at the market times the change in the farmgate price of output. The third term is the market price of the product,  $P_0$ , times the change in the marginal physical product of labor as distance increases, transport costs rise and factor proportions change. The final term is the product of the changes in farmgate output price and marginal physical product of labor. This sum, at any location  $t$ , is the maximum amount an employer will be willing to pay to hire workers. The least a locally resident worker will accept is the amount he could earn at the market minus his commuting costs, which represents the supply curve of labor facing an employer at  $t$ :

$$w(t) = w_0 - \frac{\int_0^t c(t) dt}{N^{off}(t)}, \quad (3)$$

where  $c(t)$  is the per round trip commuting cost per worker, and  $N^{off}(t)$  is the amount of farm labor (number of hours) supplied to off-farm employment at the market, per commuting trip. The more labor hours supplied

to the market per trip, the lower the commuting cost for the total wages brought home but the higher the farm wage cost. In equilibrium, the worker must be indifferent between working locally (on the farm) and commuting to the market. Thus, equating expressions (2) and (3) in (4), it is seen that the only choice variable available to the farm worker,  $N^{off}(t)$ , must be adjusted to yield a marginal physical product of labor at  $t$  which bears a specific relationship to the market wage, commuting costs and the farmgate price of local output, which includes the transport rate on that product:

$$w(t) = P(t) f_N(N^{off}, t) = w_0 - \frac{c(t)}{N^{off}(t)}, \quad (4)$$

$$\text{where } \frac{dw(t)}{dN^{off}} = -P(t) f_{NN}(N^{off}, t) = \frac{c(t)}{[N^{off}(t)]^2} > 0.$$

To summarize the setting of the model, agricultural workers live on a homogeneous plain on which rents fall more rapidly than transport costs increase over distance. Wages on the farms are related to the farmgate price of local outputs and fall with distance from market but have a floor put under their fall by the opportunity to earn wages at the market.

### 1.2 The demand for off-farm employment

It is presumed that the arbitrage of farm wages into line with off-farm wages has been accomplished but that incomes are risky, riskier in fact than nonfarm incomes. Farmers may desire to undertake some off-farm employment as a means of reducing the riskiness of their farm incomes. Beginning with some portfolio of assets which can be described as land, farm labor, and off-farm labor, they can rearrange their portfolios to include more off-farm labor and reach some preferred combination of risk and expected return, as in Figure 2. The off-farm wage,  $w_0$ , is drawn as a risk free return, although it need not be considered so. From a combination of assets which yields a risk-return combination of  $a$  in Figure 2, the farmer can put  $0c/0b$  of his total assets into off-farm employment and  $cb/0b$  into a higher return-higher risk combination of farm assets--land and on-farm labor--thus reaching the higher level of satisfaction at point  $c$ . The rate at which farmers are to exchange income for lower

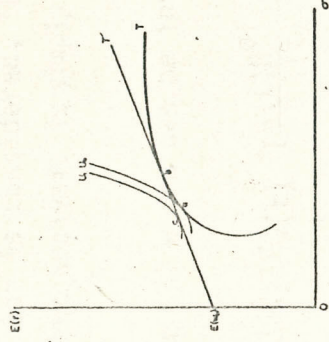


Figure 2: Diversification of farm earning assets by taking off-farm employment.

risk is the slope of the efficient frontier T at a tangency with an indifference curve at point c, which is simply the slope of the line  $E(w_0)T'$ . The price per unit of risk exchanged that a farmer would be willing to pay for the difference in risks associated with two different portfolios can be represented by the slope of that line, which will be called A.

Supposing the only goal of off-farm employment to be insurance, a demand function for such jobs would be of the form

$$D[N^{\text{off}}(t)] = A(\sigma_F(t) - \sigma_N), \tag{5}$$

where A is the marginal price of risk,  $\sigma_F(t)$  is the standard deviation of farm income at location t and  $\sigma_N$  is the standard deviation of nonfarm income, or strictly speaking, of income from the market portfolio of assets which will include farm as well as nonfarm assets. The variance of farm income is

$$\begin{aligned} \sigma^2_F(t) = & x_{N_1}^2 \sigma_w^2(t) + x_{N_2}^2 \sigma_{w_0}^2 + x_L^2 \sigma_r^2(t) + 2(x_{N_1} x_{N_2} \sigma_w(t) w_0 + x_{N_1} x_L \sigma_w(t) r(t) \\ & + x_{N_2} x_L \sigma_{w_0} r(t)), \end{aligned} \tag{6}$$

where  $N_1$  represents on-farm employment (or  $N^{\text{on}}$ ) and  $N_2$  off-farm employment ( $N^{\text{off}}$ ). The  $x_i$  are shares of the subscripted variable in farm income, with  $\sum x_i = 1$ . For example,  $x_{N_1}$ , the share of total income from on-farm labor is defined as  $w(t)N^{\text{on}}(t) / [w(t)N^{\text{on}}(t) + w(t)N^{\text{off}}(t) + r(t)L(t)]$ . The  $x_i$  as well as the  $\sigma_F^2(t)$  are functions of location, t;  $\sigma_w(t)w_0$  is the

covariance between the farm wage rate at  $t$  and the market wage rate. Since only off-farm labor is considered, the variance of nonfarm income relevant for the present analysis is simply the variance of the market wage,  $\sigma_{w_0}^2$ .

An expression for (5) which can be investigated for spatial influences

$$\begin{aligned}
 \text{is} \\
 D[N^{\text{off}}(t)] &= A \left\{ \left[ \frac{w(t)N^{\text{on}}(t)}{Y(t)} \right]^2 \sigma_{w(t)}^2 + \left[ \frac{w(t)N^{\text{off}}(t)}{Y(t)} \right]^2 \sigma_{w_0}^2 + \left[ \frac{r(t)L(t)}{Y(t)} \right]^2 \sigma_r^2 \right. \\
 &+ \frac{2w(t)N^{\text{on}}(t)w(t)N^{\text{off}}(t)}{[Y(t)]^2} \sigma_{w(t)w_0} + \frac{2w(t)N^{\text{on}}(t)r(t)L(t)}{[Y(t)]^2} \sigma_{w(t)r(t)} \\
 &\left. + \frac{2w(t)N^{\text{off}}(t)r(t)L(t)}{[Y(t)]^2} \sigma_{w_0r(t)} \right\}^{\frac{1}{2}} - \sigma_{w_0}. \quad (7)
 \end{aligned}$$

Although the motivation for taking off-farm employment is assumed to be purely risk reduction, it is of interest that a higher urban wage will increase the demand for such employment:

$$\begin{aligned}
 \frac{dD(N^{\text{off}})}{dw_0} &= \frac{A}{Y(t)\sigma_F(t)} \left\{ [x_{N_1} \sigma_{w(t)}^2 + x_{N_2} \sigma_{w(t)w_0} + x_L \sigma_{w(t)r(t)}] \frac{\partial x_{N_1}}{\partial w_0} \right. \\
 &+ [x_{N_2} \sigma_{w_0}^2 + x_{N_1} \sigma_{w(t)w_0} + x_L \sigma_{w_0r(t)}] \frac{\partial x_N}{\partial w} + [x_L \sigma_r^2(t) + x_{N_1} \sigma_{w(t)r(t)}] \\
 &\left. + x_{N_1} \sigma_{w_0r(t)} \right] \frac{\partial x_L}{\partial w_0} + \frac{1}{2} \sum_{i=1}^3 \sum_{j=1}^3 x_i x_j \frac{\partial \sigma_{ij}}{\partial w_0} \} > 0, \quad (8)
 \end{aligned}$$

$$\text{where } \frac{\partial x_{N_1}}{\partial w_0} = \frac{N^{\text{on}} - x_{N_1}}{Y(t)} > 0, \quad (9)$$

$$\frac{\partial x_N}{\partial w_0} = \frac{N^{\text{off}} - x_{N_2}}{Y(t)} > 0, \quad (10)$$

$$\text{and } \frac{\partial x_L}{\partial w_0} = \frac{-x_L}{Y(t)} < 0. \quad (11)$$

Notice that with a large number of assets, the effects of covariance terms could quickly outweigh the effects of the variance terms. On the other hand, for landless farmers who predominantly work on the farm, the sizes

of the variances of the farm wage and the covariance of the on-and off-farm wages will most influence the responsiveness of the demand for off-farm employment to the urban wage. A negative  $\sigma_{w(t)w_0}$ , which is not unlikely, will reduce the responsiveness of the demand to the urban wage.

The influence of location relative to market on  $D[N^{\text{off}}(t)]$  has a form similar to (8), where the  $\partial x_i / \partial t$  are as below:

$$\frac{\partial x_{N_1}}{\partial t} = \frac{1}{Y(t)} \{ (N^{\text{on}} - x_{N_1}) \frac{\partial w(t)}{\partial t} + w(t) \frac{\partial N^{\text{on}}}{\partial t} - x_{N_1} L(t) \frac{\partial r}{\partial t} \} \geq 0, \quad (12)$$

$$\frac{\partial x_{N_2}}{\partial t} = \frac{1}{Y(t)} \{ (N^{\text{off}} - x_{N_2}) \frac{\partial w(t)}{\partial t} + w(t) \frac{\partial N^{\text{off}}}{\partial t} - x_{N_2} L(t) \frac{\partial n}{\partial t} \} \leq 0, \quad (13)$$

$$\frac{\partial x_L}{\partial t} = \frac{x_L(1-x_L)}{r(t)} \frac{\partial r}{\partial t} < 0. \quad (14)$$

The first terms in (12) and (13) are negative, and the third terms are positive; the middle terms are negative in (12) and positive in (13), giving an unambiguously positive change in off-farm labor share with greater distance from market, but the possibility of employment switches allows the possibility that  $x_{N_2}$  may increase at the expense of  $x_{N_1}$  as well as land's share.

It is worthwhile examining at this point expressions for  $\partial w(t) / \partial t$  and  $\partial r(t) / \partial t$ . Recalling that expression (2) is used as the wage according to which on-farm labor is demanded,

$$\begin{aligned} \frac{\partial w(t)}{\partial t} \Big|_{N^{\text{on}}} &= -f_N(0)k(t) + P_0 \frac{\partial f_N}{\partial n} \frac{\partial n}{\partial k} \frac{\partial k}{\partial t} - k(t) \int_0^t \frac{\partial f_N}{\partial n} \frac{\partial n}{\partial k} \frac{\partial k}{\partial t} dt \\ &- \frac{\partial f_N}{\partial n} \frac{\partial n}{\partial k} \frac{\partial k}{\partial t} \int_0^t k(t) dt < 0. \end{aligned} \quad (15)$$

Although the wage falls over distance from market, there is a positive component to wage rate adjustments, shown in the second term of (15), which is the increasing marginal physical product of labor as the land/labor ratio increases. There is, analogously, a positive component to the change in the floor put on wages by commuting costs, the wage used to evaluate off-farm employment opportunities:

$$\frac{\partial w(t)}{\partial t} \Big|_{N^{\text{off}}} = \frac{-c(t)}{N^{\text{off}}(t)} + \frac{\int_0^t c(t) dt}{[N^{\text{off}}(t)]^2} \frac{\partial N^{\text{off}}(t)}{\partial t} < 0. \quad (16)$$

The first term is the direct increase in per labor unit commuting cost; the second is the decrease in per unit costs attributable to the additional units of labor supplied off-farm. Note also that  $\partial N^{\text{off}}(t)/\partial t > 0$  entails a fall in  $n$  and thus an increase in  $f'_N(t)$ , which increases the wage rate at a constant output price as in the second term of (15), although the farmgate output price falls in the last term of (15).

The rental rate decreases unambiguously with distance:

$$\frac{\partial r}{\partial t} = -k(t) - \frac{\partial r}{\partial n} \frac{\partial n}{\partial k} \frac{\partial k}{\partial t} < 0, \quad (17)$$

in which the first term is the direct reduction of rent by the reduction in farmgate prices by the amount of the additional transportation cost, and the second term is the reduction in rent due to changing factor proportions.

The total change in the demand for off-farm employment as distance from market increases, ignoring  $\partial \sigma_{ij}/\partial t$  terms which have ambiguous signs, is:

$$\begin{aligned} \frac{d[D(N^{\text{off}}(t))]}{dt} &= \frac{A}{Y(t)\sigma_F(t)} \{ [x_{N_1} \sigma_w^2(t) + x_{N_2} \sigma_w(t)w_0 + x_L \sigma_w(t)w_0] \frac{\partial x_{N_1}}{\partial t} \\ &+ [x_{N_2} \sigma_w^2 + x_{N_1} \sigma_w(t)w_0 + x_L \sigma_w(t)w_0] \frac{\partial x_{N_2}}{\partial t} + [x_L \sigma_r^2(t) \\ &+ x_{N_1} \sigma_w(t)w_0 + x_{N_2} \sigma_w(t)w_0] \frac{\partial x_L}{\partial t} > 0, \end{aligned} \quad (18)$$

where the  $\partial w(t)/\partial t$  and  $\partial r(t)/\partial t$  are incorporated in the  $\partial x_i/\partial t$ .

## 2. The cost of off-farm employment

To reduce the riskiness of his income by working off-farm, a farmer incurs some definite costs. There is obviously the commuting cost,  $c(t)$ . There is also the foregone wage which could have been earned on the farm,  $w(t)$ . Additionally there is the rental income sacrificed by reducing the supplies of cooperating labor. Explicitly, these costs are:

$$C[N^{\text{off}}(t)] = c(t) + w(t)N^{\text{off}}(t) + \frac{r(t)N^{\text{off}}(t)}{n(t)}, \quad (19)$$

where  $n$  is noted explicitly as a function of  $t$ . The total cost of supplying off-farm labor is the sum of the commuting cost, the local wage rate times the amount of time spent working off-farm, and the rental rate times the amount of land taken out of production per unit of labor taken off the farm. This last term warrants justification. Since the farm is a price taker, it is given the local wage rate by market wage and price conditions, given transportation costs, and production technology. If labor is taken off the farm, the land/labor ratio would rise, producing a larger marginal physical product of labor. At given prices, the local wage rate would rise, but it cannot without exogenous changes. So some land is left unutilized as part of the cost of stabilizing incomes. The analysis avoids using an assumption of surplus agricultural labor.

The cost of stabilizing incomes in this manner rises as more stabilization is undertaken:

$$\frac{d [C[N^{\text{off}}(t)]]}{dN^{\text{off}}(t)} = w(t) + \frac{r(t)}{n} > 0. \quad (20)$$

The second term is evaluated at a constant  $n$  for the reason just noted and is consequently positive, representing a movement along a ray from the origin rather than a move along an isoquant.

Off-farm employment may be either more or less costly further from markets:

$$\begin{aligned} \frac{d [C[N^{\text{off}}(t)]]}{dt} &= \frac{\partial c(t)}{\partial t} + [w(t) + \frac{r(t)}{n(t)}] \frac{\partial N^{\text{off}}(t)}{\partial t} + N^{\text{off}}(t) \frac{\partial w(t)}{\partial t} \\ &+ \frac{N^{\text{off}}(t)}{n(t)} \frac{\partial r(t)}{\partial t} - \frac{r(t)N^{\text{off}}(t)}{[n(t)]^2} \frac{\partial n(t)}{\partial t} \geq < 0. \end{aligned} \quad (21)$$

The first, second, and last terms are positive, representing: (1) additional commuting costs over distance; (2) wage costs and rent costs from land taken out of production, evaluated at constant prices and constant  $n$ , for the additional units of off-farm labor supplied; and the larger quantities of land released from production per unit of labor taken off the farm as distance from market increases. The third and fourth terms represent the reduction in farm wage and rent costs as those prices fall with distance. Higher commuting costs and larger elasticities of substitution in farm production will increase the costs of taking off-farm employment as distance



from market increases, but more costly transportation for farm output [ $k(t)$ , in  $r(t)$  and  $w(t)$ ] and more elastic substitution possibilities in agriculture will increase the size of the forces tending to reduce  $d C[N^{off}(t)]/dt$ . Influences on  $c(t)$  and  $k(t)$  are probably closely correlated, and the elasticity of substitution in production will increase the absolute values of  $\partial N^{off}/\partial t$ ,  $\partial w(t)/\partial t$ ,  $\partial r(t)/\partial t$  and  $\partial n(t)/\partial t$ . However,  $|\partial r(t)/\partial t| > |\partial n(t)/\partial t|$ , with the difference declining to zero at  $t^*$ --see Figure 1--so it is quite plausible that  $d C[N^{off}(t)]/dt$  is first negative and then becomes positive at greater distances from market as the difference between  $\partial r(t)/\partial t$  and  $\partial n(t)/\partial t$  eventually becomes outweighed by  $\partial c/\partial t$ .

### 3. Evaluating the spatial incidence of off-farm employment.

Without equating expressions (7) and (19) to solve for an equilibrium  $N^{off}(t)$ , some shifts in the  $N^{off}(t)$  at demand-supply equilibria can be inferred from the directions of movement of the demand and cost functions separately. Since  $d D[N^{off}(t)]/dt > 0$  and  $d C[N^{off}(t)]/t$  appears to be negative (to shift outward or down on a supply-demand diagram) at nearer and intermediate distances from market and positive at remote locations, a plausible spatial pattern of  $N^{off}(t)$  would first increase and then decrease.

It may be useful to consider alternative risk spreading devices at this point. Crop diversification was mentioned casually early in the paper as such a major alternative; it would reduce  $\sigma_F^2(t)$  through portfolio effects, which would tend to reduce the demand for off-farm employment. It would also be likely to more fully utilize labor on farms throughout the crop year, as well as use land more completely--through multiple cropping, for instance. Since  $N^{off}(t)$  appears to be more costly closer to markets, crop diversification, which appears to have a comparative advantage at locations more intensively utilized, should be a stronger competitor to off-farm employment closer to markets.

Some evidence from India lends support to the pattern of relatively more crop diversification closer to markets and thus indirectly to the thesis that off-farm employment is a more competitive risk spreading

device farther from markets. Presented in Table 1 are several regressions of the number of crops per ten thousand hectares of gross cropped area on some measures of distance from markets, average local town/city size, rainfall, prices and yields, and employment opportunities. The groups of districts in each regression are those districts out of a 25% sample of districts from thirteen states in India in which are grown particular crops. Specifically, the first regression uses sugar cane growing districts, the second districts in which bajra is grown, the third is for rice growing districts, the fourth for cotton districts, and the fifth, groundnuts. The distance from markets measures,  $D_i$ , are population potentials of towns and cities of 20,000 or more population in 1961, in three concentric, 50 kilometer rings around the centroid of each sample district; a larger value of  $D_i$  implies greater proximity to larger markets. The variable ATS is average town and city size within the district; RAIN is average yearly rainfall;  $P_i$  is the average farmgate price of the crop, e.g. rice in the rice districts, in each district over the period 1954/5-1970/1;  $CVP_i$  is the coefficient of variation of the price over the period;  $L_i$  is the percent of gross cropped hectareage devoted to crop  $i$ ;  $W_{uh}$  is the percent of the district labor force engaged in urban household industries, a measure of off-farm employment opportunities, not necessarily of actual off-farm employment;  $W_{unh}$  is the percent of the labor force in urban nonhousehold industry, again an employment opportunity surrogate; and  $W_{rh}$  is rural household industry employment.

In each group of districts there are positive and significant elasticities of the dependent variable with respect to the access to market variables ( $D_i$  or ATS); only in the bajra districts is there a negative coefficient on ATS, significant at .05, but in conjunction with a positive elasticity with respect to the population potential in the 50-100 kilometer zone. The interpretation of the  $W_i$  elasticities is speculative, but suggestive. In the only districts where these variables have negative coefficients, crop diversification is unaffected by population potential in the nearest zone, but there is a positive effect in the second ring. In the sugarcane districts, there is a positive  $D_i$  coefficient in the first zone and a negative one in the third zone,

Table 1: Regressions of the number of crops per ten thousand gross cropped hectares in elasticities, for groups of districts.\*

	Sugarcane	Bajra	Rice	Cotton	Groundnuts
D <sub>0-50</sub>	.2623 <sup>1</sup>		.0526 <sup>4</sup>		.1629 <sup>2</sup>
D <sub>50-100</sub>		.0737 <sup>3</sup>			
D <sub>100-150</sub>	-.2624 <sup>1</sup>			.3241 <sup>3</sup>	
ATS		-.5307 <sup>3</sup>			
RAIN		.2815 <sup>5</sup>	.5172 <sup>1</sup>		-.3626 <sup>4</sup>
P <sub>i</sub>	-1.5239 <sup>1</sup>				-2.2796 <sup>1</sup>
CVP <sub>i</sub>	1.5922 <sup>1</sup>		-.4972 <sup>1</sup>		
L <sub>i</sub>			-.2032 <sup>1</sup>		
W <sub>rh</sub>					-.5117 <sup>1</sup>
W <sub>uh</sub>	.2110 <sup>2</sup>				
W <sub>unh</sub>		-.1152 <sup>4</sup>			
R <sup>2</sup>	.8642	.3660	.5095	.2534	.5334
F(·)	(5,12)=15.27 <sup>1</sup>	(4,15)=2.17	(4,22)=5.71 <sup>1</sup>	(1,12)=4.07 <sup>4</sup>	(4,17)=4.86 <sup>1</sup>

\* from Jones and Kaul / 1981, Table 9/

1 significant at .01

2 significant at .025

3 significant at .05

4 significant at .10

5 |t| ≥ 1.20

with a positive influence of urban household industry. Could this represent crop diversification in the zones nearest markets, crop diversification crowded out in the third zone as a risk spreading device by off-farm employment going into part time work in urban household industry? The large negative coefficient on the average town size variable in the bajra districts coexists with a negative coefficient on the urban nonhousehold industry employment variable; both a larger local average town size and larger proportions of the labor force in urban non-household industries inhibit crop diversification, but larger population potentials in the second zone encourage

crop diversification. Could off-farm employment be crowding out crop diversification in near districts but not consistently enough to actually produce a negative elasticity on  $D_{0-50}$ ? The remaining variables can be considered controls for the present purposes.

These regressions are presented not as confirmation or rigorous attempts at disproof, but rather as suggestive of patterns of crop diversification and as even more remotely suggestive of coordinated patterns of off-farm employment. The interpretations offered are consistent with predictions regarding the spatially differing proclivities to take, and costs of, off-farm employment derived from the model and with the less formally developed reasoning regarding the locational competition of crop diversification and off-farm employment.

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APPENDIX: EXPLANATION OF SIGNIFICANCE OF SOME VARIABLES

- $D_{0-50}$  : Urban population potential of district k in the zone from the center of k to a ring 50 kilometers distant. It is defined as  $\sum_j P_j / d_{kj}^2$ , where  $P_j$  is the population of city j and  $d_{kj}$  is the distance from the centre of district k to city j.
- $D_{50-100}$  : Urban population potential in the zone from 50 to 100 kilometers from the center of district k.
- $D_{100-150}$  : Urban population potential in the zone from 100 to 150 kilometers from the center of district k.
- ATS : Average town and city size in district k.
- PAIN : Average yearly rainfall at reporting stations in district k.
- $P_i$  : Average farmgate price of crop i, i=sugarcane, bajra, rice, cotton, groundnuts, in district k.
- $CVP_i$  : Coefficient of variation of  $P_i$  in district k.
- $L_i$  : Percentage of gross cropped land devoted to crop i in district k.
- $W_{rh}$  : Percentage of the labor force in district k employed in rural household industry in 1961; used as a surrogate for employment opportunities in rural household in district k.
- $W_{uh}$  : Percentage of the labor force in district k employed in urban household industries in 1961; used as a surrogate for employment opportunities in these industries.
- $W_{unh}$  : Percentage of the labor force in district k employed in urban non-household industries in 1961; used as a surrogate for employment opportunities in these industries.

RISK IN CROP PRODUCTION PLANNING  
OVER A REGION

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1. Introduction

Planning of crop production both in individual enterprises and over larger territories is always subject to high production risk. Biological and spatial nature of crop raising, influence of changing soil-and-weather conditions, limited possibility of human intervention into production processes as well as lacunae in agrotechnical knowledge cause together that crop production level is a random variable. Hence, actual crop production levels may importantly differ from the ones anticipated or planned.

Agricultural practice indicates that in particular regions certain plants display a higher crop yield stability, i.e. incur less production risk, while other display lower stability, i.e. incur greater risk. Hence, risk in the whole of production over this region could be diminished by an appropriate choice of production structure.

Assuming that the production risk is a reflection of crop yield variability with regard to multiannual average or temporal trend line - it can be expressed, for a single plant, with a variance of a random vector. Knowing of distribution of these variables would enable evaluation of risk for a production structure over a relatively homogenous object.

Past analyses /e.g. Dowgiallo, 1971/ indicate that, for

instance, for north-western Poland least reliable are fodder and some industrial crops /rapeseed, sugar beets/. Variability in feedstuff production and especially its untimely shortages impinge negatively on livestock breeding, sometime over a longer period of time.

Identification and measurement of the production risk allows undertaking appropriate countermeasures. Main activity here, in the case described above, consists in creation of feedstuff reserves adequate to crop variability in the area considered.

This particular paper refers to a study of agricultural risk and risk countermeasures in the north-western "Szczecin" province of Poland, in the state farming enterprises. State farms of the area own approximately 60% of agricultural land /the rest being owned by private farmers and cooperatives/. It should be emphasized that a shift in planning and decision making process, making an enterprise a more independent and - at the same time - more self-responsible unit, introduces more room for consideration of risk sources and magnitudes, and of decreasing risk. According to Heady, 1961, risk identified and accounted for in planning is no longer a source of threat.

## 2. Goal of the study

The concept of risk conscious crop production planning, being the object of the study here reported amounts to a procedure consisting of:

- definition of crop yield distributions and their parameters for main crops in the state farms of the region, as well as estimation of the yield covariance matrix S,
- estimation of possible one-year crop deviations from expected - planned - values for the whole crop production and for the individual plants and plant groups,
- determination of the optimal crop production structure for the object considered assuming risk minimisation.

The planning procedure obtained, as well as specific results may be of use for individual farming enterprises, larger agricultural aggregates, or for local authorities carrying out an active agricultural policy.

### 3. Method applied

Let  $Y_i$  be a random variable describing yields of the  $i$ -th plant,  $i=1, \dots, r$ , in an agriculturally homogenous object, over a certain period of time. Distribution function of  $Y_i$  is unknown. On the basis of yield data from  $n_i$  enterprises in which this plant was raised over  $p$  years hypothesis could be verified postulating normal yield distribution and parameters of this distribution could be estimated. The  $\chi^2$  test might be used for hypothesis verification.

Assume now that the verification had been performed with no grounds for rejecting the hypothesis for  $r$  plants raised in the region. Yields can be treated as normally distributed, then. Observations of yield allow estimation of their covariance matrix  $S$ , valid for a given period of time. Hence, for the crop structure planned for the object considered,  $x = (x_1, \dots, x_r)^T$ , where  $x_i$  is the area of land under the  $i$ -th crop, and known matrix  $S$ , variance of the production plan can be estimated through

$$\sigma^2 = x^T S x$$

where  $\sigma^2$  would be expressed in some unified measure, based upon monetary or protein units. Variance can be analogously defined for a group of plants, e.g. cereals.

Expected value of the production plan defined through  $x$  is expressed as

$$m = c^T x$$

where  $c = (c_1, \dots, c_r)$  is the vector of expected values of random variables  $Y_1, \dots, Y_r$ . The same sort of units as for  $\sigma^2$  can be used.

Crop production actually realised is a random variable  $Y$  having normal distribution  $N(m, \sigma)$ . Knowing properties of this distribution function one can relatively precisely determine plan realisation deviation probabilities.

In order to minimise the crop production plan variance for the object considered, a quadratic programming (QP) problem



can be analysed having the following form:

$$x^T S x \rightarrow \min$$

$$A x \leq b$$

$$x \geq 0$$

where:  $x = (x_1, \dots, x_r)^T$  is the decision variables vector, describing areas under particular crops,

A : is the coefficient matrix, whose elements specify balances of: labor force, feedstuffs - both voluminous and protein-rich, arable land, grassland, processing capacities etc.,

b : right hand side vector,  $b = (b_1, \dots, b_m)^T$ , whose components describe: feedstuff demand, arable land available, crop area limitations resulting from agrotechnical considerations, labor force available etc.

Solution to this QP problem gives a vector  $x^0 = (x_1^0, \dots, x_r^0)^T$ , constituting a least-variance crop production structure plan, satisfying resource balances and other constraints, defining feasible structures.

Certainly, minimisation of plan variance does not lead to complete elimination of risk. That is why on higher organisational level some reserves should be created for the case of poor harvest, primarily reserves of cereals as the basic component of protein-rich feedstuffs. Reserves of voluminous feedstuffs, because of their technical characteristics, should be maintained in particular economic units /enterprises, farms/.

Knowing demand for cereals /generated by all enterprises/ in a region, and planned crop structure, production variance  $\sigma_c^2$  of cereals can be calculated, and then appropriate reserve, according to the formula:

$$R_c = u_\alpha \cdot \sigma_c$$

where  $u_{\alpha}$  is from tables of  $N(0,1)$  with  $\alpha$  being the risk level coefficient, indicating the risk that the reserve thus created may turn out insufficient.

Other crop reserves can be calculated in the similar simple way.

#### 4. Empirical study

The study was performed for 34 state farming enterprises in the Szczecin province, the data accounting for the period 1973-1978. Test  $\chi^2$  was used to verify whether 14 main crops /fall and spring wheat, rye, fall and spring barley, oats, sugar beets, potatoes, rapeseed, maize, alfalfa, pulse mixtures, clover and meadows/ could be normally distributed. At the  $\alpha=0.01$  level it was conjectured that no grounds existed for rejecting this hypothesis. More detailed results are contained in Dowgiałło et al., 1981.

On the basis of these results expected value and variance of an annual crop production plan were determined. These parameters were determined both for the whole of structure and for the particular plant groups /cereals, root crops, forage crops/. The results were compared with actual plan realisation, both in terms of protein-like units and areas under particular crops. A rough overview of the comparison is given in Table 1.

It can easily be seen that the greatest divergences occur in root crops, both with regard to crop yields and areas under these crops. 1980 was very bad for root crops, so that probability of thus low yields falls below 0.01. This situation was reflected disadvantageously upon the whole of agricultural production, especially on livestock breeding. Adequate reserves being prepared in individual units, low yield would not influence that much the final economic effects. It is of great importance therefore to determine volumes of necessary reserves, in particular of feedstuffs, to be kept in the whole region and in individual enterprises. According to previous considerations reserves should depend upon planned crop structure /see Tables 1 and 2/ and risk level accepted. With the level  $\alpha=0.1$  reserves should be in individual plant groups, in terms of grain units,

their yields in grain units, in 1980.

Table 1. Planned and actual crop structure in terms of areas under particular crops and

Crops	Planned			Actual		
	Area in hectares	% of arab-land	Yield in grain units	Area in hectares	% of arab-land	Yield in grain units
1. Fall wheat	12,500	6.3	34.2	11,925	6.1	20.3
2. Spring wheat	5,000	2.5	31.8	2,925	1.5	28.7
3. Rye	27,500	13.9	31.7	30,229	15.5	26.7
4. Fall barley	8,700	4.4	34.0	9,251	4.7	30.3
5. Spring barley	25,100	12.7	31.4	17,959	9.2	28.0
6. Oats	24,200	12.2	29.7	20,139	10.3	26.9
7. Maize	17,300	8.8	48.1	19,103	9.8	36.1
8. Potatoes	12,700	6.4	43.5	11,903	6.1	28.2
9. Sugar beets	8,500	4.3	72.2	6,593	3.5	55.0
10. Rapeseed	20,700	10.5	45.2	19,655	10.0	42.0
11. Pulse mixtures	2,700	1.1	28.2	16,338	8.3	25.6
12. Red clover and grasses	6,100	3.1	41.5	5,746	2.9	38.4
13. Alfalfa	19,100	4.6	47.2	8,620	4.4	46.0
14. Other	17,500	8.9	-	14,375	7.3	-
Total cropped land	197,000	100.0	X	194,750	100.0	X
Permanent grassland	49,000	100.0	43.8	47,500	100.0	37.7

Table 2. Characteristics of crop production plan of the object considered.

Crops	Expected value	Standard deviation	Plan variability coefficient, (%)	Actual production	Production expected relative to
	(10 <sup>3</sup> grain units)	(10 <sup>3</sup> grain units)	(%)	(10 <sup>3</sup> grain units)	value (%)
Total	8,988	1,148	12.8	7,604	84.6
Cereals	3,261	498	15.2	1,565	78.6
Root crops	1,166	192	16.4	698	59.9
Forage crops	3,625	602	16.6	3,515	96.9

as follows:

- of cereals:  $R_c = 1.28 \times 498 = 637$ , i.e. 19.5% of total demand,
- of voluminous fodder:  $R_f = 1.28 \times 602 = 771$ , i.e. 21% of total demand,
- of root crops:  $R_r = 1.28 \times 192 = 246$ , i.e. 21% of total demand.

Comparison of theoretically calculated reserves with actual production results of 1980 indicated that such reserves would ensure satisfaction of demand on cereals in 90%, on voluminous fodder in more than 100%, while root crop forage demand would still not be satisfied to an important degree. The latter is obviously due to exceptionally low yield in 1980.

Minimal variance solution of crop structure planning problem gives higher than actual preference to fall crops and oats among cereals, pulse mixtures among forage crops, while in root crops the structure proposed does not diverge importantly from the actual one.

The above results were questioned from the point of view of agrotechnical practice, and therefore they were treated merely as first approximation.

#### 5. Concluding remarks

The method of risk-conscious planning of crop structures outlined in the present paper is founded upon essential simplifications of agricultural reality. The most important aspect which has to be added to the procedure shown consists in delimitation of subregions /subsystems/ homogenous with regard to crop yield variability, which would add reliability to results achieved. This, in turn, requires, however, an important increase of the number of statistical data items used in analysis.

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