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ИНСТИТУТ СИСТЕМНЫХ ИССЛЕДОВАНИЙ ПОЛЬСКОЙ АКАДЕМИИ НАУК

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INTEGRATED RURAL/SPATIAL DEVELOPMENT: ELEMENTS OF SYSTEMS ANALYTIC APPROACH

Edited by

Andrzej Straszak and Jan W. Owsinski

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V. LOCATION/ALLOCATION PROBLEMS

REGIONAL ANALYSIS AND LOCATION/ALLOCATION PROBLEMS: AN INTRODUCTION

Jan W. Owsiński

The two preceding chapters emphasized the importance of adequate allocation of efforts and resources in defining a development outline. The choice of particular allocation scheme to be followed, prescribing certain resources, effort or other available controls to individual activities and/or economic and social actors, should be guided by goals existing within the system, by resource capacities and by the specifics of system's functioning. Some of the analytic and planning approaches presented introduce, in addition to economic dimension of allocation, i.e. in addition to activities and economic agents /producers/, also a spatial, geographic dimension. Thus allocation can have varying quality depending upon a choice of locations for which particular allocation schemes are envisaged. The simplest example is diversification of crop activities depending upon the soil-and-climatic conditions of various regions.

The spatial dimension is of primary importance in location analysis, regional analysis and planning. There, the resources are mainly regarded from the viewpoint of their geographic distribution, and needs and goals as well. Land, as a foremost spatial resource is therefore concerned in the first place. Adequate land use is aimed at. If a narrow set of activities or resources is considered to be assigned to a geographical situation, one speaks of location problems. Certainly, geographical space, just as it happens to the economic one, can be represented through its very fine subdivisions, or through certain aggregates, sometimes quite abstract, such as e.g.

urban and rural sector. Therefore, in addition to allocation of resources and effort to individual activities and producers, the location of these should be defined, whether in particular points or in parts of geographical space. The locations may be, for instance, individual settlements - villages or other, subdivisions of space may come in the form of regions, urban and rural zones, etc. Resources to be allocated may be such as manpower, and accompanying costs of its attraction, investment capital in general or of particular kind, for instance into irrigation, drainage and water system construction, or any adequate physical input such as seeds of newly introduced crop varieties. It may sometimes be difficult to identify appropriate activities into which allocation is made, or to distinguish them from the resources which are being allocated. In fact, it is often assumed in the location problems that one locates activities themselves, meaning an adequate package of investments into additions to existing or setting up of new establishments. The activities considered would range from "economy in general", through "production" or "service" down to particular sectors, branches or even finer distinctions such as crops in case of agriculture.

While location analysis is concerned with relatively narrow scope of resources and activities /for even considering "economy in general" one sees it as turning out just one product/, regional analysis and planning addresses a possibly broad spectrum of economic and social activities, even though the resource side might be represented by one or few resources, mainly land, manpower, capital, water, raw materials, transportation facilities, other infrastructure. It is obvious, therefore, that regional analysis and planning has to link closely location with allocation aspects, since limited and geographically immobile, or, if mobile, then with relatively high cost, resources are to be allocated among competing uses so that an array of goals is possibly well attained.

The above outlined difference between location analysis and regional analysis and planning has an important bearing on application of formalized methods, attributed to systems analysis, in solving of these two kinds of problems. The more particular, well defined problems of location analysis find appropriate formal solution methods, mainly within the domain of mathematical /integer, mixed/ programming, combinatorial analysis and graph theory. These methods have reached the stage when they can be in a straightforward way applied in planning, hence also in rural development planning, and they can be referred to as techniques. On the other hand the complex and comprehensive nature of the regional analysis and planning problems makes it much more difficult to apply to them all-embracing formal methods. There are two main lines of introduction of system analysis into this type of problems. First, more theoretical, consists in trying to define and solve the whole problem. Second, more pragmatic, introduces certain formal, or system analytic, methods for dealing with particular aspects or parts of the regional problematique. This state of art does not preclude anyhow, though, that some sort of comprehensive regional modelling method will not become a handy tool for analysis and/or planning in the near future. The methods in view may belong to input/output, systems dynamics or any other species. Since they have not yet reached the stage of routine planning implementability, it was not deemed appropriate to present them in this volume. Instead, a discussion on planning generalities was offered.

MODELLING AND OPTIMIZATION OF EMPLOYMENT AND INCOME DISTRIBUTION POLICIES

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1. Introduction

Many controversial issues arise in connection with sectoral and regional income distribution. In the system where the authority regulates the basic wages and prices, the wage differences influence the employment pattern. In the common interest of all the parties concerned (i.e. employers, trade unions and authority) is to set these differences in such a way that an efficient allocation of labour resources follows (since in that case only the total product subject to distribution, is maximum, see Abbot and Ashenfelter, 1976, Gilbert and Pfouts, 1958; and Kulikowski 1978, 1980a,b). That requires, however, that the demand, as well as the supply, of labour force is taken into account and, generally, higher wages are paid in these sectors which achieve greater (as compared to the rest) labour efficiences (value added per worker). When the wages are properly choosen they stimulate the intersectoral or interregional flow of labour and minimize the possible unemployment.

Generally, the labour transfer incurs transportation, housing, retraining etc. costs. The optimum strategy follows when the economic benefits minus labour transfer costs are maximized. However, the implementation of optimium strategies is frequently difficult due to the lack of reliable information regarding benefits, costs and optimum strategies. Since the necessary computations in the real systems are not simple the computerized, employment-policy models, which enable fast computation of different alternatives, are very useful (in the wage negotiations in particular).

In the present paper the employment policy model for the rural-urban system is studied, as the concrete example. First of all the supply functions for labour forces are evaluated. They depend on the rural-urban migration and commuting flow. The migration and commuting rates depend, in turn, on the urban-rural income parity, which is regarded as the main policy variable. When the parity increases the supply of labour at urban (rural) areas increases (decreases). The optimum parity can be derived maximizing the value added increments minus the labour transfer costs. An extension of such an approach for the multisectoral economy is also described.

2. Labour supply functions

Assume the labour supply in the given region and the planned year L^s to depend on the last (basic) year labour stock $\overline{L} = \overline{P}\,\overline{\kappa}$, [*] where $\overline{P}$=population, $\overline{\kappa}$ = professional activity index (labour participation); as well as : rate of demograhic growth d, rate of migration μ, rate ofcommuting δ.

Introducing the r = rural, u = urban subscripts, one can write for the closed (with respect to migration) rural-urban system:

$$L_r^s = \overline{L}_r \,|1 + d_r - \mu - \delta| \qquad (1)$$

$$L_u^s = \overline{L}_u \,|1 + d_u + 1(\mu + \delta)|,\; 1 = \frac{\overline{L}_r}{\overline{L}_u} \qquad (2)$$

It is also assumed that professional activity depends on age a, though it does not change much between basic and planned interval, so $\kappa(a)$ $\overline{\kappa}(a)$ for all a. Then one can estimate the values of d_r, d_u using the standard demographic forecasts of cohorts $P_r(a)$, $P_u(a)$:

$$d_r = \sum_a \frac{P_r(a)\,\kappa_r(a)}{\overline{P}_r\,\overline{\kappa}_r} - 1 \simeq \sum_a \frac{P_r(a)\,\overline{\kappa}_r(a)}{\overline{P}_r\,\overline{\kappa}_r} - 1, \qquad (3)$$

$$d_u \simeq \sum_a \frac{P_u(a)\,\overline{\kappa}_u(a)}{\overline{P}_u\,\overline{\kappa}_u} - 1 \qquad (4)$$

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[*] Upper bars will be used henceforth to denote the last year values of variables.

As shown in Fig.1, the $\kappa_u(a)$, $\kappa_r(a)$, graphs for Polish urban-rural labour (according to 1978 census) differ mainly for increasing age. The rural population (owners of small farms) stay active up to an old age (unless they exchange the land for retirement pay). According to the census $\kappa_u = 0.48$, $\kappa_r = 0.61$, $1 = 0.836$.

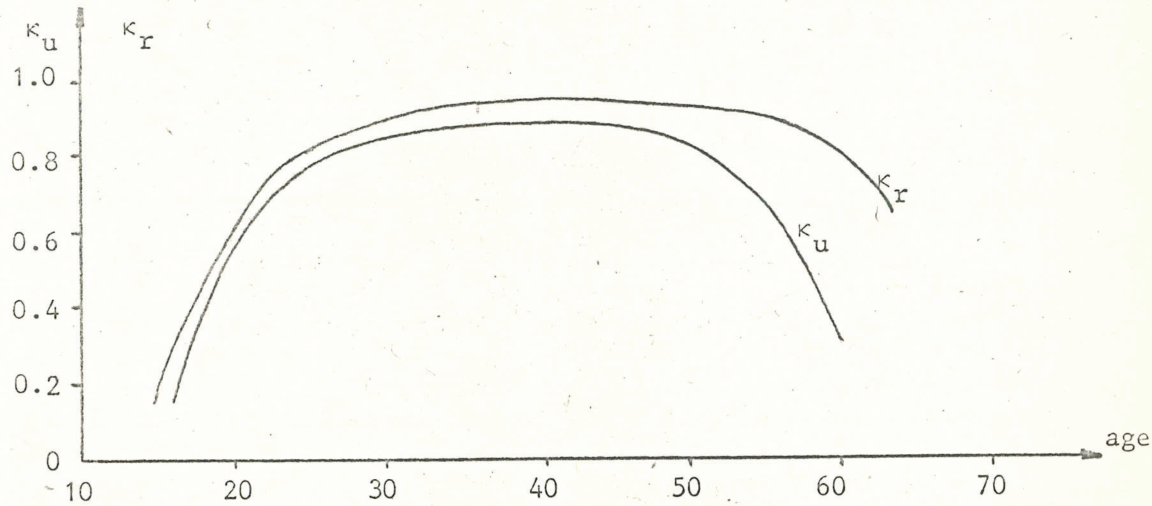

Fig. 1.

The numerical values of d_r, d_u for recent years (e.g. for 1978 $d_u = -0{,}00129$, $d_r = 0{,}173$) indicate that the national demographic growth is due mainly to the relatively high rural fertility. The great value of rural-urban migration (μ) and commuting (δ) decrease, however, the rural labour supply L_r^s.

Since μ, as well as δ, depend on income parity $\P = \frac{\omega_u}{\overline{\omega}_r}$, where ω_u = average planned urban wage, $\overline{\omega}_r$ = last year rural wage or income (for farm owners), it is necessary first of all to derive $\mu(\P)$, $\delta(\P)$. Generally, $\mu(\P)$, $\delta(\P)$ are nonlinear functions (see Kulikowski, 1980a). In the present paper, for the purpose of simplicity, the linear approximations of these functions will be used. Following Kulikowski, 1980a the econometric model

$$\mu(\P) = a_1\P - a_2 \tag{5}$$

where a_1, a_2 = coefficients determined by statistical data, will be used to describe the rual-urban migration. Using the statistical data for 1970 ÷ 1979, see Table I, and linear regression technique, one finds

$a_1 = \bar{a}_1 : \zeta_r$, $\bar{a}_1 = 0.019$, $a_2 = 0.014$ with the correlation coefficient $\rho = 0.80$. In Table I the real agriculture income (in consumption and nonproductive investments) and real wage outside agriculture has been used. Since the statistical data used indicate the relative change $[\zeta_r = \omega_r/\bar{\omega}_r,\ \zeta_u = \omega_u/\bar{\omega}_u]$ of income and wages the values : $\bar{\omega}_r \doteq 1867$ zł (last year average wage in state farming sector), $\omega_u = 2277$ zł (average wage outside agriculture) has been taken for the initial (1970) year. Since the data on s (migration rate for whole population) are commonly available, in the conversion $\mu = \frac{\kappa_m}{\kappa_r} \cdot s$ the value $\kappa_m/\kappa_r = 0.79$ has been used.

In order to derive the δ(¶) relation a simple commuting model should be introduced. Assume the commuter's spatial density function $\bar{G}(x)$, shown in Fig. 2, where x is the distance of commuters residence place from the city. One can approximate $\bar{G}(x)$ by the linearized function:

$$G(x) = \begin{cases} \overline{bL_r}\left(1 - \frac{x}{R}\right), & x \subset [0,R] \\ 0\ , & x > R \end{cases} \qquad (6)$$

where R is the maximum distance of commuting, $\bar{b}$ = const.

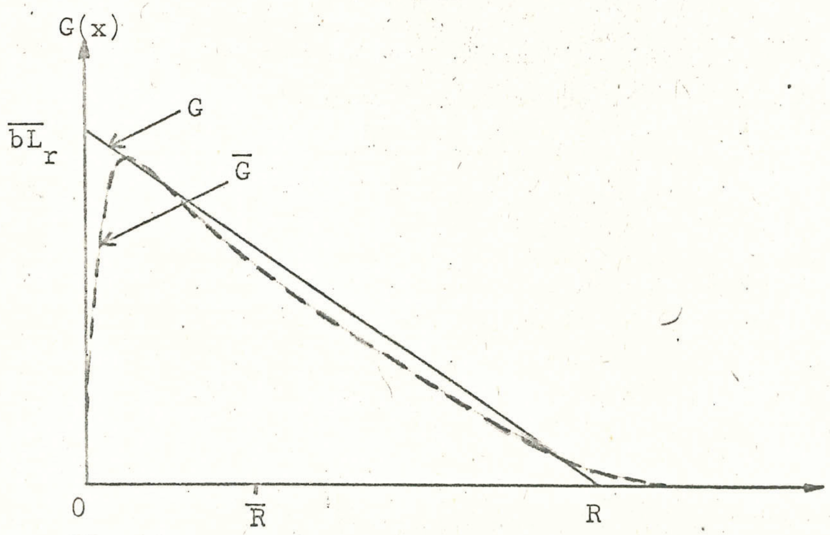

Fig. 2.

The value of R can be derived from the condition that the urban-rural wage difference $\omega_u - \omega_r$, is not less the commuting cost $R(\gamma c_1 + c_2/v)$, where c_1 = fare per km, γ = commuter's participation rate in full transport cost c_1 $(0 < \gamma < 1)$, c_2 = cost of 1 hour of labour lost for transportation, v = transportation speed. Then one gets

$$R = \frac{\omega_u - \omega_r}{\gamma c_1 + c_2/v} \tag{7}$$

One can observe that by decreasing γ (i.e. increasing subsidies for transportation) it is possible to increase R and consequently ÷ increase the number of commuters:

$$D = \int_0^R G(x)\,dx = \frac{1}{2}Rb\bar{L}_r . \tag{8}$$

Assuming in (7) $\omega_r = \zeta_r \overline{\omega}_r$, where ζ_r = given (exogeneous) one gets

$$\delta = \frac{D}{\overline{L}_r} = b(\P - \zeta_r), \quad b = \frac{\overline{b}\,\overline{\omega}_r}{2(\gamma c_1 + c_2/v)} \tag{9}$$

According to the 1978 census in Poland there were D = 1609299 (rural--urban, professionally active net commuters) while $\overline{L}_r$ = 8124800. Since the value of ¶ was 1.29 while ζ_r = 0.98, from (9) one gets the estimate of b = 0.64.

Then instead of (1), (2) one can write:

$$L_r^s(\P) = \overline{L}_r \; |1 + d_r - (a_1+b)\P + a_2 + b\zeta_r| ,$$

$$L_u^s(\P) = \overline{L}_u \; |1 + d_u + 1(a_1+b)\P - 1(a_2+b\zeta_r)|$$

Assuming 1977 to be the basic year and fitting the correponding data the reconstructed supply functions (for 1978) become:

$$L_r^s(\P): \overline{L}_r = 1.66 - 0.67\P ,$$

$$L_u^s(\P): \overline{L}_u = 0.45 + 0.55\P ,$$

see Fig. 3 a,b.

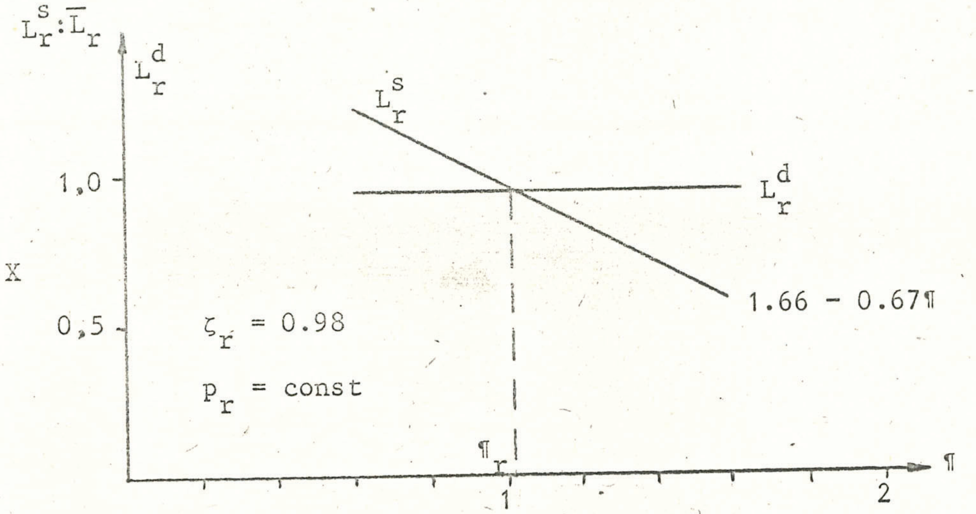

Fig. 3a.

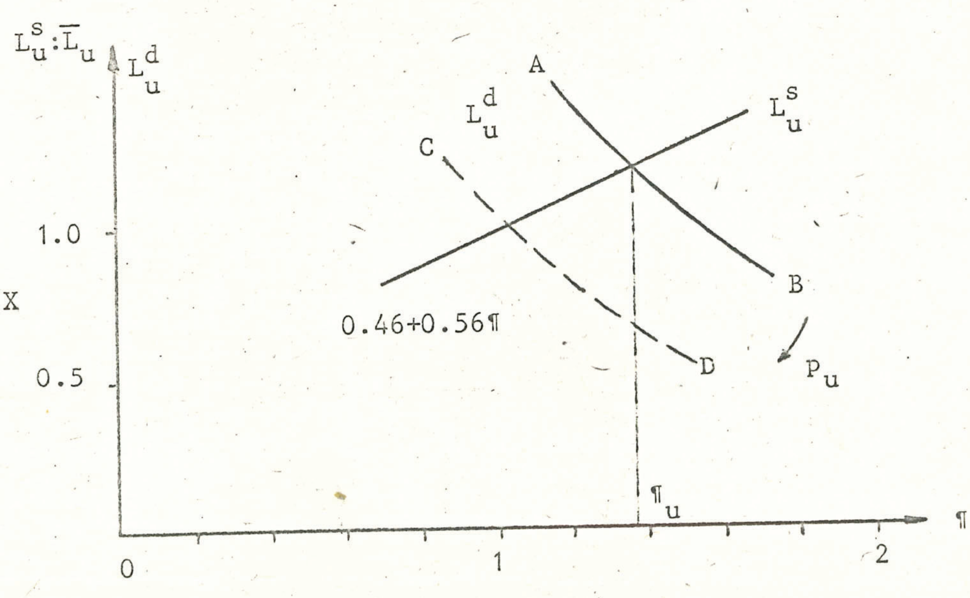

Fig. 3b.

3. Labour demand functions and labour markets

Due to the fast technical progress and capital investments in urban economy the labour demand in rural areas in Poland for the past decade was steadily decreasing in time. The policy objective was to keep fixed basic food prices (which stabilized also the farmers income) and keep the income parity $\P > 1$.

With fixed ζ_r (i.e. ω_r), cultivated land and modest capital investment & technological progress, the agricultral labour demand L_r^d may be assumed constant (i.e. not depending on $\P$) as shown in Fig. 3 a.

In order to derive the urban labour demand $L_u^d(\P)$ assume the urban economy to be approximated by the Cobb-Douglas production function:

$$Y = p_u A\, e^{\lambda t} L_u^{\alpha} K^{1-\alpha}, \tag{10}$$

where Y = value added (net product),

p_u = price for urban product (exogeneous in the model),

λ = Harrod and Hicks neutral technical progress,

K = capital stock, A = const., $0 < \alpha < 1$.

From the profit maximization condition: $\frac{\partial}{\partial L_u}(Y - \omega_u L_u) = 0$ one gets the optimum employment (demanded by the employers):

$$L_u = \frac{\alpha}{\omega_u} Y \tag{11}$$

Solving (10), (11) for Y one obtains

$$Y = p_u A\, e^{\lambda t} \left(\frac{\alpha}{\omega_u}\right)^{\alpha \frac{1}{1-\alpha}} K$$

Then the labour demand (11) becomes

$$L_u^d = \overline{A}\, \P^{\frac{1}{\alpha-1}}, \quad \text{where} \quad \overline{A} = \left(p_u A\, e^{\lambda t} \left(\frac{\alpha}{\omega_u}\right)^{\alpha}\right)^{1-\alpha} K. \tag{12}$$

The graph of $L_u^d(\P)$ function for $\alpha = 0.7$, $\overline{A}$ = const. is shown in Fig. 3 b.

It can be observed that in the situation depicted it is impossible to obtain the equilibrium at rural and urban labour markets simultaneously (by changing parity $\P$). Setting $\P$ in such a way that $\P_r \leq \P \leq \P_u$ one can achieve, however, $L_r^d \geq L_r^s$ and $L_u^d \geq L_u^s$, which corresponds to the full employment. Such a situation existed for a long time in Poland. For example, the number of new jobs registered in 1976 at Employment Offices (82654) was over 6 times greater the registered jobs wanted.

After the last economic reform in Poland the authorities would control some basic prices only (with the rest left to the market mechanisms) as well as basic income distribution (i.e. mainly wages and taxes). The employment is not, according to the reform, controlled directly as it was the case in the past.

As a result of changing market prices etc. the situation described in Fig. 3 may change easily to $\P_r \geq \P_u$ case (L_u^d may shift from AB to CD) and unemployment in one or both sectors may occur. In such a situation the social costs, connected with relief and retrainning of unemployed labour force, should be taken into account.

4. Optimization of rural - urban labour transfer

The transfer of one worker between two regions or sectors (e.g. rural and urban) characterized by different labour efficiencies, i.e., values added per worker (D_u, D_r), results in an economic benefit. The $\delta \overline{L}_r$ commuters produce e.g. the increase of $\delta \overline{L}_r(D_u - D_r)$ of value added. The $\mu \overline{L}_r$ migrants, who setle down in urban area, produce in addition $\mu \overline{L}_r(D_u - D_r)$ each year so the present, discounted benefit becomes $\frac{\mu}{r} \overline{L}_r(D_u - D_r)$, where r is the discount rate. The total gain in terms of value added becomes

$$\Delta(\P) = (D_u - D_r)\left[\delta(\P) + \frac{\mu(\P)}{r}\right]\overline{L}_r \qquad (13)$$

Since the labour transfer, in form of migration and commuting, incurs the cost $C(\P)$ the resulting benefit function becomes

$$\Phi(\P) = \Delta(\P) - C(\P).$$

When $\Phi(\P)$ is a concave function the optimum value of $\P$ can be derived by solving the equation $C'(\P) = \Delta'(\P)$.

In the case when D_u is not affected by change of $\P$ (e.g. when the change of $\P$ is executed by imposing income tax τ on gross wage $\tilde{\omega}_u$, so $\tilde{\omega}_u(1-\tau)$) one gets:

$$\Delta'(\P) = (D_u - D_r)(b + \frac{a_1}{r})\overline{L}_r = \overline{\Delta}\,\overline{L}_r = \text{const.}$$

Due to the existing pricing policy in Poland, before 1982 (aimed at keeping the food prices down) the D_u was much bigger D_r. For example, in 1976 in agriculture D_u = 44000 zł, while in industry D_r was 170000 zł, in transportation it was 115000 zł etc.

When $C(\P)$ is evaluated the following main components should be taken into account:

a. Consumption increase cost. Each migrant coming to the city increases his personal and aggregate (services) consumption, in the present as well as next years to come. The increase of consumption for a commuter is due mainly to the increased urban salary though he (and his family) does not benefit much out of urban services.

The transfer of $P_r s$ migrants from rural to urban area increases their discounted consumption value by

$$C_\mu = P_r s(Z_u - Z_r)\, r^{-1},$$

where

$s = \frac{\kappa_r}{\kappa_m}\mu$ = migration rate (κ_m = migrant's activity index)

$Z_u = \omega_u \frac{\kappa_u}{\theta_u}$ = total urban consumption (personal income and services, $\theta_u Z_u$ represents personal income),

$Z_r = \omega_r \frac{\kappa_r}{\theta_r}$ = total rural consumption,

r = discount rate.

Since $\zeta_r = \dot{Z}_r : \overline{Z}_r = \omega_r : \overline{\omega}_r$, is exogenous one gets

$$C_\mu(\P) = \overline{\omega}_r \overline{L}_r a \left(\P - \zeta \frac{a_2}{\bar{r} a_1}\right) (\P - \zeta_r \overline{g}) ,$$

where

$$a = \frac{\kappa_u a_1}{r \kappa_m \theta_u} , \qquad \overline{g} = \frac{\theta_u \kappa_r}{\theta_r \kappa_u}$$

The marginal cost becomes

$$C'_\mu(\P) = 2 \overline{\omega}_r \overline{L}_r a \left[\P - \zeta_r \left(\frac{a_2}{2a_1} + \frac{\overline{g}}{2}\right)\right]. \tag{14}$$

In a similar way one derives the increased consumption of commuters

$$C_\delta(\P) = \overline{L}_r \delta(\omega_u - \omega_r) = \overline{\omega}_r \overline{L}_r b (\P - \zeta_r)^2 ,$$

and the corresponding marginal cost, which becomes

$$C'_\delta(\P) = 2 \overline{\omega}_r \overline{L}_r b(\P - \zeta_r) \tag{15}$$

b. <u>Cost of commuting</u>. The total cost of commuting is

$$C_t = \delta \overline{L}_r c_1 \overline{R}.$$

where R is the average distance (see Fig. 2):

$$\overline{R} = \int_0^R x\, G(x) dx : D = R/3.$$

Then the subsidized cost of commuting becomes

$$C_s = (1 - \gamma) C_t = (1 - \gamma) \delta \overline{L}_r c_1 R/3 =$$

$$= \overline{\omega}_r \overline{L}_r B \quad (\P - \zeta_r)^2, \quad \text{with} \quad B = b \frac{1 - \gamma}{3\left(\gamma + \frac{c_2}{c_1 v}\right)} \tag{16}$$

The marginal commuting cost becomes

$$C'_s(\P) = 2\,\overline{\omega}_r \overline{L}_r B\,(\P - \zeta_r) \tag{17}$$

c. Urbanization cost. The migrants and commuters increase also the rate of urbanization, housing and enviromental pollution. It may be assumed that the corresponding cost C_u is proportional to $\frac{\mu}{r}\overline{L}_r$ and $\delta\,\overline{L}_r$ so that the marginal cost becomes:

$$C'_u = (c_m \frac{a_1}{r} + c_d b)\,\overline{L}_r = \overline{C}_u \overline{L}_r\,, \tag{18}$$

where $c_m, c_d, \overline{C}_u$ = given constants.

The marginal cost of extending the urban facilities only was in the period studied around 2000 zł per capita per year. The cost of the new apartment per migrant was around 10^5 zł. However, a part only of that cost was paid by migrants. The rest is usually paid by employers or municipalities. The low percentage credits is also granted for housing.

Commuters contribute also to C_u and it is assumed usually that $c_d = (0.1 \div 0.5)\,c_m$.

d. Unemployment and retraining costs. In the case when the labour supply is greater than the corresponding demand the cost C_r, connected with unemployment arises. Part of that cost is the retraining process, while the rest goes to the unemployed relief.

Assuming the unemployed graphs $\phi(\P) = L^s(\P) - L^d(\P)$ (see Fig. 3) to be given one can approximate them by linear functions:

$$\phi_r = \begin{cases} L_r(1 - \P/\P_r)\,c_r, & \P \le \P_r \\ 0, & \P > \P_r \end{cases}, \quad \phi_u = \begin{cases} L_r(1 + \P/\P_u)\,c_u, & \P \ge \P_u \\ 0, & \P > \P_u \end{cases},$$

and derive the corresponding marginal cost

$$C'_r = \begin{cases} c_u/\P_u\,; & \P \ge \P_u \\ 0, & \P < \P_u \end{cases} \overline{L}_r - \begin{cases} c_r/\P_r\,, & \P \le \P_r \\ 0, & \P > \P_r \end{cases} \overline{L}_r = \overline{C}_r \overline{L}_r, \tag{19}$$

where $c_u/\P_u$, $c_r/\P_r$, can be regarded as unemployment cost (per capita) at urban and rural areas respectively. The numerical values of $\P_u$, $\P_r$ depend on the supply - demand relations, i.e. prices, technological change etc.

As follows from (15) ÷ (19) the marginal cost

$$C'(\P) = C'_{\mu}(\P) + C'_{\delta}(\P) + C'_{s}(\P) + C'_u + C'_r$$

is a linear function of ¶. It is then possible to find such $\P = \hat{\P}$, which satisfies the optimality condition

$$\Phi'(\P) = \overline{\Delta} - C'(\P)\, L_r^{-1} = 0. \tag{20}$$

Since

$$a_{2/2\overline{a}_1} + \overline{g}/2 \simeq 1$$

one gets

$$\P \simeq \zeta_r + \frac{\overline{\Delta} - \overline{C}_u - \overline{C}_r}{2\overline{\omega}_r(a+b+B)} \tag{21}$$

It should be noted that the optimum parity may change in time and space as a result of changes in:

a. production efficiences $\overline{\Delta}$, i.e. prices, technical progress etc.

b. cost parameters, which take into account demographic, enviromental, transportation, housing etc. parameters.

When durihg the planning negotiations the expected $\Phi' > 0$ ($\Phi' < 0$) the best policy is to increase (decrease) the parity. A similar effect may be also obtained by changing other control parameters such as prices, taxes or transportation subsidies (γ). Since $\Phi(\gamma)$ is a concave function one can find the optimum value of γ by solving the equation $\frac{d}{d\gamma}\Phi = 0$.

One can also observe that when the value of ¶ is decided it is possible to derive the expected employment and other socioeconomic factors within the planning interval.

5. Multisector model

The optimization of employment, studied so far in the rural-urban model, can be easily extended to the multisector (or multiregional) hierarchic model, shown in Fig. 4. The national economy (NE) consists here of rural (R) and urban (U) subsystems, which in turn can be split in private farming (RP) + rural rest (RR) and - industry (UI) etc.

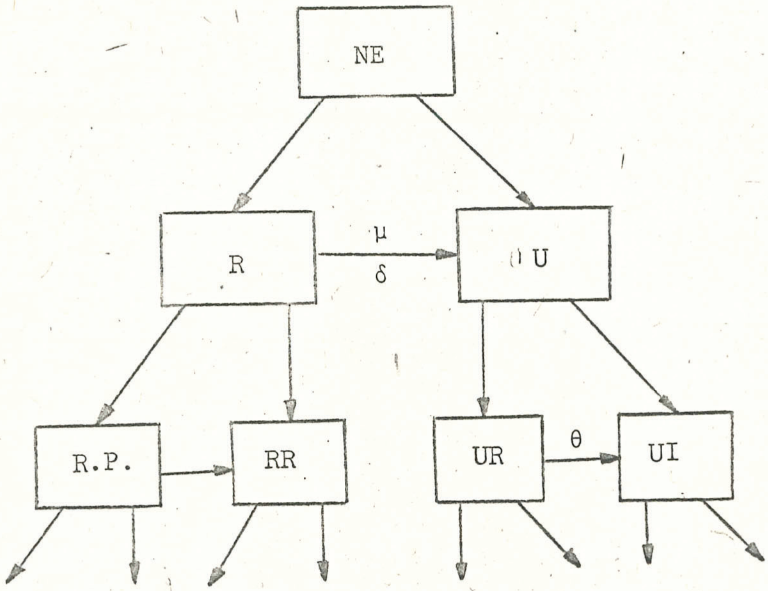

Fig. 4.

In order to explain how to use the methodology, described in sections 2 ÷ 4, consider the UI-UR subsystem. The supply functions of labour for industry (L^s_{ui}) and rest (L^s_{ur}) according to (1), (2) become:

$$L^s_{ur} = \overline{L}_{ur}(1 + d_{ir} - \phi_{ir}) \tag{21}$$

$$L^s_{ui} = \overline{L}_{ui}(1 + d_{ir} + l_{ir}\,\phi_{ir})$$

where

$$d_{ir} \doteq d_u + l_{ir}|\mu(\P) + \delta(\P)|,$$

ϕ_{ir} = rate of labour transfer between UI-UR

$l_{ir} = \overline{L}_{ur} : \overline{L}_{ui}$.

Since the cost due to labour transfer ϕ_{ir} constitutes mainly the "increased consumption" the formula (14) can be used, and one gets

$$C_{\phi}^{\prime}(\P_{ir}) = 2\,\overline{\omega}_{ur}\,\overline{L}_{ur}\,a_{ir}\,(\P_{ir} - \zeta_{ur}g_{ir}) \qquad (23)$$

where

$\overline{\omega}_{ur}$ = average wage in UR subsystem,

$\P_{ir} = \frac{\omega_{ui}}{\overline{\omega}_{ur}}$ - wage parity (industry-urban rest),

$\zeta_{ur} = \omega_{ur} : \overline{\omega}_{ur}$ = exogeneous parameter,

$a_{ir} = \frac{\overline{a}_{1ir}}{\theta_u}$, $g_{ir} = [\frac{a_{2ir}}{a_{1ir}} + 1] : 2$

The $\overline{a}_{1ir}$, a_{2ir} coefficients can be determined by linear regression using the past data for ϕ_{ir} and $\P_{ir}$.

Another cost component, which may enter into account, when labour supply is greater than demand, is equivalent to (19) and can be denoted by $C_{ur}^{\prime}$.

One finds the optimum value of $\P_{ir} = \hat{\P}_{ir}$ by solving the equation

$$C_{\phi}^{\prime}(\P_{ir}) = \overline{L}_{ur}(\overline{\Delta}_{ir} - \overline{C}_{ur}), \qquad (24)$$

where $\overline{\Delta}_{ir} = (D_{ui} - D_{ur})\,a_{ir}$, D_{ui}, D_{ur} = labour efficiencies of UI, UR economies respectively.

One can observe that the optimum solution

$$\hat{\P}_{ir} = \zeta_{ur}\,g_{ir} + \frac{\overline{\Delta}_{ir} - \overline{C}_{ur}}{2\,\overline{\omega}_{ur}}\,\theta_u \qquad (25)$$

consists of two components. The first one depends on the relative job atractiveness in sector UI (compared to UR) expressed by $\frac{a_{2ir}}{a_{1ir}}$. The second component depends on net benefit resulting from transfer of one worker from UR to UI. The optimum value (25) depends also on the value of the exogeneous parameter ζ_{ur}. That parameter should be choosen in such a way that the average urban wage

$$\omega_{ui} \frac{L_{ui}}{L_u} + \omega_{ur} \frac{L_{ur}}{L_u} = \bar{\omega}_{ur} \hat{\P}_{ir}(\zeta_{ur}) \frac{L_{ui}}{L_u} + \zeta_{ur} \bar{\omega}_{ur} \frac{L_{ur}}{L_u} \quad (26)$$

equals ω_u, which has been already derived for R-U system.

When the wages for the complex system, shown in Fig. 4, are being derived, the minimum wage, say ω_m, can be determined (adjusting the corresponding ζ_m parameter) in such a way that it yields the given minimum standard of living. All the rest of sectorial wages can be then determined in a unique fashion.

It should be also noted that the methodology described can be also used when the employment concerns a particular sector or region and the corresponding wage is compared with the average wage for the rest of the country. An extension of the methodology used for the skilled and unskilled or male-female labour is also possible.

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Table I:

Rural - Urban Migration Model

	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979
Migration rate Δ^{*}	0.0106	0.0118	0.0109	0.0111	0.0121	0.0167	0.0166	0.0146	0.0154	0.0151
Agricultural income** dynamics ζ_r		1.090	1.099	1.033	0.981	1.004	1.095	1.052	0.982	0.997
Urban income dynamics** ζ_u		1.057	1.087	1.087	1.066	1.085	1.039	1.023	0.973	1.020
Parity of income $\P/\zeta_r$	1.220	1.183	1.145	1.205	1.310	1.415	1.343	1.306	1.294	1.323

$$\mu = 0.791 = a_1\P - a_2 \quad , \quad a_1 = \overline{a}_1 : \zeta_r ; \quad \overline{a}_1 = 0.019 \qquad a_2 = 0.014$$

Data sources:

*) Roczniki Demograficzne (Demographic Yearbooks) (1970 - 1979). Główny Urząd Statystyczny, Warsaw.

**) Roczniki Statystyczne (Statistical Yearbooks) (1970 - 1979) Główny Urząd Statystyczny, Warsaw.

OPTIMIZATION OF THE TERRITORIAL SERVICE SYSTEM

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1. Introduction

In practice we often deal with service systems, in which calls are geographically distributed over a certain territory. In such cases it is necesary to take into account - besides the service time - also the travel time to site of call. If a bounded territory is considered, then a problem to determine necessary number of servicepersons in that territory arises. If the territory is large, it is usually divided into smaller parts and service systems are set up independently in every part. Therefore the next problem appears - the determination of the best division of territory.

Well organized service system should have following two properties:

A. It should secure prompt services.

B. Operating costs of a system should be possibly small.

The problem of optimal division of territory is now clear. The division into small parts is undesirable because of large amortization and exploitation costs per one serviceperson. Moreover in the case of a small number of servicepersons in such parts, servicepersons will not be well employed - the situation will often occur that in one part there will be a long queue of unserviced calls, whereas in other parts there will be idle servicepersons. On the other hand if we divide territory into too large parts, travel times will increase and obviously also travel costs will increase.

Consequently the conclusion could be made, that we deal with a specific optimization problem, which combine elements of the theory of queues and location theory.

2. Formulation of the problem

We take into account the bounded territory with uniformly distributed points, in which calls could appear (it means, that the expected number of calls in every subterritory is proportional to its area). The problem is to divide the whole territory into equal, independent parts (subterritories). We asume, that if there is an idle serviceperson in the part, in which a call appeared, then service of the call begins immediately. If there is no idle serviceperson, then call waits in queue. Calls are served in the order of appearance ("first come first served" - FCFS rule).

The optimization problem is to find such a division of territory into equal parts, that:

A. With the probability not less than q service of each call should be finished during L days from the day of its appearance.

B. Operating costs of the system should - under above restriction - be minimal.

The use of such two-stage criterion causes that presented model becomes more general because in many applications it happens that losses conected with unserved call, could not be expressed as a cost. Examples of such situations are various social services, health-nurses service etc..

3. Interpretations

The form of criterion A determines the kind of service systems, for which the model presented is adequate. It is clear, that it could not be used for optimization of systems, in which service must be immediate, as the ambulance service, fire-brigades etc.. There are many papers on these problems and a bibliography could be found in [2]. Presented model is applicable for systems in which calls are served generally in the FCFS order and for which an effect of travel times to sites of calls is important. The examples of such systems are:

1. service systems for technical devices of general use (TV sets, receivers, refrigerators etc.),
2. social service systems, health-nurses systems,
3. computer systems, telecommunication equipment, copiers, dictating equipment.

An a priori assumption is often made, that the territory is to be divided into small, one-serviceperson parts. It is often advisable in spite of losses described in section 1, because the personal responsiblility for services impels servicepersons to work better. Also in this case presented model - after slight modifications - is useful (see [5]).

4. Description of main variables

There are two main elements of service systems:

- calls,
- servicepersons.

Calls would be described by random variable N_k - random number of calls, which appears during the k-th day. By limit theorems it was proved in [6], that numbers N_k , k=1,2,... form a sequence of independent random variables with the same, approximately Poisson, distribution:

$$P(N_k=j) = \frac{(\bar{N})^j}{j!} e^{-\bar{N}}$$

where $\bar{N}$ is an expected value

$$\bar{N} = E(N_k)$$

Let us assume, that there are K points in the territory, in which calls appears with rates λ_i (i=1,2,...,K). We call

$$\bar{\lambda} = \frac{1}{K}\sum_{i=1}^{K} \lambda_i$$

the average rate of calls. If the territory is divided into n equal parts, then (see [6]) calls from every part will form Poisson process with the parameter $\frac{K \cdot \bar{\lambda}}{n}$.

The activity of servicepersons is described by service time of one call. It is also a random variable, consisting of:

- random time D of travel to the site of call from the site of foregoing call,
- random time T of the service "on-site",
- constant time a, necessary for formalities.

Thus the total service time is equal $D+T+a$.

We assume, that D and T are mutually independent, independent also of N_k $(k=1,2\ldots)$, and

$$P(D>0) = P(T>0) = 1$$

Time T of the service "on-site" could be every random variable with known distribution function. It is proper to pay more attention to the probability distribution of the travel time. Let us assume, that servicepersons travel with a constant travel rate v. Consequently, if the distance between two consecutive call sites is $\bar{D}$, then

$$D = \frac{\bar{D}}{v}$$

Therefore the form of a distribution function of the travel time is the same, as of the distribution function of random variable $\bar{D}$. It was assumed, that calls appear uniformly at the whole territory, so $\bar{D}$ is just a random distance between two points, chosen independently in accordance with the uniform distribution on this territory. The distribution function of $\bar{D}$ depends on the area of territory and also on its shape. For instance, for the circle with radius R the distribution function of D is (see [1]):

$$P(\bar{D}<x) = \frac{\pi x^2 + \alpha(R^2-x^2) - \sin\alpha \cdot (R^2-x^2)/2}{\pi R^2} \qquad /4.1/$$

where $\frac{x}{2R} = \sin\frac{\alpha}{2}$, and for the square with a side "a" (see [4]):

$$P(\bar{D}<x) = \begin{cases} \left(\frac{x}{a}\right)^2 - \frac{8}{3}\left(\frac{x}{a}\right)^3 + \frac{1}{2}\left(\frac{x}{a}\right)^4 & 0 \leqslant x \leqslant a \\ \frac{1}{3} + 4\sqrt{\left(\frac{x}{a}\right)^2 - 1} - \left[\pi + 2 - 4\arcsin\left(\frac{a}{x}\right)\right]\left(\frac{x}{a}\right)^2 + & \\ + \frac{8}{3}\left[\left(\frac{x}{a}\right)^2 - 1\right]^{\frac{3}{2}} - \frac{1}{2}\left(\frac{x}{a}\right)^4 & a < x \leqslant a\sqrt{2} \end{cases} \qquad /4.2/$$

The general method for computing these distribution functions for ovals was presented by Gaciauskas [3].

These formulae for territories with different shapes seem to be very different, but in fact they differ a little: in Fig. 1 probability densities of $\bar{D}$ are presented for a circle and a square with the same area.

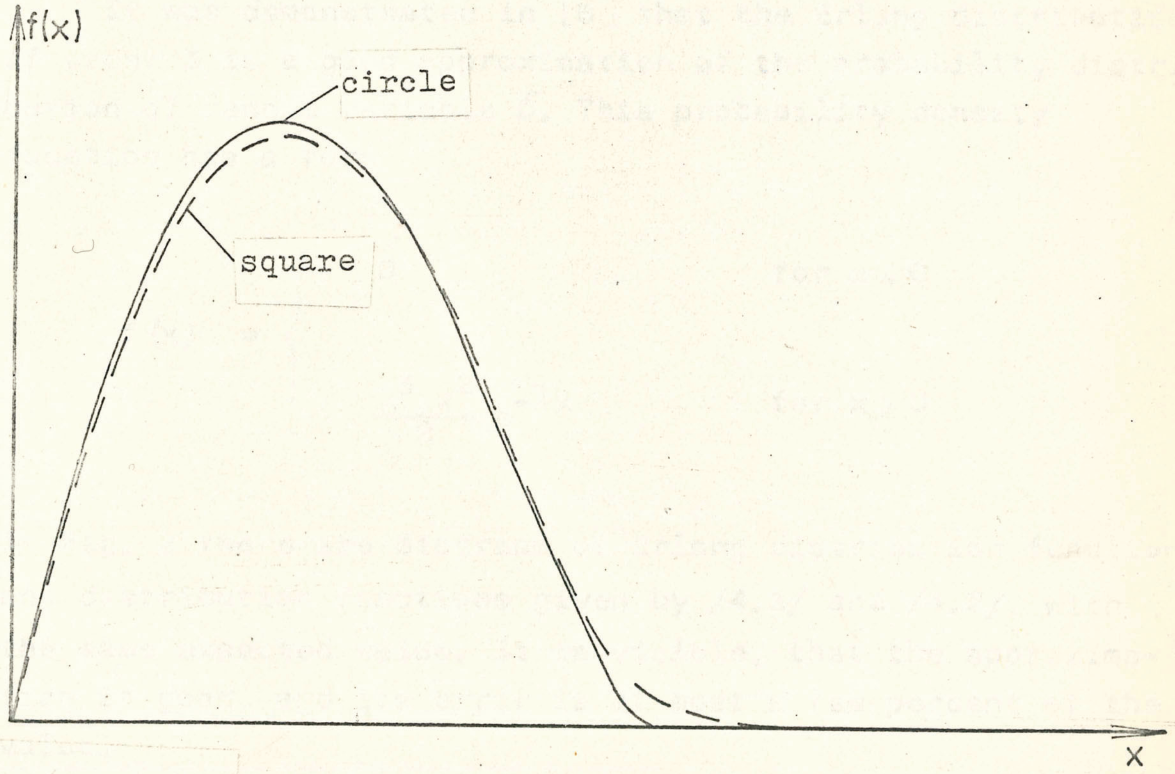

Figure 1.

The differences are insignificant. The average distance between two random points of the territory with the area A^2 is:

for a circle

$$E\bar{D} \approx 0.511\ A$$

for a diamond

$$E\bar{D} \approx 0.53\ A$$

for an equilateral triangle

$$E\bar{D} \approx 0.554\ A$$

for a square

$$E\bar{D} \approx 0.521\ A$$

These numbers were computed by Santalo ([7] -p.49, see also [8]).

Also in this case differences are insignificant. Therefore it is convenient to find one distribution function, which approximates all these distribution functions, for using exact formulas is troublesome. Moreover in real world territories with such simple shapes, as a circle, square or triangle do not occur, while for territories with more complicated it is impossible to compute exact distribution functions.

It was demonstrated in [6] that the Erlang distribution of order 3 is a good approximation of the probability distribution of random variable $\bar{D}$. This probability density function has a form

$$f(x) = \begin{cases} 0 & \text{for } x \leqslant 0 \\ \dfrac{\lambda^3 x^2}{2} e^{-\lambda x} & \text{for } x > 0 \end{cases}$$

In Fig. 2 there are diagrams of Erlang distribution function and distribution functions given by /4.1/ and /4.2/, with the same expected value. It is visible, that the approximation is good, and its error is at most a few percent of the value.

Further it will be convenient to describe servicepersons by their service abilities, i.e. random number Z of calls that one serviceperson is able to serve during a day. Let us denote the daily worktime by t. Then obviously

$$P(Z \geqslant i) = P\big((T_1+D_1+a)+\dots+(T_i+D_i+a) \leqslant t\big) =$$

$$= P(T_1+\dots+T_i+D_1+\dots+D_i \leqslant (t-i)\,a)$$

It is clear, that the maximal service ability is

$$l_{max} = \text{entier}\left(\frac{t}{a}\right)$$

and the probability distribution of Z is finally given by formulae

$$P(Z=l_{max}) = P(Z \geqslant l_{max})$$

$$P(Z=i) = P(Z \geqslant i) - P(Z \geqslant i+1) \qquad \text{for } i=1,2,\dots,l_{max}-1.$$

$$P(Z=0) = 1 - P(Z \geqslant 1)$$

If there is m servicepersons in a subterritory, then their total service ability is a sum of independent random variables

$$Z(m) = Z_1 + \dots + Z_m$$

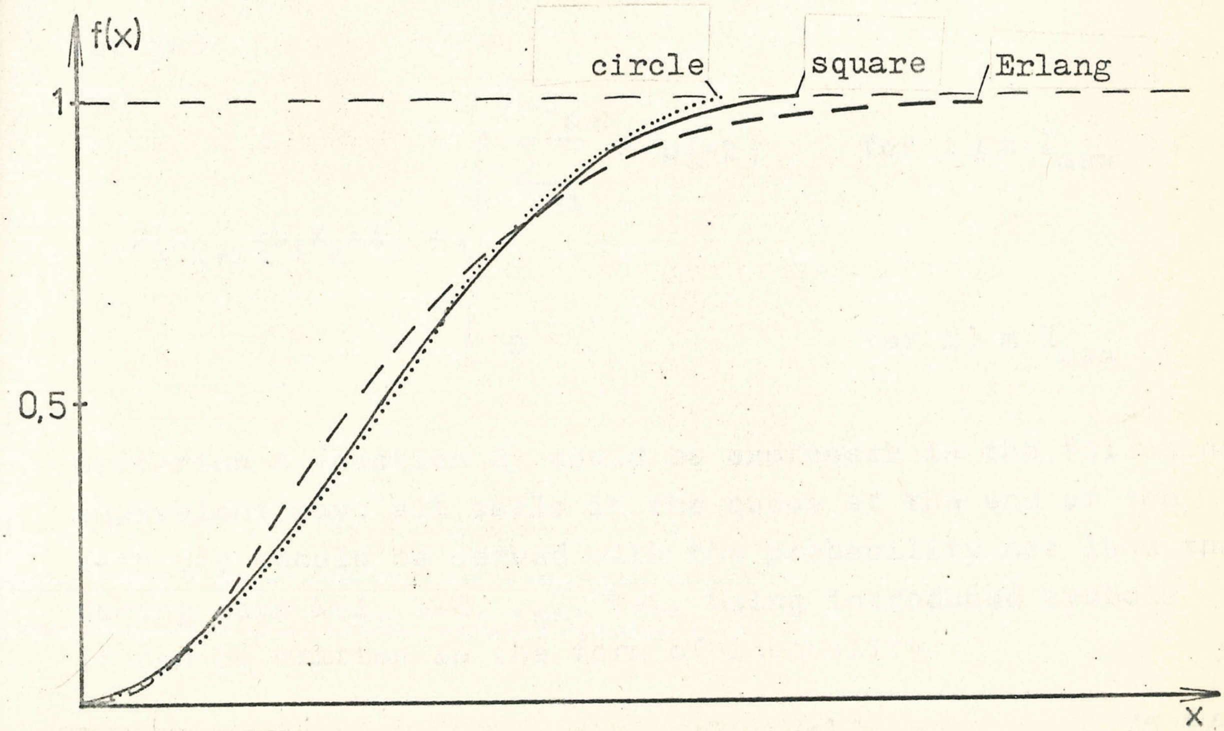

Figure 2.

5. Solution of the problem

It follows from the description of the system (section 2), that every part of the system works independently of other parts. Thus let us divide the whole territory into parts and take into consideration one part with m servicepersons. The probability distribution of the number of calls from such subterritory during the k-th day is

$$P(N_k=j) = \frac{\left(\frac{K\cdot\bar{\lambda}}{n}\right)^j}{j!} e^{-\frac{K\bar{\lambda}}{n}}$$

Let X_k $(k=0,1,\ldots)$ be a number of calls in a queue at the end of the k-th day. $\{X_k\}$, $k=0,1,2,\ldots$ forms a homogenous Markov chain with the following transition probabilities:
for $j > 0$

$$P(X_{k+1}=j \mid X_k=i) = p(j-i) = \begin{cases} \sum\limits_{l=0 \vee (j-i)}^{m\cdot l_{max}+j-i} P(Z(m)=l-j+i)\,P(N=l) & \text{for } i-j \leqslant m\cdot l_{max} \\ 0 & \text{for } i-j > m\cdot l_{max} \end{cases}$$

for $j=0$

$$P(X_{k+1}=0 \mid X_k=i) = \begin{cases} \sum\limits_{l=i}^{m\cdot l_{max}} p(-l) & \text{for } i \leqslant m\cdot l_{max} \\ 0 & \text{for } i > m\cdot l_{max} \end{cases}$$

Criterion A (section 2) could be expressed in the following, equivalent way: all calls in the queue at the end of the k-th day should be served with the probability not less than q during days k+1, k+2, ..., k+L. Using introduced symbols it can be written in the form of inequality:

$$P(X_k+N_k) \leqslant Z_{k+1}(m)+\ldots+Z_{k+L}(m) \geqslant q \qquad /5.1/$$

The probability distribution of the Markov chain $\{X_k\}$ varies in time (i.e. in k), however if the inequality

$$E(Z(m)) > E(N) \qquad /5.2/$$

holds, then this distribution tends to the limit distribution

$$\lim_{k\to\infty} P(X_k=i) = p_i$$

irrespective of the initial state.

In the purpose of a valuation of system work the initial period can be neglected and the optimization can be performed for the period of a stable work. Thus the limit distribution will be used in the inequality /5.1/. The method for determining this distribution was presented in detail in [6].

Let us notice, that condition /5.2/ is intuitive. It states that the length of queue tends to the equilibrium if on average all servicepersons can service more calls than appear. If inequality /5.2/ does not hold, then the length of queue tends to infinity with probability 1, i.e.

$$\lim_{k\to\infty} P(X_k=i) = 0 \qquad i=0,1,2,\ldots$$

If we use the above limit probability distribution, then - after slight simplifications - criterion A can be written in the form

$$\sum_{j=0}^{m\cdot l_{max}\cdot L-1} p_j \cdot P(Z_1(m)+\ldots+Z_{L-1}(m)\geqslant j) \geqslant q$$

and for L=1 in a simple form

$$p_0 \geqslant q$$

It follows from these formulae, that for fixed subterritory criterion A determines necessary, minimal number of servicepersons in this subterritory. The method for determining this number consists of following steps:

1. Choice of an interval $\langle m_1, m_2 \rangle$, which includes the solution i.e. for m_1 criterion A does not hold, whereas for m_2 it holds .
2. Verification of criterion A in a middle of the interval $\langle m_1, m_2 \rangle$, and reduction in this way of this interval to the half of it.

Detailed description of this algorithm and numerical program were presented in [6].

Now let us return to the problem of optimal division of the territory into equal parts. Operation costs of the system consists of four main elements:

- salaries of servicepersons,
- amortization costs,
- exploitation costs,
- transportation costs.

The first component is obviously proportional to the number of servicepersons:

$$b \cdot n \cdot m(n)$$

Amortization and exploitation costs depend on a number of servicepersons in every subterritory. If $I(m)$ and $J(m)$ would be these costs per one territory, then these components of operating costs will be expressed in a form

$$n \cdot I(m(n)) + n \cdot J(m(n))$$

Transportation costs are proportional to average distance, that servicepersons travel every day, so that they are equal

$$c \cdot n \cdot \frac{K}{n} \cdot \bar{\lambda} \cdot \bar{D}(n)$$

where $\bar{D}(n)$ is an average travel time between two consecutive calls in the case of division of the territory into n parts. It was proved in [6], that $\bar{D}(n)$ changes as the square root of the territory area. Thus

$$\bar{D}(n) = \frac{d}{\sqrt{n}}$$

where d is an average travel time between two consecutive calls at the whole territory.

Finally the cost function is of the form

$$C(n,m(n)) = n\left[\,b\cdot m(n) + I(m(n)) + J(m(n))\right] + \frac{c\cdot K\cdot\bar{\lambda}\,d}{\sqrt{n}}$$

In this way the optimization problem was reduced to the search for a minimum of a function with one argument. However, it is specific problem, for no general properties of function $C(n,m(n))$ are known, it is not expressed in a known form and it is only possible to compute its value individually for every argument. If a number of possible values of n is small, the minimum could be found by computing and comparing all values of the function. For the case of many possible values of argument n the simple stochastic algorithm will be proposed, because of the long time of computations of values $m(n)$.

Let $\bar{n}$ be such an integer, that $m(\bar{n})=1$, and

$$\tilde{n} = \left(\frac{c\cdot K\cdot\bar{\lambda}\cdot d}{2\cdot[b+I(1)+J(1)]}\right)^{\frac{2}{3}}$$

It is easy to prove, that for $n > n' = \max(\bar{n},\tilde{n})$ the $C(n,m(n))$ is increasing, so the minimum falls to the interval $\langle 1,n'\rangle$. The proposed algorithm consists in independent choice of k points according to the uniform distribution on a set of all integers in the interval $\langle 1,n'\rangle$. This one $/n^*/$ of chosen points is admitted as optimal, for which $C^* = C(n^*,m(n^*))$ is the least (among chosen values). The accuracy of the algorithm is given by the equation:

$$P[\mathcal{P}(\{n : C(n,m(n)) < C^*\}) < \gamma] = 1-(1-\gamma)^k$$

The equation states, that with the probability $1-(1-\gamma)^k$ the part of these points from the interval $\langle 1,n'\rangle$, for which values of $C(n,m(n))$ are less then C^*, is less than γ.

It is reasonable to make slight modifications of this algorithm in the following way:

1. If for some chosen point n_i the necessary number of servicepersons $m(n_i)=1$, then the next point should be chosen according to the uniform distribution on the interval $\langle 1,n_i\rangle$.

2. If follows from a description of a problem (section 1), that the distance between arguments, for which $C(n,m(n))$ is smallest, is small. Therefore it seems reasonable to check a few points near the optimum after finishing of the above algorithm.

6. Conclusions

The presented method of optimization of service systems is useful in practice, however it does not solve the whole problem. Generally, mathematical methods for analysis of public service systems àre not panacea for obtaining optimal solutions, because it is impossible to get perfectly precise mathematical description, for the real world is too complicated. Therefore optimal solutions are in fact optimal for a model, but not necessarily for a real system. However the use of the model makes possible a better understanding of the problem, and also it indicates "good" solutions, from among which we can select the best one.

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NORMATIVE APPLICATION OF CLUSTERING METHODS IN LOCATION/ALLOCATION PROBLEMS

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Abstract:

The paper considers the service points' location/allocation problem over a rural settlement network. The typical formulations of this general question leading to mathematical programming problems are shown as corresponding to appropriate assumptions about the settlement metwork, service provision and transportation. It is then shown that for a wide range of these assumptions it becomes proper to reformulate the problem into an intuitively simpler one, that of grouping or cluster analysis. Principles of cluster analysis are illustrated for the considered case of the rural settlement system. Conditions on the clustering algorithms which would make it possible to explicitly solve the initial location/allocation problem are stated. An appropriate algorithm is presented and an example given.

The course of the paper resides on the assumption that the mathematical questions arising out of the socio-économic problems are highly sensitive to the problem formulations and so are the solution algorithms thereof. By slightly changing initial assumptions one can therefore obtain formulations which are sill adequate, simpler to solve, and may even offer better interpretability in planning practice.

Keywords: location/allocation, mathematical programming, cluster analysis, rural settlement network, services.

1. Introduction: from the initial question to a planning procedure

The main question taken up in this paper is as follows:

- given a set of villages in a certain area, along with distances among them and demands for certain services, how to best (optimally) locate service centers among these villages and allocate other villages to them.

It can easily be seen that the question, thus generally formulated requires more precise definitions of its main elements. As it is stated above, no planner would be able to properly address it in practice, to say nothing of answering. Any design or planning approach to be applied here requires thus a series of additional assumptions, defining the actual problem out of the initial question . Hence, it is to be clarified:

* whether all the villages may be chosen as centers, or perhaps only someu of them, or some other points?
* how are distances defined?
* how are service demands defined?
* can various types of service be separately located/allocated, and if so, under what conditions?
* can individual villages be allocated to more than one center?
* with respect to what criterion shall we measure the quality of solutions obtained, i.e. what does "best (optimally) locate/allocate" mean?

These additional clarifications are just the most essential ones. There are, on the top, many more detailed ones, like: given the subset of eligible centers, i.e. villages or other points in which service centers can be located, for one type of service, can other service types be located in the same centers?

Having had specified all these additional assumptions one has in fact formulated a concrete location/allocation problem, which can take on a definite mathematical shape.

Once mathematical formulation, or, in other words, once a formal model of the planning situation established, algorithms for solving the problem can be looked for. In many cases it may turn out that for specific problem formulations efficient algorithms that find the global optima cannot be obtained. In some other cases reaching of the global optima can amount to a lengthy and cumbersome process.

Simultaneously, it can easily be observed that additional assumptions which complete the definition of the planning problem are to some extent axbitrary. Thus, without any loss to adequacy the problem could be reformulated in such a way that it would become much easier to set up an efficient algorithm for solving it.

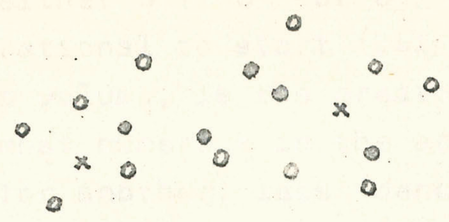
Hence, the actual planning procedure involves both the solution algorithm and full recognition of the significance and role of additional assumptions, completing the model of the planning situation. The procedure consists of model building and solution activities interlocked in a feedback loop.

In its formal considerations, this paper refers to a large extent to Domański [1981] and Leonardi [1980].

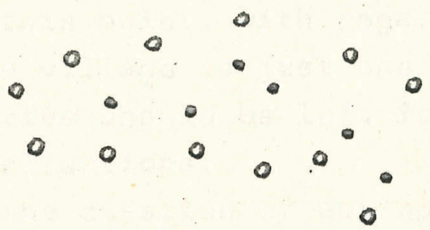
2. Model specification

According to previous explanations model specification would consist of answers to questions asked in the introduction, and meant to provide a precise definition of the problem. Let us how outline the answers to these questions while introducing appropriate notation:

- the villages, numbered i, belong to the set I, $i \in I$, so that $i=1,2,\ldots,$ card I, while the eligible centers, numbered j, belong to the set J, $j \in J$, so that $j=1,2,\ldots,$ card J, and nothing specific is assumed about the relation of I and J, hence generality may be preserved , and we might have either of: $J = I$, $J \not\subset I$, $I \cap J \not\subset J$, $I \cap J = \emptyset$, see Fig. 1,
- regarding distances it may be assumed that for each village i and each eligible center j the shortest route between i and j over the existing transportation network is taken, amounting to distance d_{ij}, and in case of multiple transportation modes the d_{ij} would correspond to the route with the lowest transportation cost,
- service demand of each village is simply determined as proportional to the equivalent adult population of the village, with the rule of transformation into equivalent adults for all age groups and the coefficient of proportionality corresponding to a particular service type, the demand volume thus defined for village i denoted a_i, or a_i^k if service types are numbered k, $k \in K$,
- with regard to separation of location/allocation tasks for various service types it may be assumed that for any two service types k' and k'' if we denote by $J^{k'}$ and $J^{k''}$ the aporopriate sets of eligible centers there should be



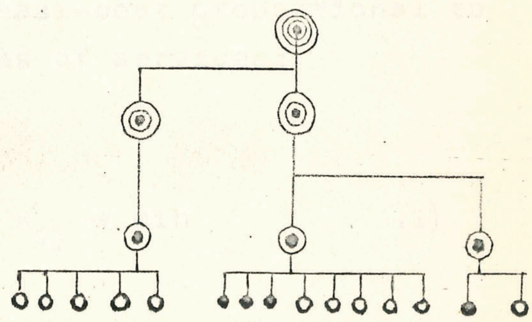
A. $J - I \cap J \neq \emptyset$, $I \cap J \neq \emptyset$



B. $J = I \cap J$,

Fig. 1. Two examples of the sets of villages, I, and eligible centres, J. ○ - $i \in I$
● - $i \cap j \in I \cap J$
× - $j \in J$

Resulting hierarchy:



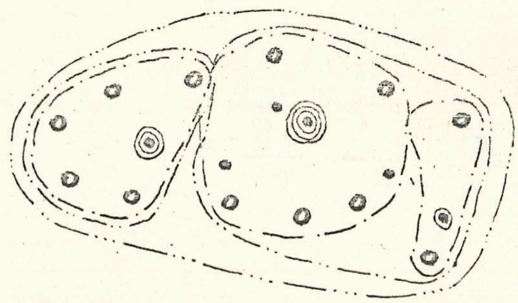


Fig. 2. An example of a solution for the B. case of Fig. 1, when a village i can be allocated to one centre j, and there are three types of services, k = 1,2,3.

◎ —·— first order center and the boundary of its service area,
◎ —··— second order center and the boundary of its service area,
◎ —···—third order center and the boundary of its service area,

either $J^{k'} \subset J^{k''}$ or $J^{k''} \subset J^{k'}$; in particular, it seems rational to start $(k=1)$ with services whose demand, in a volume, is the greatest, i.e. whose centers would be most numerous in the area, and choose as J^{k+1} each time for another, less "dense" demand, the set of centers which have before been found as optimal, J^{k*} , so that a hierarchy of centers is created, see Fig. 2,

- while previous assumptions might be regarded as straightforward simplifications, intuitively reasonable enough, at this point, with regard to the alternative of allocating a village to just one or at least one center, an alternative should be left for choice for individual planning situations,
- the question of appropriate design of the criterion for measurement of quality of various location/allocation schemes deserves special attention, since it usually constitutes the most arbitrary and discussable element of the model, while on the other hand it determines to a large extent the outlook of the location/allocation scheme eventually obtained; for the sake of brevity let us exemplify a few of the criteria formulated, together with additional characteristics of the modelling situations in which these quantitative criteria of quality may be applied:

  ∘∘ Minimum transportation quasi-cost proportional to distances travelled and volumes of services:

$$Q_1 = \sum_j \sum_i a_i d_{ij} x_{ij} \rightarrow \min \qquad (1)$$

where x_{ij} denote allocation of i to j, with x_{ij} subject to one of the following conditions:

$$\sum_j x_{ij} = 1 , \quad x_{ij} \in [0, 1], \qquad (1A)$$

$$\sum_j x_{ij} = 1 , \quad x_{ij} \in \{0, 1\} \qquad (1B)$$

$$\sum_j x_{ij} \geqslant 1 , \quad x_{ij} \in \{0, 1\} \qquad (1C)$$

It can easily be seen that in cases (1A) and (1C) it is assumed that each village i can be allocated to more than one center j.

°° Minimum transportation and functioning cost:

$$Q_2 = Q_1 + \sum_j b_j x_{jj} + \sum_j c_j \sum_i a_i x_{ij} = Q_1 + Q_{12} \rightarrow \rightarrow \min \qquad (2)$$

with conditions on x_{ij} as before. Another variant of the problem formulation is

°° Minimum transportation cost:

$$Q_1 \rightarrow \min \qquad (3)$$

with conditions as in (1), and also with $Q_{12} \leqslant C$ (3A) as an additional financial constraint.
Another, simple form of the criterion is:

$$°° \; Q_3 = \sum_j x_j \rightarrow \min \qquad (4)$$

where x_j denotes service location at center j, with

$$\sum_{j \in N_i} x_j \geqslant 1, \quad x_j \in \{0, 1\} \qquad (4A)$$

and N_i denotes the set of eligible centers $j \in J$, $N_i \subset J$, to which i-th village can be allocated by virtue of some condition, e.g.

$$N_i = \left\{ j \in J \,\middle|\, d_{ij} \leqslant s \right\} \qquad (4B)$$

s being a certain threshold distance. This formulation is, of course, a simple expression of the will of obtaining a full coverage of the rural settlement system by the service network distance-wise, with the capacity and cost issues altogether abandoned.

There is more of such various formulations, differing with regard to criteria and conditions thereof, the ones presented here, however, illustrate the principles governing the transformation of the real problem into its mathematical model form. The variety given here, mainly after Domański [1981] allows creation of other, more refined forms.

3. Mathematical programming algorithms

Most of the models shown in the previous section are treated as representing mathematical programming type problems, whose solutions are sought with the respective type of algorithms. In short, a set of values of x_{ij} or x_j is sought such that minimizes or maximizes a Q, with x_{ij} / x_j subject to certain conditions. In mathematical programming language the Q is called objective function and conditions on variables x_{ij}/x_j are called constraints.

Effectiveness of programming algorithms varies, however, essentially, according to the type of problem. The most effective, linear programming (LP) algorithms, can be applied in cases of criteria (1) or (2) or (3) with constraints (1A) and/or $Q_{12} \leqslant C$. These algorithms give relatively quickly optimal solutions in terms of x_{ij}, with appropriate solution procedures existing as standard software equipment with most of the computer systems. Even relatively large - as measured by card I and card J - problems can be easily solved with these procedures.

More difficulties are offered by the 0-1-programming problems, i.e. those ones, in which x_{ij} or $x_j \in \{0,1\}$. Unless dimensions of such problems are relatively small, there may arise computational difficulties, due to the non-continuous nature of the problems. The same sort of obstacles is encountered in mixed-programming problems, i.e. such in which some x_{ij} are allowed to take on values from the continuous set constituted by the segment $[0,1]$, while other may only take on either 0 or 1 as their value.

Notwithstanding, however, the degree of computational facility related to particular problem or algorithm, there is another question, related indirectly to the assumption of continuity or discontinuity of x_{ij}/x_j. It is the question of sensitivity and stability of results when the parameters, such as a_i, b_j etc. are slightly changed. It may quite often turn out that such slight changes in parameters bring about important modifications in the structure of results. These situations are not only caused by the objective nature of the data used, but, quite often, by -- almost inevitable -- mistakes in formal and numerical specification of the problem. The faults may include e.g. too big differences in magnitudes of parameters -- i.e. measurement unit misspecification, so that some of the parameters yield singularities in computations, or near-colinearity of some constraints, leading to similar difficulties.

These questions and, indeed, a reinterpretation of basic assumptions leads to a reassessment of the whole problem.

4. Some features of the optimal location/allocation solution

According to assumptions concerning sets I and J, allocatability of i to j, and presented LP problem formulations, an important feature of the problem's solution can be approached in the following way.

Assume that in the optimal solution a subset $J^{opt} \subset J$ of centers is chosen. Centers belonging to it, j^{opt}, are numbered $j^{opt} = 1,2,\ldots,$ card J^{opt}. To each center j^{opt} a set I_{jopt} of villages is allocated, such that

$$I_{jopt} = \{ i \in I \mid x_{ijopt} > 0 \}$$

i.e. it is the set of villages i whose inhabitants are acquiring service at j^{opt}. The sets I_{jopt} can be further broken down into two subsets:

$$I_{jopt} = I^{o}_{jopt} \cup I^{+}_{jopt}, \quad I^{o}_{jopt} \cap I^{+}_{jopt} = \emptyset, \text{ so that}$$

$$I^{o}_{jopt} = \{ i \in I_{jopt} \subset I \mid x_{ijopt} = 1 \} \text{ and } I^{+}_{jopt} = \{ i \in I_{jopt} \subset I \mid x_{ijopt} \in (0,1) \}$$

i.e. inhabitants of $i \in I^{o}_{jopt}$ are getting served solely at jopt, while those from $i \in I^{+}_{jopt}$ have choice of either jopt or some other center(s). Of course, in case of 0-1-programming, sets I^{+}_{jopt} are empty, and $I_{jopt} = I^{o}_{jopt}$ for each jopt.

Shapes of J^{opt}, I^{o}_{jopt}, I^{+}_{jopt} are defined by actual parameters of the problem. It can be safely assumed that composition of J^{opt} is more stable than the composition of I^{o}_{jopt}, which, in turn, is more stable than the levels of x_{ijopt} for i -- and jopt -- within the I^{+}_{jopt}. Thus, as inevitable errors are made in numerical specification of parameters, resulting output errors are the greatest in the levels of x_{ijopt}, for $i \in I^{+}_{jopt}$.

In general $j \in J^{opt} \Leftrightarrow I_j \neq \emptyset$, it may happen, however, that $I^{o}_{jopt} = \emptyset$, and in that case the very presence of this jopt in J^{opt} is importantly vulnerable to errors in parameters.

5. A choice of formulation

Since the only way to cope with the errors in data is to analyze resulting hypothetical variability of results -- via variance or some other magnitude -- it seems equally plausible to perform such analysis assuming $x_{ij} \in [0,1]$ and assuming $x_{ij} \in \{0,1\}$. In fact, in the first case the analysis is very cumbersome, while in the second rather simple tests can be set up in order to invalidate the hypothesis initially accepted.

Denote by I^{01}_{jopt} the sets of i's related to jopt obtained with condition $x_{ij} \in \{0,1\}$ for the same task, for which with condition $x_{ij} \in [0,1]$ the sets I^{o}_{jopt} and I^{+}_{jopt} were obtained.

Then $I^{o}_{jopt} \subset I^{O1}_{jopt}$ for all jopt if both J^{opt} are the same, which may not be the case, in general.

With that in mind, let us come back to the objective function formulations contained in (1) to (4). According to previous considerations, problem formulations (1) - (1B) and (1) - (1C) lead to $I^{+}_{jopt} = \emptyset$, with, however, important difference in that an i which belongs to an I^{o}_{jopt} in the (1) - (1C) case might, by virtue of the constraint requirement, belong also to another I^{o}_{jopt}. It cannot, however, since that would mean addition of a deletable element $a_i d_{ijopt}$ to Q_1, which should be minimized. Further on we shall therefore consider I^{o}_{jopt} as sets of i's which are fully and uniquely getting service from a jopt.

Now let us consider conditions of non-emptiness for I^{+}_{jopt} in the (1) - (1A) case. Assume a I^{+}_{jopt} non-empty, and denote J^{opt}_{i} the set of jopt providing service to an i. When $i \in I^{+}_{jopt}$, then card $J^{opt}_{i} \geqslant 2$. Obviously, for the jopt $\in J^{opt}_{i}$ there exists a $\overset{*}{j}opt$ such that $a_i d_{i\overset{*}{j}opt} \leqslant a_i d_{ijopt}$, for all jopt. Hence, by setting $x_{i\overset{*}{j}opt} = 1$ and all the other $x_{ijopt} = 0$, one obtains at least a non-worse value of Q_1. Thus, in order to obtain non-empty sets I^{+}_{jopt} one would have to consider the condition:

$$\sum_{i} x_{ij} \leqslant X_j \qquad (1D)$$

which is the capacity condition, or, which amounts to about the same:

$$c_j \sum_{i} a_i d_{ij} x_{ij} + b_j x_{jj} \leqslant C_j \qquad (1E)$$

i.e. the setup and running cost limitation for centers j.

There is still another side-effect of the (1) etc. formulation. Namely, there exists a tendency towards includning as many $j \in J$ into J^{opt} as possible, provided $I^{min}_{j} = \{ i \in I \mid d_{ij} < d_{ij'}, \forall j' \in J \} \neq \emptyset$. When $I^{min}_{j} = \emptyset$, which may happen only when $j \notin I$, this j would not be included into J^{opt}. All the other j will enter J^{opt}.

Hence, it seems rational to focus attention on problems of type (2) - (1B) which:

- allow easy analysis of stability and sensitivity of results,
- avoid unnecessary abritary conditions on individnal centers, like (1D) and (1E),
- avoid a priori biasses in composition of J^{opt}, and
- provide a broadly based criterion for comparing solutions.

6. Another approach: a reformulation

Since we are now dealing with the case when any village i is univocally allocated to just one center j, the problem boils down to finding $\{I_{jopt}, jopt\}$, i.e. the optimal set $J^{opt} \ni jopt$ of centers, and optimal partion of I, $\bigcup_{jopt} I_{jopt} = I$;

$I_{j'opt} \cap I_{j''opt} = \emptyset$, $j'opt \neq j''opt$ as exemplified in Fig. 2.
The set of pairs $\{I_{jopt}, jopt\}$ of subsets of I and centers $jopt \in J^{opt}$ corresponding to them should minimize a criterion similar to (2).

Let us therefore denote:

$$\delta_{ij} = a_i (d_{ij} + c_j), \text{ for } i \neq j, \text{ and}$$

$$\delta_{jj} = b_j, \text{ otherwise}$$

so that

$$Q_2 = \sum_j \sum_i \delta_{ij} x_{ij}, \qquad i \in I,\ j \in J,\ x_{ij} \in \{0,1\}$$

where δ_{ij} could be interpreted as sort of generalized distance measures. The question of metricity does not apply here, for $_{ij}$ is fully asymmetric. With this interpretation, one can relatively easily set up a following suboptimization algorithm:

1. for each $i \in I$ form a sequence of j,

$$J^{min}_{(i)} = \left\{ j_i^1, j_i^2, \dots, j_i^{card\ J} \right| $$

$\left| \delta_{ij_i^k} \leq \delta_{ij_i^{k+1}} \right\}$, so that each subsequent element $j \in J$ in such a sequence is more "distant" from i in terms of δ_{ij}, and therefore from a local viewpoint it would be most advantageous to allocate i to j_i^1;

2. simultaneously form for each j a set $I_j^{min} = \left\{ i \in I \middle| j_i^1 = j \right\}$, i.e. the set of i's "allocatable" to given j;

3. for each $j \notin I$, such that $I_j^{min} \neq \emptyset$, either modify δ_{ij},

$i \in I_j^{min}$, so that $\delta^1_{ij} = \delta_{ij} + \dfrac{b_j}{card\ I_j^{min}}$

and return to 1, or, if δ_{ij} have already been modified, check whether any changes occurred to I_j^{min}, and if there were no changes or the only changes consisted in decreasing of card I_j^{min}, then go to 4, otherwise modify δ_{ij} appropriately and return to 1;

4. classify $j \in J^o = \left\{ j \in I \middle| j \notin I_j^{min} \right\}$ according to whether

   a. $I_j^{min} = \emptyset$, and then $j \in J^{oo}$, otherwise, if

   b. $I_j^{min} \cap J = \emptyset$, and then $j \in J^{O1}$, otherwise, if

   c. $I_j^{min} \cap J \neq \emptyset$, and then $j \in J^{O2}$;

5. omit J^{oo} from further considerations;

6. for each $j \in J^{01} \cup J^{02}$ construe an alternative:

$a1 \equiv j \in I_j^{min}$ or $a2 \equiv I_j^{min} \subset \overset{*}{I}{}^{min}_{j_j^1}$, as a set $a(j) = \{a1(j), a2(j)\}$

7. form a set of all alternatives over $j \in J^{01} \cup J^{02}$ as the cartesian product

$$\prod_{j \in J^{01} \cup J^{02}} a(j) = A(J^{01} \cup J^{02})$$

8. check whether for $j \in J^{02}$ there are no closed loops, i.e. in a chain $\{j, j_j^1, j_{j_j^1}^1, \ldots\}$ where all elements belong to J^{02} no element is repeated, and if so, delete from $A(J^{01} \cup J^{02})$ alternatives corresponding to $a2(j)$ for all j, in such a chain.

9. search for min Q^2 over resulting A.

This algorithm may seem a bit cumbersome at a first glance, for practical cases, however, both step 3 and analysis of A should not present serious difficulties. Since we are not dealing here directly with computer applications, let us consider this simplistc heuristic suboptimization algorithm for an example depicted below.

7. An example

In this example we have card I = 10, card J = 6, i.e. we are dealing with 10 villages and 6 eligible centres. $I \cap J = \{a, c, d, e, f\}$, where $\{a, b, c, d, e, f\} = J$.

Numerical characteristics are:

$i \in I$	1	a	3	4	5	c	d	e	9	f
a_i	3	10	4	4	5	12	7	6	3	8

where a_i represent the volume of service per village,

$j \in J$	a	b	c	d	e	f
c_j	3	3	2	3	3	4
b_j	20	30	17	29	27	28

where c_j and b_j are expressed in appropriate monetary units,

D:

i	a	b	c	d	e	f
1	5	8	11	13	13	16
a	ø	2	7	9	10	10
3	2	1	5	8	8	9
4	3	1	5	7	8	8
5	6	3	2	5	6	7
c	7	5	ø	2	4	6
d	9	7	2	ø	1	4
e	10	8	4	1	ø	3
9	10	9	4	3	2	1
f	10	10	6	4	3	ø

where d_{ij} are expressed in monetary units corresponding to proportional transportation costs,

on the basis of the above we can set up the table of δ_{ij}, Δ:

Δ:

i \ j	a	b	c	d	e	f
1	24	33	39	48	48	60
a	20	50	90	120	130	140
3	20	16	28	44	44	52
4	24	16	20	40	44	48
5	45	30	20	40	45	55
c	120	96	17	60	84	120
d	84	70	28	29	28	56
e	78	66	36	24	27	42
9	39	36	18	18	15	15
f	104	104	64	56	48	28

where δ_{ij} are expressed in monetary units.

In the review phase, step 1 and 2, we obtain: $I_a^{min} = \{a,1\}$, $I_b^{min} = \{3,4\}$, $I_c^{min} = \{c,5,d\}$, $I_d^{min} = \{e\}$, $I_e^{min} = \{d,9\}$, $I_f^{min} = \{f,9\}$. Passing over to step 2, one modifies δ_{3b} and δ_{4b}, so that $\delta_{3b}^1 = 31$ and $\delta_{4b}^1 = 31$ and they are no longer minimal over their rows. Modified result of steps 1 and 2 now takes on the form: $I_a^{min} = \{1,a,3\}$, $I_b^{min} = \emptyset$, $I_c^{min} = \{4,5,c,d\}$, $I_d^{min} = \{e\}$, $I_e^{min} = \{d,9\}$, $I_f^{min} = \{9,f\}$.

Evidently, $b \in J^{00}$, and may not be considered further on. On the other hand, I_d^{min} and I_e^{min} belong to J^{02}. The set of alternatives A is as follows:

$\{\{I_d^{min} = \{d\},\ I_e^{min} = \{e,9\}\},\ \{I_d^{min} = \{d,e9\},\ I_e^{min} = \emptyset\},$

$\{I_d^{min} = \emptyset,\ I_e^{min} = \{d,e,9\}\},$

$\{I_d^{min} = \emptyset,\ I_e^{min} = \emptyset \Rightarrow I_c^{min} = \{4,5,c,d,e,9\}\},$

$\{I_d^{min} = \{e\},\ I_e^{min} = \{d,9\}\}\}$

The fourth alternative is not necessary since no improvement in Q_2 can be achived with it, for the sake of completeness, however, it can be generated. The last alternative will be omitted as a result of step 8. Thus, the following values of Q_2 are to be compared:

$$Q(A_1) = 220$$
$$Q(A_2) = 220$$
$$Q(A_3) = 219$$
$$Q(A_4) = 231$$

Hence, since $Q(A_3)$ is minimal, the solution is as follows:

$$I_a = \{1,a,3\}, \quad I_b = \emptyset, \quad I_c = \{4,5,c\}, \quad I_d = \emptyset,$$

$$I_e^{min} = \{d,e,9\}, \qquad I_f = \{9,f\}$$

In this solution the village 9th can be allocated either to centre e or to centre f.

8. A broader perspective

The simplistic algorithm here presented allows reaching of optimal solution in most practical cases and is intuitively easily interpretable. Moreover, it may be manually operated, in contradistinction to mathematical programming techniques, which may, additionally, turn out inapplicable to this particular problem. There is, however, one important difficulty in the wider usage of the algorithm outlined, related to the algorithm's generality.

In fact, although the objective function Q_2 is general enough, the main question resides in the divergence from optimality inherent to the algorithm. This divergence shall in reality depend upon changes to problem formulation and assumptions thereof. It can be observed, however, that the assumptions previously mentioned can be relatively easily integrated into the Q_1 framework.

Let us now reformulate:

$$Q_2 = \sum_j \sum_{i \in I_j^{min}} \delta_{ij}$$

This structure is indeed a structure of grouping or clustering of $i \in I$ around $j \in J$. A general method developed by Diday [1975] could be used here, instead of the algorithm presented. Minimization of Q_2, albeit suboptimal, can be achieved via application of appropriate cluster analysis methods, such that allow in principle for separation of I and J, and for alternative paths in the algorithm, as corresponding to steps 4 to 9 in the algorithm proposed. A method suitable for that purpose was presented by Owsiński [1981]. This method would weigh δ_{ij} for $i \in I-J$ against δ_{jj}, and stepwise form clusters increasing in numbers of elements.

Thus, a relatively general problem of locating service centers in a rural setting can be satisfactorily solved via quite simple methods, when only appropriate interpretation of basic assumptions is made. Such simple methods may better serve planning purposes than the more complicated ones.

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