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Figura solida in the Early Medieval Manuscripts of *De natura rerum* by Isidore of Seville as a Circular Diagram According to the Geometric Ratio[1]

ABSTRACT: The Platonic (or more appropriately the Neo-Platonic) doctrine of the elements of matter, their properties, and their harmonious interrelations received significant attention during the Middle Ages. One example is the treatise *On the Nature of Things* (*De natura rerum*) by Isidore of Seville. In the chapter entitled *De partibus mundi*, the Bishop of Seville dealt in more detail with the Aristotelian doctrine of the elements (*De natura rerum* XI, 2–3), but he also introduced the (Neo-)Platonic theory (*De natura rerum* XI, 1). He supplemented both descriptions of the teachings regarding the elements of matter with a diagram illustrating the theories presented in words. In the case of the drawing to illustrate Plato's doctrine, his diagram is called *figura solida secundum geometricam rationem*, and it is very difficult nowadays to determine what this image was intended to look like, what exactly it was to show, what it was supposed to be used for, and what its explanatory value was. The main reason for these uncertainties is the great variety of forms in which the diagram has been preserved; moreover, in many cases it is difficult to uncover the nature of the geometric ratio and harmony, or the spatial character of the diagram in the extant forms.

In this paper, the focus is on the diagram *figura solida* and some of its most frequently preserved forms. The aim of the paper is to show that the widely differing variants of the *figura solida* can be explained in accordance with period views of solid geometric figures and the nature of the geometric ratio. For geometric solids, the explanation presented by Macrobius in *Commentary on the Dream of Scipio* is used in this article. Here, plane parallelograms characterise the parallelepipeds, and the three dimensions of solids can be adequately represented by two plane quadrilaterals placed one above the other. Geometric ratios were described by Calcidius (and also, e.g., by Boethius or Martianus Capella), who paid considerable attention to them in his interpretation of Plato's dialogue *Timaeus* and also put them in direct reference to the elements and their properties and mutual relations.

Using these sources (no doubt familiar to Isidore), this paper offers an interpretation of two often appearing variants of the *figura solida* in the manuscripts. These two forms both respect Calcidius' conclusions regarding plane geometric analogies and the justification of

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the application of plane ratios (with one mediator) to solids representing elements (the need for two mediators). Moreover, this interpretation makes it possible to explain the diagram of the *figura solida* as a circular diagram, which would also make this diagram fully compatible in with the nature of the other diagrams contained in Isidore's *De natura rerum*, which is why the name *Liber rotarum* was adopted for this work in the Middle Ages.
KEYWORDS: Isidore of Seville • elements • *figura solida* • Calcidius • Macrobius

1. Introduction

The treatise *De natura rerum* by Isidore of Seville was probably completed in 612 or 613[2], and it can be assumed that it quickly gained considerable popularity, which remained to varying degrees throughout the Middle Ages. At least this can be judged from the numerous extant medieval manuscripts of this work[3], the oldest of which dates from the 7th century[4]. Isidore's book contains a set of figures and pictures (*figura*, *pictura*, *formula*)[5], most of which are circular in their composition (*rota*), which gave the work the alternative and often appearing title *Liber rotarum* in the earliest manuscripts[6].

These figures or diagrams from Isidore's treatise receive relatively extensive attention these days. Discussions are held about their purpose, their source, or their originally intended form[7]. Besides the so-called

[2] Cf., for example, A. Fear, *Putting the Pieces Back Together. Isidore and De Natura Rerum*, [in:] *Isidore of Seville and his Reception in the Early Middle Ages. Transmitting and Transforming Knowledge*, eds. A. Fear & J. Wood, Amsterdam 2016, p. 79; M. J. Carlos, *Isidorus Hispalensis ep.*, [in:] *La trasmissione dei testi latini del Medioevo. Mediaeval Latin Texts and their Transmission*, vol. II., eds. P. Chiesa & L. Castaldi, Florence 2005, p. 353.

[3] Most of the extant manuscripts are dated to the 9th century, but the 8th and 10th–12th centuries are also frequently represented – cf. C. B. Kendall & F. Wallis, *Introduction*, [in:] Isidore of Seville, *On the Nature of Things*, transl. C. B. Kendall & F. Wallis, Liverpool 2016, pp. 66–99.

[4] The earlier Fontaine's *stemma codicus* was followed up by (especially during the 7th–9th centuries) M. J. Carlos or M. M. Gorman. See J. Fontaine, *Introduction*, [in:] Isidore de Séville, *Traité de la nature / De natura rerum*, ed. & trad. J. Fontaine, Bordeaux 1960, p. 70 bis; M. J. Carlos, *op.cit.*, p. 357; M. M. Gorman, *The Diagrams in the Oldest Manuscripts of Isidore's «De natura rerum» with A Note on the Manuscript Traditions of Isidore's Works*, „Studi Medievali", 2001, t. 42, p. 541.

[5] For more details see, e.g., F. Wallis, *What a Medieval Diagram Shows: A Case Study of Computus*, „Studies in Iconography", 2016, t. 36, p. 37.

[6] Details can be found, e.g., in C. B. Kendall & F. Wallis, *op.cit.*, pp. 27–28, or pp. 36–38.

[7] Cf., for example, B. Obrist, *Le diagramme isidorien des saisons, son contenu physique et les représentations figuratives*, „Mélanges de l'Ecole française de Rome. Moyen Âge", 1996, t. 108/1, pp. 95–164; B. S. Eastwood, *The diagram of the four elements in the oldest manuscripts of Isidore's «De natura rerum»*, „Studi medievali", 2001, t. 42/2, pp. 547–70; B. Teyssèdre, *Un exemple de survie de la figure humaine dans les manuscrits précarolingiens: Les illustrations du De natura rerum d'Isidore*, „Gazette des Beaux-Arts", 1960, t. 56, pp. 19–34; and so on.

T–O diagram[8], particular attention is paid to the so-called *figura solida*, which, in Isidore's own words, is supposed to clarify (*expressit*) the (Neo-) Platonic doctrine of the properties of the elements and the relations between them, as discussed in *De natura rerum* XI, 1[9].

For the sake of brevity, it is sufficient to state that Isidore speaks of the four parts of the world (*quattour partes mundi*), namely the traditional elements, i.e., fire (*ignis*), air (*aer*), water (*aqua*), and earth (*terra*). Each of the four elements has in its natural constitution three key properties: fire is thin (*tenuis*), sharp (*acutus*), and mobile (*mobilis*); air is mobile (*mobilis*), sharp (*acutus*), and thick (*crassus*); water is thick (*crassa*), blunt (*obtunsa*), and mobile (*mobilis*); and earth is thick (*crassa*), blunt (*obtunsa*), and immobile (*inmobilis*). Although all four elements are characterised differently, they are nevertheless interconnected (*conmiscere*). Earth and water are linked by bluntness and thickness; water and air by thickness and mobility; and air and fire by mobility and sharpness. Thus, earth and water are completely distinct and separate from each other (*separare*), but through two mediators (i.e., water and air), they are connected (*iungere*) to each other. In order to make this clear and not confusing (*ne confusa*), Isidore attached a diagram that is labelled as the solid figure according to a geometric ratio (*figura solida secundum geometricam rationem*)[10].

This diagram is preserved in widely varying forms in the extant manuscripts of *De natura rerum*. B. S. Eastwood attempted to reconstruct its primary form, which might be the closest to Isidore's original picture[11]. In principle, he suggests that Isidore drew two vertical quadrilaterals, that are parallel to each other, which Isidore attempted to connect through the base (horizontal quadrilateral), creating the appearance of a spatial shape. While the front vertical quadrilateral carries the information that it is a solid figure according to geometric ratio, the back vertical quadrilateral is divided into four triangles according to diagonal lines, and each triangle represents one element, including their key characteristic properties – see Fig. 1, which attempts to capture this diagram in a schematic form[12].

[8] Cf., e.g., C. Mauntel, *The T-O Diagram and its Religious Connotations. A Circumstantial Case*, [in:] *Geography and Religious Knowledge in the Medieval World*, ed. C. Mauntel, Berlin – Boston 2021, pp. 57–82; W. M. Stevens, *The Figure of the Earth in Isidore's "De natura rerum"*, „*Isis*", 1980, t. 71/2, pp. 268–77; etc.

[9] Isidore de Séville, *Traité de la nature* XI, 1, p. 213: *Haec itaque ne confusa minus colligantur, subiecta expressi picture* […].

[10] *Ibidem*.

[11] See, B. S. Eastwood, *The diagram of the four elements*, pp. 547–70.

[12] According to the classification of diagram forms in Eastwood's article, it is Group I, which means, e.g., manuscripts: Paris, Bibliothèque nationale de France, ms. Lat. 6400G, f. 122r (https://gallica.bnf.fr/ark:/12148/btv1b105154335/f251.item); Laon, Bibliothèque munici-

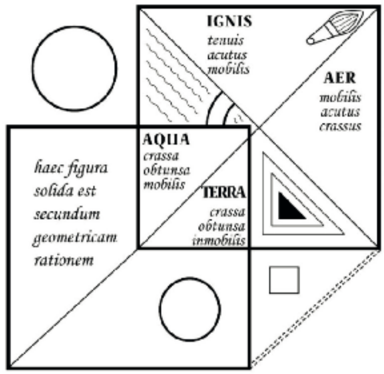

Fig. 1. Reconstruction of the form of the diagram of the *figura solida*, which could be close to Isidore's original picture, according to the interpretation of B. S. Eastwood.

Earlier, John E. Murdoch offered a different explanation of the *figura solida*[13]. He assumed that the diagram should represent a cube (the so-called *cybus elementorum*, although Isidore did not use this term despite Murdoch's opposite assertion), where the six faces of the cube represent the six properties of the elements (always two opposing properties are symbolised by two parallel opposing faces of the cube). Since each element has three constituent physical properties, Murdoch placed these elements on four of the eight vertices of the cube, with each element located on the vertex where its three properties meet. Fig. 2 tries to capture Murdoch's interpretation.

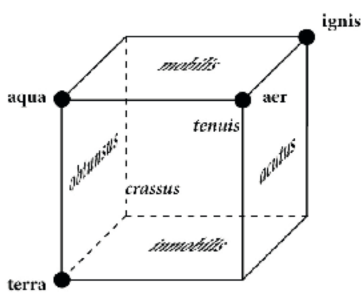

Fig. 2. Murdoch's representation of the *figura solida*.

pal, ms. 423, f. 11v (https://bibliotheque-numerique.ville-laon.fr/viewer/1465/?offset=#page=32&viewer=picture&o=bookmark&n=0&q=); Köln, Dombibliothek, Hs. 83/II, f. 131r (https://digital.dombibliothek-koeln.de/hs/content/zoom/168782); Bern, Burgerbibliothek, Cod. 224, f. 167v (https://www.e-codices.unifr.ch/de/bbb/0224/167v); Karlsruhe, Badische Landesbibliothek, Aug. 229, f. 152v (https://digital.blb-karlsruhe.de/blbhs/Handschriften/content/pageview/17242); Laon, Bibliothèque municipal, ms. 422, f. 8v (https://gallica.bnf.fr/ark:/12148/btv1b8492138z/f28.item).

[13] J. E. Murdoch, *Album of Science. Antiquity and the Middle Ages*. New York 1984, pp. 280–81.

Murdoch's short text suggests his belief that Isidore was following the older Neo-Platonic tradition and may have been inspired by a now-lost source. This would certainly be possible (e.g., Barbara Obrist has presented a similar view[14]), but contemporary early medieval sources do not permit more than regarding these claims as hypothesis. On the other hand, it can also be assumed, as, e.g., Jacques Fontaine, editor of the *De natura rerum*, has already claimed, that Isidore created this scheme himself[15]. Then, it might be expected that if he wanted the diagram to look like a cube it would indeed look like this[16]. This would mean that Isidore's intended image of the *figura solida* could have looked as Eastwood's interpretation implied, but it could also have corresponded to other extant examples of the diagram from the earliest manuscripts of the *De natura rerum*. Which in some cases omit the three-dimensional illusion of the diagram (horizontal square) and place the two quadrilaterals overlapping each other or close together[17] – see Fig. 3.

[14] See, in particular, B. Obrist, *Wind Diagrams and Medieval Cosmology*, „Speculum", 1997, t. 72/1, p. 64; eadem, *La cosmologie médiévale. Textes et images. I. Les fondements antiques*. Firenze 2004, pp. 276–277; for a critical viewpoint, see, e.g., M. M. Gorman, *op.cit.*, p. 532.

[15] J. Fontaine, *Introduction*, p. 17. However, even Fontaine acknowledged that it is equally possible that Isidore was inspired by the older tradition of commentaries on *Timaeus* – see, J. Fontaine, *Isidore de Séville et la culture classique dans l'Espagne wisigothique*, Paris 1959, pp. II/657–59.

[16] Despite the illusion of a cube in many extant early medieval representations of the diagram and the possible connection with similar illustrations of the cube in period manuscripts (cf. B. S. Eastwood, *The diagram of the four elements*, pp. 553–54), there is reasonable doubt that the diagram of the *figura solida* is indeed a cube.

[17] According to Eastwood's classification of the oldest preserved versions of the diagram, this would be mainly Group III and IV. Manuscript references can be made to the following: for Fig. 3a – Bamberg, Staatsbibliothek, Patr. 61, f. 88v (https://www.bavarikon.de/object/SBB-KHB-00000SBB00000157); El Escorial, Real Biblioteca del Monasterio se San Lorenzo, Cod. R.II.18, f. 6r (https://rbme.patrimonionacional.es/s/rbme/item/14202#?xywh=-1319%2C-1%2C4290%2C2339&cv=11); Vatican City, Bibliotheca Apostolica Vaticana, Reg. lat. 255, f. 6v (https://digi.vatlib.it/view/MSS_Reg.lat.255). For Fig. 3b – Bamberg, Staatsbibliothek, Nat. 1, f. 15r (https://www.bavarikon.de/object/bav:SBB-KHB-00000SBB00000139?p=1&cq=Nat%201&lang=de); Vatican City, Bibliotheca Apostolica Vaticana, Pal. lat. 834, f. 60r (https://digi.vatlib.it/view/bav_pal_lat_834); Besançon, Bibliothèque municipale, ms. 184, f. 18v (https://bvmm.irht.cnrs.fr/iiif/7746/canvas/canvas-1255143/view). For Fig. 3c – Paris, Bibliothèque nationale de France, Lat. 10616, f. 27v (https://gallica.bnf.fr/ark:/12148/btv1b9066990b/f29.item); Cambrai, Bibliothèque municipale, ms. 937, f. 45v (https://bvmm.irht.cnrs.fr/iiif/10506/canvas/canvas-1297244/view) – bottom left image.

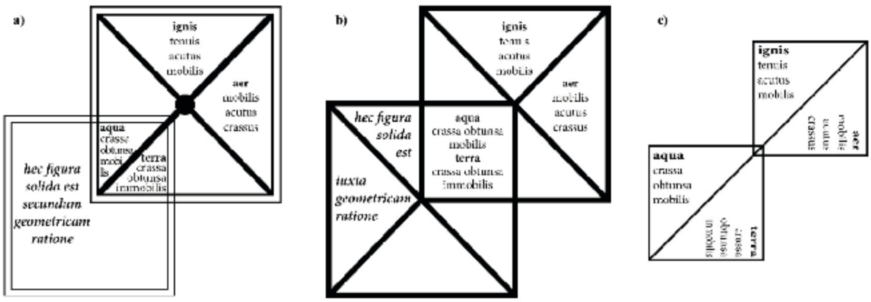

Fig. 3. *Figura solida* as two quadrilaterals with elements written in one or both of them.

The various interpretations of Isidore's *figura solida* that have been offered, and were briefly shown here, demonstrate how difficult it is to identify the initial form, the sources of inspiration, and the intended purpose of the diagram. Since there is no clear evidence of Isidore's original intent, we can only speculate on all these issues. The present paper tries to demonstrate that many of the extant manuscript versions of the *figura solida* to *De natura rerum* XI, 1, can be sufficiently interpreted by considering several aspects of the context of the period. These are mainly: Isidore's (and common at the time) geometric reasoning about three-dimensional solids (see Part 2 of this article); Calcidius' commentary on his own translation of part of Plato's dialogue *Timaeus*, especially the considerations there about the analogical bond, i.e., geometric proportion according to geometric ratio (see Part 3); the diagrams in *De natura rerum* themselves, which are circular in their character, so that it is possible to interpret the *figura solida* as a circular diagram as well (see Part 4).

In this respect, it can be said that the aim of this paper is to show that the *figura solida*, even though it may not always seem to be, it can read as a *rota* diagram, and therefore belongs fully to the so-called *Liber rotarum*.

2. *Figura solida*

In his loose translation of Nicomachus of Gerasa's *Introduction to Arithmetic*, Boethius defined the subject of geometry as an immobile magnitude (*magnitudo immobilis*), which can be expressed, for instance, by means of geometric forms (*formae geometricae*)[18]. This conception of the subject of geometry was adopted by Cassiodorus in *Institutions of Divine and Secular*

[18] A. M. S. Boethius, *De arithmetica* I, 1, eds. H. Oosthout & I. Schilling, Turnhout 1999, pp. 10–13.

Learning, where he stated that this discipline investigates immobile magnitude and forms (*magnitudo immobilis et formae*)[19]. Both also specified that geometry deals, among other things, with solids, i.e., three-dimensional objects (*figurae solidae*), which are characterized by length, width, and height[20].

Isidore immediately followed this specification of geometry and its subject when he described this discipline in *The Etymologies*. He derived the name of this art in the traditional way from the surveying of land during the Nile floods in Egypt, and that therefore, its subjects are lines (*lineamenta*), intervals (*intervalla*), magnitudes (*magnitudines*), and forms (*figurae*) expressed in dimensions (*dimensiones*) and numbers (*numeri*)[21]. He then repeated almost verbatim Cassiodorus' words that geometry investigates, among others, solid figures (*figurae solidae*), which always involve length, width, and height[22]. One of the examples of three-dimensional solids is the cube (*cubus*), which Isidore denoted as a characteristic (*propria*) solid form. The cube is always determined by the triad of length, width, and height[23].

Thus, the *figura solida* was one of the types of geometric forms in the common interpretation of the time, since it represented solid objects, i.e., spatial geometric shapes occupying three dimensions. Since each of the three dimensions can be thought of as a surface, the typical surface form is a quadrilateral or a square. It is clear that space was defined by a combination of quadrilaterals or squares in the geometry.

Of the texts that dealt with this in more detail and were also available in Isidore's time, we may mention Macrobius' commentary on the last book of Cicero's *De re publica*, the so-called *Scipio's Dream*. Macrobius stated that bodies (*corpora*) are defined by surfaces (*superficies*), which constitute their extremal limits (*termini*), and although they are the limits of something corporeal, we conceive them as incorporeal (*incorporei*)[24]. This allows us to distinguish between geometric forms in and of themselves, such as surfaces based on lines (*lineae*) or lines based on points (*puncti*), which are generally referred to as mathematical solids (*corpora mathematica*) and are the subject of geometry, and material solids, which include geometric plane figures as

[19] Cassiodorus Senator, *Institutiones* II, 6, 2, ed. R. A. B. Mynors, Oxford 1937, p. 151.

[20] Cf., e.g., Boethius, *op.cit.* II, 20, p. 129; Cassiodorus, *op.cit.* II, 6, 2, p. 152.

[21] Isidorus Hispalensis episcopi, *Etymologiae sive Origines* III, 10, 1–3, ed. W. M. Lindsay, Oxford 1911.

[22] *Ibidem* III, 11, 1–2.

[23] *Ibidem* III, 12, 3.

[24] A. T. Macrobius, *Commentarius in Somnii Scipionis* I, 5, 5, ed. I. Willis, Stuttgart–Leipzig 1994, p. 15.

their boundaries[25]. Since all solids have three dimensions (length, width, and height), geometric solids (*corpora solida*) also have these three dimensions.

Macrobius mentioned the cube (*tessera*, κύβος) as an example of a spatial geometric figure. This figure can be imagined by placing two similar squares (*quadrata*) above each other, and in this way, the height is added to the length and width (which is typical for plane forms)[26]. It is these three dimensions (*dimensiones*) that properly define a spatial figure. Moreover, Macrobius offered a connection to numbers: The number one represents a point (*punctus*); the number two expresses a line (*linea*) – since it is defined by two points; the number four characterizes a square (*quadrum*) – since two equal lines correspond to four points, which are appropriately placed to form the vertices of the four right angles of the quadrilateral; the number eight is then the number of the cube (*cybus*) – which is formed by doubling the square when one is placed above the other, thus forming a height, resulting in a cubic figure meaning a solid with eight vertices[27].

It is important to the topic of this paper that Macrobius (Isidore most likely knew his work) defined a solid geometric figure as two similar squares placed one above the other, and it is this placement that incorporates the third dimension of solids, thus expressing the whole triad of length, width, and height. Indeed, similar expressions can also be found in Calcidius[28] or Martianus Capella[29], and of course the association with Plato's dialogue *Timaeus* is evident too[30].

3. *Ratio geometrica*

Nicomachus of Gerasa, in his *Introduction to Arithmetic*, distinguished between ratio (λόγος) and proportion (ἀναλογία). While the former is a relationship of two items or terms to each other, the latter is a relationship of ratios, so that a minimum of three items is needed to produce a proportion. An example of a ratio would be, for instance, a double (1 : 2 ratio) that is between two numbers 1 and 2, or between two numbers 2 and 4, etc.

[25] *Ibidem* I, 5, 7, pp. 15–16.

[26] *Ibidem* I, 5, 9, p. 16.

[27] *Ibidem* I, 5, 10–11, pp. 16–17. Macrobius also described it very similarly in *ibidem* II, 2, 3–11, pp. 100–101.

[28] Calcidius, *On Plato's Timaeus* II, 32–33, ed. & transl. J. Magee, Cambridge, Mass.–London, UK 2016, pp. 168–72.

[29] Martianus Capella, *De nuptiis Philologiae et Mercurii* VI, 707–710 or VI, 721–722, ed. I. Willis, Leipzig 1983, pp. 251–52 or pp. 257–58.

[30] See, for instance, Plato, *Timaeus* 32B, [in:] idem, *Platonis opera*, vol. IV, ed. J. Burnet, Oxford 1902.

However, if we have three or more numbers (e.g., 1, 2, 4, 8, etc.), then it is a proportion where two numbers are always linked to each other by the same ratio[31].

Boethius used the terms *proportio* (ratio) and *proportionalitas* (proportion) in his loose translation of Nicomachus' treatise[32], whereupon he discussed the different types of proportions in accordance with the Greek text being translated. He referred to geometric proportion (*proportionalitas geometrica*) in the proper sense of the word 'proportion', since all items in it are arranged according to the same ratio (*proportio aequa*), thus ensuring that all kinds of inequality (*omnes species inaequalitatis*) arise from equality (*aequalitas*)[33].

Hence the distinction that differentiates geometric proportions from other proportions. It is only in the case of geometric proportion that the product of their extreme values is equal to either the square of the mean of the proportion (in the case of three items in the so-called continuous proportion[34]) or the product of the two means (in the case of a discrete proportion[35], but also in the case of a four-item continuous proportion). So, if we have a continuous proportion, e.g., doubles of 6, 12, and 24 (where the first is to the second as the second is to the third), then $6 \times 24 = 12 \times 12 = 144$. If we have a discrete proportion, e.g., given by the ratio 2 : 3, i.e., the numbers 6,

[31] Nicomachus Gerasenis, *Introductio arithmeticae* II, 21, 2–4, ed. R. Hoche, Leipzig 1866, p. 120.

[32] Boethius, *op.cit.* II, 40, pp. 172–73.

[33] *Ibidem* II, 44, pp. 184–90.

[34] A continuous proposition (*proportionalitas continua*), according to Boethius, is one where there is a series of numbers and each number that has other numbers on either side is in the same ratio to these numbers, the first called the leader (*dux*), the second the follower (*comes*). Examples of such a ratio are the numbers 2, 4, 8, 16, etc.; four is twice the number two and half of the number eight; eight is twice the number four and half of the number sixteen, etc. In this case, all numbers are continuously connected by the same ratio. See, *ibidem* II, 40, p. 173. Calcidius described the continuous proportion (*continuum competens*) as composed of at least three terms, where: as the first is to the second so the second is to the third – cf. Calcidius, *op.cit.* I, 16, p. 144.

[35] A discrete proportion (*proportionalitas disiuncta*) is specified by Boethius as a proportion that has at least four items, where two items are always related according to the given ratio and the other two items are related according to the same ratio. For example, the numbers 2, 4, 8, and 16 represent a fourfold ratio: the latter is between the numbers 2 and 8, and at the same time between the numbers 4 and 16. The same is the case when these numbers represent a double ratio, since the numbers 2 and 4 have the same ratio to each other as the numbers 8 and 16 (cf., Boethius, *op.cit.* II, 40, p. 174). In Calcidius' words, in a discrete proportion (*distans competens*): as the first is to the second, so the third is to the fourth, e.g., the numbers 3, 4, 6, 8, where the numbers 3 and 4 are related by the ratio 3 : 4 and the same ratio is between the numbers 6 and 8 – see, Calcidius, *op.cit.* I, 16, p. 144.

8, 9, and 12 (the first is to the third as the second is to the fourth), then $6 \times 12 = 8 \times 9 = 72$[36].

Isidore also mentioned this fact when he explained the difference between arithmetic and geometry (and also music) in *The Etymologies*. Isidore identified the difference between these *artes* via only the different ways of finding the means of different types of proportions. For a geometric proportion, the product of the extremes (*extrema*) is equal to the product of the means (*medians*). The example he used was the same as that stated by Boethius, i.e., the sequence of numbers 6, 8, 9, and 12[37]. Martianus Capella also presented the same example in *On the Marriage of Philology and Mercury*. He, moreover, used the same words to describe the geometric ratio that emerged in Isidore's diagram *figura solida*, i.e., *ratio geometrica*[38].

However, Isidore's main source of information was probably Calcidius, who also dealt with this issue in detail. Commenting on Plato's dialogue *Timaeus*, Calcidius first emphasised the perfection of the material world (*mundi corpus perfectum*), whose perfection is provided by the perfect solids composed of length, breadth, and height[39]. If we are looking for a transition between two different but similar solids, it is necessary to find two mediators (*medietates*) between them. As an example, he gives the transition from a cuboid with side lengths 2, 3, and 4 to a cuboid with side lengths 4, 6, and 8. Thus, the transition from a cuboid with a volume of 24 to a cuboid with a volume of 192 is made using two mediators, which are cuboids with a volume of 48 and 96[40]. Thus, an analogous proportion (*competens*) based on geometric ratios (*rationes geometricae*) is postulated between two distinct solids[41].

At this point in his commentary, Calcidius made a terminological and classificatory note in which he distinguished between ratio (*ratio*), which is the correlation between two extreme items (*fines*), and proportion (*competens*), which the Greeks called *analogia*, which is characterised by the comparison (*comparatione*) of several of these ratios (*rationes compluria*). As already mentioned (Boethius made a similar distinction), proportions can be either continuous (*continuum competens*) or discrete (*distans competens*)[42]. Calcidius added that God (*deus*) used ratios of continuous proportions

[36] Boethius, *op.cit.* II, 44, pp. 188–89.
[37] Isidore, *Etymologiae* III, 8, 2.
[38] Martianus Capella, *op.cit.* VII, 737, p. 266.
[39] Calcidius, *op.cit.* I, 8, p. 133.
[40] *Ibidem* I, 13–15, pp. 141–143.
[41] *Ibidem* I, 16, p. 143.
[42] *Ibidem* I, 16, pp. 143–145.

during the creation of the world, since these ratios are by their very nature uniting different items.

For this reason, in creating the sensible world (*mundus sensibilis*), God placed two mediators in the form of water and air between the disparate and extreme items (*limites extimi*), i.e., earth and fire[43]. To this Calcidius provided an example of how two parallelepipeds that do not share any sides are connected by two other (middle) parallelepipeds when the ratio of a continuous proportion (*iuxta rationem continui competentis*), i.e., an analogical bond (*analogia*, *nexus*) is used[44]. It is in this way that the elements of the material world are connected to each other, as evidenced by their six basic natural properties: as sharpness is to bluntness (*acumen* / *obtunsitas*), so thinness is to thickness (*subtilitas* / *corpulentia*), and so mobility is to immobility (*mobilitas* / *immobilitas*), or in other words, as fire is to air, so air is to water, and so water is to earth[45].

It is clear then, that Isidore – when he described his diagram of the *figura solida* as arranged according to geometric ratio – most likely had in mind the analogous bond which is a continuous proportion that is the mathematical basis for binding the distinct elements of earth and fire by the mediators of water and air. The three dimensions of the solids and the three natural characteristics of each element are an obvious indication of the use of the proportion that postulates two mediators between the two extreme items and thus harmoniously joins them.

4. *Figura solida secundum geometricam rationem*

As a partial summary of the previous two subparts, it can be said that the *figura solida* for Isidore most likely meant that it has three dimensions (length, width, and height). A typical instance of a solid figure is the cube, which can be imagined or represented by two superimposed squares (each of which thus includes length and width), and the area between them expresses the third dimension, i.e., height. However, in the case of solids, if it is necessary to transition from one extreme item to another that is different from the first, then there is an optimal solution. This solution is in accordance with how God created the material world using geometric ratios, which are the basis for a continuous proportion that connects the different extreme items by harmonious analogy. Isidore thus had in the aforementioned texts a fairly clear manual on how to understand the elements of the material world, their

[43] *Ibidem* I, 17, p. 145.
[44] *Ibidem* I, 18–19, pp. 145–149.
[45] *Ibidem* I, 21–22, pp. 151–153.

natural physical qualities, and the correlations and transitions between them. It can therefore be assumed that either Isidore himself, or those who copied his work and whose drawings are preserved in manuscripts, used this knowledge and applied it to the diagram of the *figura solida*.

Last but not least, it should not be forgotten that the diagram *figura solida* is contained in the treatise *De natura rerum*, which also contains a set of other diagrams that are mostly circular in their character. This is also referred to in the 11th chapter of the so-called *Liber rotarum*, in which the properties of the elements are discussed, since in parts XI, 2–3[46], the Aristotelian doctrine of the qualities of the elements is presented, and this theory is accompanied by a very well-known *rota* diagram *mundus–annus–homo*[47]. It can therefore be hypothesised that the *figura solida* may have been similarly intended as a specific form of a *rota* diagram.

A short presentation of Isidore's theory of the elements of matter from the thirteenth book of *The Etymologies* might serve as a possible justification of the hypothesis. Here the Bishop of Seville briefly mentioned the initial formation of the originally formless matter (ὕλη, *materia*, *silva*) by means of the elements (στοιχεία, *elementa*), in order to explain the origin of the Greek name of this term. The elements are so called because they come together (*convenire*) on the basis of harmonious concord and intercommunity (*communio*). All the elements are intertwined by natural order (*ratio naturalis*), in which they may derive their origin (*origo*) from fire and proceed gradually down to earth; at other times they may originate at earth and terminate at fire. For it is evident that fire ends in air, air is condensed into water, and water is transformed into earth by thickening; conversely, earth loosens into water, water is rarefied into air, and by further thinning of air fire is formed[48].

These words of Isidore clearly show that elements can always pass between each other from both extremes when a part of their properties is changed. Moreover, Isidore further added that elements are actually distinguished from each other and named differently according to what they have more of (*amplius*) in them. He did not fail to point out that Divine Providence (*divina providentia*) also provided typical animals for the elements – e.g., fish are typical for water and birds for air[49], which are frequently used as

[46] Isidore, *De natura rerum* XI, 2–3, pp. 215–217.

[47] See, e.g., S. Cohen, *Transformations of Time and Temporality in Medieval and Renaissance Art*, Leiden & Boston 2014, pp. 68–69; or N. Germann, *De Temporum Ratione: Quadrivium und Gotteserkenntnis am Beispiel Abbos von Fleury und Hermanns von Reichenau*, Leiden & Boston 2006, pp. 123–125.

[48] Isidore, *Etymologiae* XIII, 3, 1–2.

[49] *Ibidem* XIII, 3, 3.

illustrations for the characteristics of the elements in the *figura solida* in *De natura rerum* XI, 1, as preserved for us in the above-mentioned medieval manuscripts.

As mentioned, the extant forms of the *figura solida* considerably differ in the manuscripts. Therefore, attention will now be focused exclusively on the two variants whose forms have been presented in Figs. 1 and 3c. The reasons are obvious: while Murdoch's *cybus elementorum* (see Fig. 2 above) seems unlikely to be the originally intended form of the *figura solida* in regard to both the text of *De natura rerum* XI, 1 and the diagram in the extant manuscripts, the various depictions in Figs. 3a–b are substantially complementary to Fig. 1. Other forms of *figura solida* that appear in medieval manuscripts (e.g., circle[50] or table[51], four squares[52], etc.) do not – unlike Fig. 1 and Fig. 3c – seem to be able to satisfy the requirements for *figura solida* as they have been presented.

4.1 One square

First, then, to that form of the *figura solida*, which is visually close to a cube and represents two (or three) squares, but the elements and their properties are placed only in the back square (see Fig. 1). Part of the diagram, of course, is the information that it is a solid figure that is arranged according to a geometric ratio.

Although we do not know whether this may have been a certain prototypical form of the scheme, it has been hypothesised by B. S. Eastwood, who has also suggested the key to its interpretation[53]. Here, then, let it be added that solids such as cubes or cuboids can be thought of as two quadrilaterals. If the elements are inserted into one of these quadrilaterals when it is divided by diagonals into four triangles, this naturally indicates a reference to the analogy between plane figures.

Calcidius also dealt with this topic in detail in his commentary on Plato's *Timaeus*. One case is finding a mediator in relation to the harmonic

[50] Cf., e.g., Einsiedeln, Stiftsbibliothek, Codex 360(177), f. 38v (https://www.e-codices.unifr.ch/de/sbe/0360/38v); or, in a partly different form, Oxford, Bodleian Library, MS. Auct. F. 2. 20, f. 5v (https://digital.bodleian.ox.ac.uk/objects/86e72632-da7e-4ba2-861a-473ae729d3ca/surfaces/c77d3178-9682-4825-bb81-0ac1733a5bf7/).

[51] See the aforementioned manuscript Cambrai, Bibliothèque municipale, ms. 937, f. 45v (https://bvmm.irht.cnrs.fr/iiif/10506/canvas/canvas-1297244/view) – top left image.

[52] Cf. St. Gallen, Stiftsbibliothek, Cod. Sang. 238, 337 (https://www.e-codices.unifr.ch/en/csg/0238/337).

[53] B. S. Eastwood, *The diagram of the four elements*, pp. 548–51.

geometric ratio between two similar triangles (*triangulares similia*)[54]. In this particular instance, it is the isosceles triangles ΑΒΓ and ΔΒΕ, which have a common vertex Β, where the sides ΑΒ and ΒΓ, or ΔΒ and ΒΕ, have equal angles (*aequi anguli*). It is also true that the length of side ΑΒ is related to side ΒΕ in the same way as sides ΓΒ and ΒΔ are related to each other. If side ΑΒ is then extended by side ΒΕ and the vertices Α and Δ are connected, the analogous triangles ΒΔΕ and ΔΒΑ are formed. Similarly, by extending the side ΓΒ by the side ΒΔ, the analogous triangles ΔΒΑ and ΑΒΓ are obtained. It follows that the mediator between the triangles ΑΒΓ and ΔΒΕ is the triangle ΔΒΑ, since (ΑΒΓ : ΔΒΑ) = (ΔΒΑ : ΔΒΕ) – see Fig. 4.

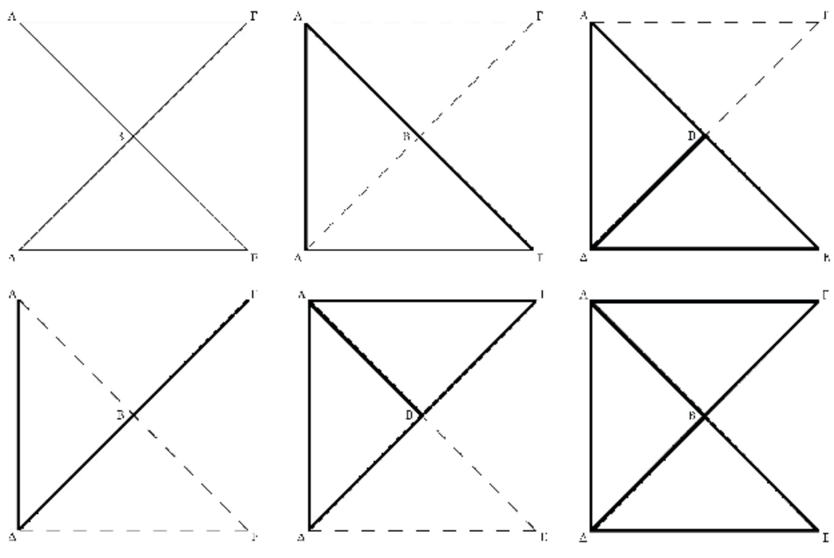

Fig. 4. Finding the mediator using the geometric ratio in the plane for two triangles according to Calcidius.

It is probably needless to say, that if we were to solve the same exercise by constructing the side ΓΕ and the triangle ΕΒΓ, it would be the mediator, since (ΑΒΓ : ΕΒΓ) = (ΕΒΓ : ΔΒΕ). This fact was known in the Middle Ages, as evidenced by the extant manuscript[55], where this triangular analogy is presented in a visual representation and both mediation possibilities are marked, i.e., the triangles ΔΒΑ and ΕΒΓ – see Fig. 5. This effectively turns

[54] Calcidius, *op.cit.* I, 12, p. 138.

[55] See Paris, Bibliothèque nationale de France, ms. Lat. 2164, f. 28r (https://gallica.bnf.fr/ark:/12148/btv1b10073861t/f32.item).

the whole drawing into a square with four isosceles triangles, two of which are mediators between the other two.

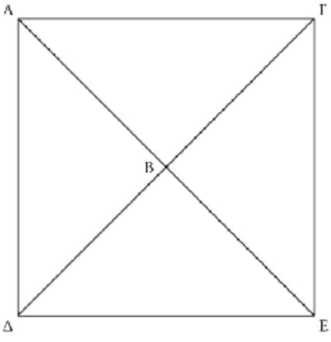

Fig. 5. Two possible mediators using the geometric ratio in the plane for a triangles according to Calcidius as depicted in the manuscript Paris, Bibliothèque nationale de France, ms. Lat. 2164, f. 28r.

This point, together with the fact that the three sides of each triangle correspond appropriately to the three natural properties of each element, could serve as a basis for a relevant interpretation of this form of the *figura solida*. Since each element always has two properties in common with the two neighbouring elements and one property in which they differ, a natural cycle of these qualities is created. Fire shares the same legs with air (sharpness) and water (mobility), but its hypotenuse is unique (thinness). Air touches fire (sharpness) and earth (thickness), but it is also mobile. Earth shares with air a thickness and with water a bluntness (i.e., both legs), and as a single element it is unmovable (the hypotenuse). Whereas water unites with earth (bluntness) and fire (mobility) and has thickness as its hypotenuse (see Fig. 6).

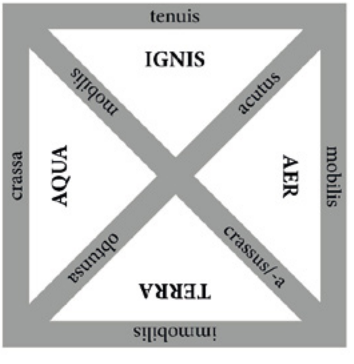

Fig. 6. *Figura solida* as a circular diagram with two mediators between fire and earth.

If the solid figure is interpreted in this way, then not only will we obtain a circular form of the diagram, but it will also show us why two mediators are required for three-dimensional solids. The basic shape of solids is a square, and two mediators between two extreme triangles naturally form a square.

4.2 Two squares

The variant of the *figura solida*, which represents two squares, each of which is divided by a diagonal into two triangles, is very close to the plane analogy according to the art of geometry, which Calcidius dealt with. He also showed how a mediator can be found between two similar parallelograms (*similia parallelogramma*) that have one common vertex. Calcidius used the parallelograms ΑΒΓΔ and ΓΗΘΖ as an example. These are defined by the diagonals ΑΓ and ΓΘ, as well as the equal angles (*angules aequales*) ΒΓΔ and ΗΓΖ. This means that the two quadrilaterals' sides are of equal length. To find the middle parallelogram, Calcidius used the extended sides of the original parallelograms and the construction of the point Ε, as well as the segments ΔΕ and ΖΕ, thus obtaining the parallelograms ΑΒΖΕ and ΔΗΘΕ. He then found a harmony in the lengths of the segments connecting the two vertices of the parallelogram with the centre of the opposite side of the

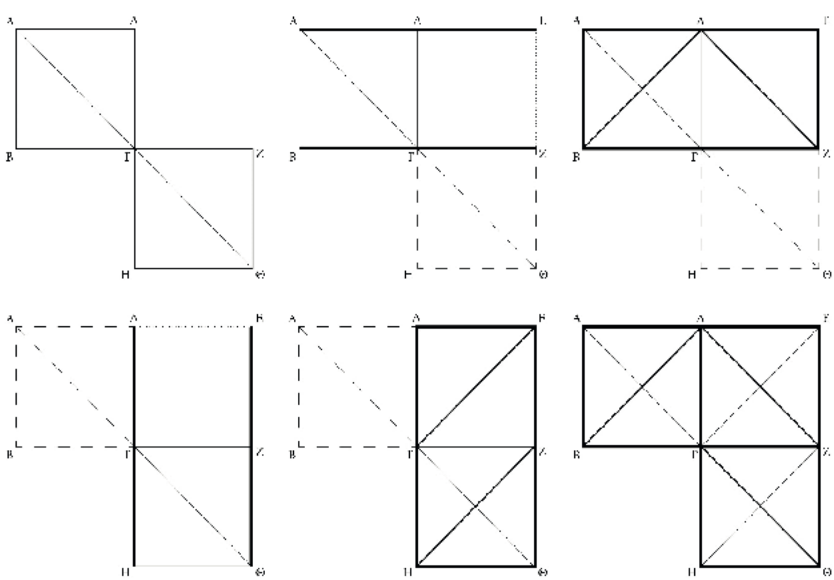

Fig. 7. Finding the mediator using the geometric ratio in the plane for two parallelograms according to Calcidius.

quadrilateral, i.e., BΔ and ΔZ, respectively, ΓE and ZH, i.e., the diagonals of the original parallelogram and the newly found quadrilateral. As a result of this harmony, it is feasible to determine that the parallelogram ΔΓZE is the mediator of the parallelograms ABΓΔ and ΓHΘZ (Calcidius referred to them as AΓ, ΓΘ, and ΔZ by their diagonals). This is because it is true that (ABΓΔ : ΔΓZE) = (ΔΓZE : ΓHΘZ) – see Fig. 7. It would also be true that a similar procedure can be followed on the opposite side and a mediating parallelogram can be constructed in the lower right part of the diagram.

This example of analogy corresponds quite appropriately to the preserved diagram of the *figura solida* as two squares. The diagonals divide the two quadrilaterals into four triangles, which is the number corresponding to the number of elements. According to the geometric analogy of triangles in one square (see subpart 4.1.), it could be ascertained that the connection of the three properties of the elements relates to the sides of each triangle. In the case of two squares, a solution using vertices is plausible. This is because each triangle always shares two vertices with the other, whereas one vertex is not in common with the others, which is consistent with the three-fold nature of the element's properties. The suggestion of a blending of the two squares would correspond surprisingly well with this, as it would suggest an identity of vertices (and therefore a common bonding of properties) between air and water – see Fig. 8a–b.

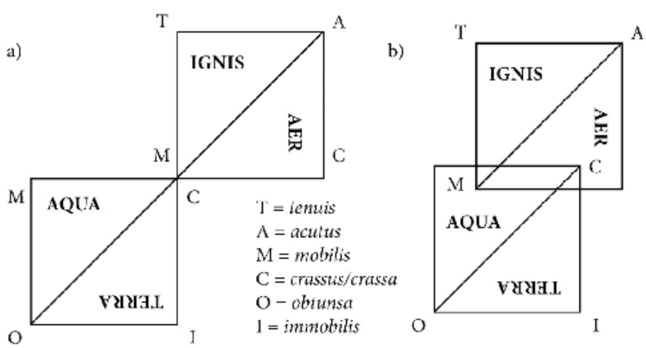

Fig. 8. *Figura solida* as two (possibly intersecting) squares, where the vertices represent the properties of the elements.

The harmonic connection of the opposite properties is also quite clear in this diagram: Since the opposite properties of thinness and thickness, or mobility and immobility, are located in the opposite vertices of the quadrilaterals; the remaining two vertices of the quadrilaterals, which in both cases also connect the two triangles of the quadrilateral by a common hypotenuse, are in the role of mediators, i.e., sharpness and mobility for density thin and thick, and thickness and bluntness for the presence or absence of motion. Similarly, between sharpness and bluntness, which are on opposite sides of the diagram, there are clearly visible mediators in the forms of mobility and thickness.

Even in Calcidius' finding of the medium between two parallelograms, the mediating character of the diagonals of the original quadrilaterals can be seen. Moreover, it can easily be used to explain this form of the *figura solida* as a circular scheme. If the original quadrilateral TMCA (in the notation of Fig. 8) is enlarged into a rectangle according to Calcidius' instructions by adding the same quadrilateral T'M'C'A', then the segments drawn from the vertex of the bottom side of the rectangle to the centre of the opposite side, i.e., the segments TC and M'A', will be analogously congruent (see Fig. 9a). The same procedure can be followed for the lower quadrilateral MOIC, to which is added the identical quadrilateral M'O'I'C', to which it is convenient, again in accordance with Calcidius' procedure, to use axial symmetry according to the side MC, thus producing the analogous segments (diagonals of the original and the expanding quadrilateral) MI and O'C' – see Fig. 9b. However, if we enlarge the lower quadrilateral in the same way as the upper quadrilateral, only to the other side, i.e. to the bottom right part of the diagram (see Fig. 9c), then the analogous connection of opposites closes into a circular form – see Fig. 10.

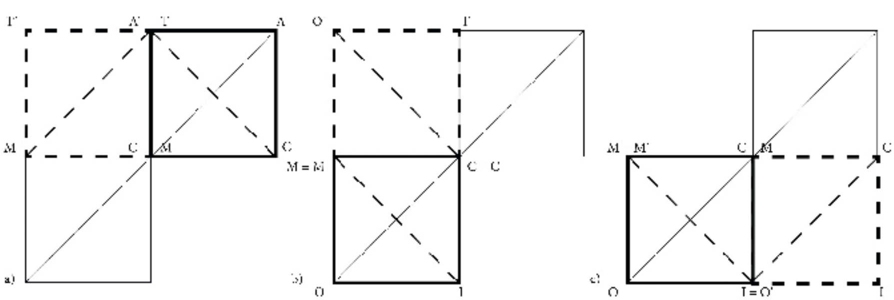

Fig. 9. Mediators between opposing properties of elements according to the Calcidus plane analogy of parallelograms.

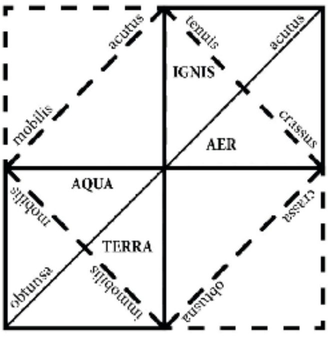

Fig. 10. *Figura solida* as a circular diagram with mediators between the opposite properties of elements.

5. Conclusion

The *figura solida*, a diagram illustrating the brief statement of *De natura rerum* XI, 1 of Isidore of Seville, is sometimes described as the oddest diagram[56]. Although we can only guess what the original form of the diagram was (as intended by Isidore) and what exactly its purpose was, it seems very likely that it is intrinsically linked to Calcidius' *Commentary on Plato's Timaeus* and the analogical bonds, the geometric ratios, used to find the mediator between extremes, presented there in detail. This is the basis for the main thesis of this paper, which seeks to show that the various preserved forms of the *figura solida* may be the result of the use of different examples of analogy in Calcidius' *Commentary.*

However, it should be added that in Isidore's time, a geometric three-dimensional figure was often interpreted as a composition of plane figures. A cube, for instance, is described as two superimposed squares separated by the same distance as the side length of these squares. If the cube is then reduced to a set of squares, it is easy to understand that the elements are represented as part of the square (or squares) in the diagram *figura solida*. However, this can lead to considerable confusion in the interpretation of the topic: the analogical bond for solid figures requires two mediators, whereas for plane figures only one.

This disproportion may be one of the main reasons for the confusion surrounding the preserved forms of the *figura solida*. If the elements are placed in one square, which is divided by its diagonals into four right-angled

[56] See, for example, A. Geritz, *Diagrams in Isidore of Seville's De natura rerum*, [in:] "Monks and Monkeys and other medieval & early modern curiosities", https://medievalmonkeys.wordpress.com/2020/05/, 28. 3. 2024.

triangles, then the sides form a natural cycle of elements according to their common and unique properties. At the same time, the mediating character of the elements of water and air between the opposing elements of earth and fire is respected. Using Calcidius' analogy between triangles (including the second possibility of a mediating triangle, which Calcidius did not mention), the *figura solida* can thus illustrate the circular character of the natural order of the material world.

Similarly, the *figura solida* as two (sometimes overlapping) squares divided by one diagonal into four triangles that represent the elements can be seen. These four triangles always have one vertex not in common and two vertices in common with the neighbouring triangle, which can clearly refer to two common and one distinct natural property of the neighbouring elements. Using Calcidius' analogy between parallelograms, this scheme can also be interpreted as a harmonic linkage of elements, showing the analogy of transitions between their opposite properties. If both possibilities are constructed for these transitions (Calcidius again presented only one), then the circular form of the transitions and the analogous bonds between the opposite properties of the elements will also be shown.

Since the *figura solida* is contained in Isidore's treatise, which is characterized by its circular diagrams, these interpretations offer reasons for the relevance of the hypothesis that Isidore or the copyists of his work were interested in showing the circular character of this diagram as well. The main complication, however, is that three-dimensional figures have two mediators, whereas plane figures have only one. For this reason, in the case of four triangles in one square as well as in the case of four triangles in two squares, it is important to postulate both alternative possibilities of mediators in the plane, which will allow the diagram *figura solida* to be properly interpreted as a *rota* diagram.

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